

## Classical Mechanics, Autumn 2019

Assignment #2, Due 24/09/2019

1. Consider a double pendulum constructed with equal masses  $m$  and massless rods of equal lengths  $\ell$ . Take any appropriate values of  $m$  and  $\ell$  in arbitrary units. Set up and solve the Hamilton's equations of motions (numerically is OK) with initial conditions of your choice. Plot the trajectories of the two masses in  $(\theta_i, p_{\theta_i})$  space. (Separate diagrams for the two masses). Comment on the features of these solutions. Can you choose initial conditions such that the trajectories look simple ?
2. Two points  $(\theta_1, \phi_1)$  and  $(\theta_2, \phi_2)$  on the surface of a sphere are to be connected by the curve  $\theta(\phi)$  of minimum length along the sphere.
  - (a) Using the variational principle, show that this curve is a great circle, and find its length.
  - (b) An ant at  $(\theta_1, \phi_1)$  wants to start to walk along this shortest curve. Find the direction it should start walking in, in terms of the basis vectors  $\hat{\theta}_1, \hat{\phi}_1$  and the parameters  $\theta_1, \phi_1, \theta_2, \phi_2$ .
3. Consider the motion of two wheels (each with mass  $m$ , radius  $b$ ) connected with a massless axle of length  $a$ , rolling without slipping on an inclined plane with slope  $\alpha$ . Employ the same coordinates defined in the class:  $x$  horizontal along the inclined plane,  $y$  along the slope of the plane,  $z$  normal to the plane,  $\phi_1, \phi_2$  the angles of rotation of the wheel,  $\theta$  the angle made by the wheel with the  $x$  axis. [Some sign conventions are arbitrary here, feel free to use your own.]
  - (a) Write down the holonomic and non-holonomic equations of constraint.
  - (b) Write down the equations of motion using the method of Lagrange multipliers.
  - (c) Solve the EoMs numerically, for any non-trivial initial conditions of your choice, plot  $x, y, \theta$  as functions of time.
  - (d) Use the substitution  $\dot{\hat{s}} \equiv \dot{x} \hat{x} + \dot{y} \hat{y}$  and get an analytic solution of the problem. Explain the features of your numerical solution using this.

4. Consider a relativistic particle of rest mass  $m$  under the influence of the electromagnetic 4-potential  $A_k = (\phi/c, -\vec{A})$ . The “action” to be minimized can only be a linear combination of the two Lorentz-invariant quantities  $ds = \sqrt{(dt)^2 - (dx)^2 - (dy)^2 - (dz)^2}$  and  $A_k dx^k$ , where  $dx^k = (dt, dx, dy, dz)$ . Take the action to be

$$S = \int_1^2 (\alpha ds + \eta A_k dx^k)$$

- (a) Find  $L$  in terms of the  $\alpha, \eta, \phi, \vec{A}$ , and the velocity  $\vec{v}$  of the particle.
  - (b) Find the value of  $\alpha$  using the condition that, in the absence of any electromagnetic field,  $L$  should “effectively” reduce to the corresponding non-relativistic Lagrangian.
  - (c) Find the relativistic generalized momenta corresponding to the coordinates  $(t, x, y, z)$ , and write down the corresponding equations of motions using Euler-Lagrange equation.
  - (d) Matching the EoMs to the Lorentz force law, determine  $\eta$ .
  - (e) Using the values of  $\alpha$  and  $\eta$  obtained above, calculate the Hamiltonian, and write down Hamilton’s equations of motion.
5. Consider a Lagrangian  $L(q_1, \dot{q}_1, q_2, \dot{q}_2, t)$ . Perform the Legendre transform to the Routhian  $R(q_1, p_1, q_2, \dot{q}_2, t)$ .
- (a) Find equations of motion in terms of this Routhian.
  - (b) If  $R$  does not have an explicit time dependence, find the conserved quantity corresponding to energy in terms of  $R$ .

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