Classical Mechanics, Autumn 2019

Assignment #3, Due 08/10/2019

1. For the attractive potential $V = ar^{n+1}$, write the differential equation for θ as a function of $u \equiv 1/r$. Hence, find the analytical form of the orbit $r(\theta)$ in polar coordinates, for $n = +1, -1, -2, -3$. For all these cases, plot examples of possible orbits in the polar coordinates (r, θ) by choosing suitable sets of parameter values that will bring out interesting features.

2. For the Kepler potential, plot the trajectories corresponding to the circular, elliptical, parabolical and hyperbolical orbits (suitable choice of parameters)

   - in the 3D space $(r, \dot{r}, p_r)$ (use Mathematica 3D plot feature) and its projections in the $(r, \dot{r})$ and (r, p_r) planes
   - in the 3D space $(r - \theta - p_r)$ [which is the projection of the 4-d phase space $(r - \theta - p_r - p_\theta)$ at constant $p_\theta = \ell$].

3. A particle of mass m moves under the influence of a central force

$$\vec{F} = -\frac{k}{r^2}\left(1 - \frac{\alpha}{r}\right)\hat{r} \ .$$

   Show that this particle will move in a precessing ellipse. Analytically find the precession angle per revolution, in terms of α (which you may take to be small as compared to the axes of the ellipse). Plot $r(t)$, $\theta(t)$ and the motion in the (r, θ) plane, to bring out the "precession of perihelion" features.

4. Calculate and plot
   (i) the scattering angle $\Theta(s)$ for specific energy values, and
   (ii) the scattering cross section $d\sigma/d\cos\Theta$, for the same energy values, with the folllowing potentials:

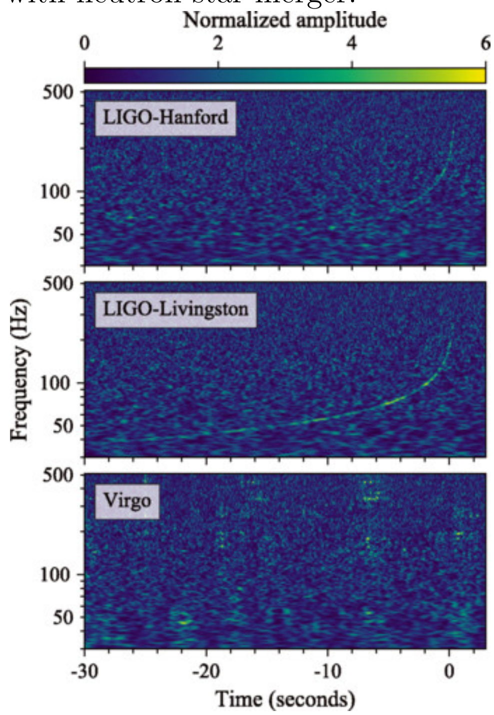
   (a) Repulsive Coulomb potential

   (b) Hard sphere potential

   (c) A combined attractive and repulsive potential, as in Goldstein Fig. 3.17 (edition 2).

   Use "interesting" parameter values that may bring out important features, and comment on them. In the last case, try to tune your parameter values to get the "large negative Θ" behaviour.

5. Data analysis with neutron star merger:



The figure shows the data from gravitational wave detectors, in the form of the frequency of gravitational waves as a function of time. Let us assume for the sake of simplicity that these are two identical neutron stars of mass 1 $M_\odot$ each, they move in almost circular orbits, and non-relativistic approximations (Kepler's laws) are valid. The frequency of gravitational waves is twice the period of rotation of the two stars around each other. The highest frequency corresponds to the epoch of merger.

From the data (use only LIGO-Livingston), calculate (as closely as you can) and plot:

(a) The distance between the neutron stars as a function of time.

(b) The rate of loss of energy of the system as a function of time

Estimate the radii of the stars.

$\infty \quad \infty \quad \infty$