Classical Mechanics, Autumn 2019

Assignment #5, Due 20/11/2019

1. Consider the Foucault pendulum oscillating at a latitude θ_l.

   (a) Write down the *exact* equation of motion in the Earth's reference frame, for the coordinates x, y, z, where z is the local vertical.

   (b) Choose two initial conditions: one C1 where the pendulum will pass through $x = y = 0$, and another C2 where the pendulum will nor pass through this point. Solve the equations of motion numerically, and plot the motion in the $[x - y]$ plane by choosing appropriate values of parameters, for both these cases.

   (c) In the approximation of small angular velocity of the Earth (specify small with respect to what), and small oscillations, calculate the motion in the $[x - y]$ plane analytically for the condition C1. Show that its features correspond to the expected behaviour of the Foucault pendulum.

2. Consider a cone of radius a and height h.

   (a) Find the principle axes, and the principle moments of inertia, with the vertex as the fixed point of rotation.

   (b) The vertex of the cone is kept fixed, its axis is held horizontal, and it is given an angular momentum $\vec{L}$ about its axis. The cone is then released, with its vertex still fixed. Calculate the frequency of gyroscopic precession for "sufficiently" large $|\vec{L}|$. Define "sufficiently large".

   (c) Plot the direction of the axis of the cone in the θ–ϕ plane, for appropriately chosen values of parameters. Show at least two plots, one for "large" $|\vec{L}|$ and one for "small" $|\vec{L}|$. Describe and interpret the plots.

3. Goldstein Edition 2, problem 27 (charged symmetric top)

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