Electrodynamics II : Autumn 2011

Assignment 1

Given: Saturday Sep 3, Expected: Wednesday Sep 21

This assignment has two sections. You need to submit answers to only the questions from Section I. The questions from Section II are strongly recommended for practice / understanding of concepts, however you need not submit them and will not be graded on them.

Section I

1. Inside a conducting medium, an EM wave will propagate as well as decay. Let the form of the plane wave solution be

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_0 e^{-\kappa x} e^{ikx} e^{-i\omega t} \; .$$

   (i) Find $c\kappa/\omega$ and ck/ω as a functions of $\omega\tau$, where τ is the relaxation time. Show your results in the form of a plot that brings out all the relevant features. Comment on the plot.

   (ii) For an EM wave reflecting normally from the surface of a conductor with finite σ, calculate the surface current $\vec{\mathbf{K}}$ and the time averaged value of the Poynting vector $\vec{\mathbf{N}}$ into the surface. Hence determine the "surface resistance" R_s.

2. The polarization of a light beam is defined as

$$P = \frac{I_1 - I_2}{I_1 + I_2}$$

   where I_1 and I_2 are the intensities of the two orthogonal polarizations. If an unpolarized beam of light is incident on a dielectric with refractive index n and permeability μ_0, calculate the polarization of the reflected beam of light as a function of the angle of incidence θ_I. Plot this dependence for $n = 1.5$. You may use the results on reflection and transmission coefficients calculated in class and in Section II of this Assignment.

3. Consider a transmission line consisting of two long co-axial cylinders of radii a and b with empty space in between ($a < b$). An AC voltage $V = V_0 e^{-i\omega t}$ is applied between the cables at one end.

   (i) Calculate the electric field $\vec{E}$, the magnetic field $\vec{B}$, and the current I flowing through the transmission line in the form of a TEM wave, as functions of time. What is the average power P transmitted ?

   (ii) For $a = 1$ cm and $b = 2$ cm, determine the capacitance, inductance and conductance of the transmission line for the transmission of TEM modes in SI units. (Some of these quantities may have to be defined per unit length, specify these clearly.)

   (iii) What are the smallest frequencies at which the TE and TM modes will be transmitted ?

4. A pulse of plane electromagnetic wave, whose time profile is a square-wave pulse of plane width $\Delta t = \tau$, is incident on a waveguide with a square cross section, each side of which is a. ($a \ll c\tau$). Determine the shape of the pulse exiting the waveguide after travelling a distance $d \gg c\tau$ through the waveguide. Plot the output pulse by taking appropriate numerical values for the parameters that will bring out the relevant features. Interpret these features in terms of propagation of different frequencies.

5. Current is slowly turned on in an infinite straight wire, such that

$$I(t) = \begin{cases} 0 & (t < 0) \\ I_0(t/\tau) & (0 \le t < \tau) \\ I_0 & (t \ge \tau) \end{cases}$$

   Calculate the resulting electric and magnetic fields as functions of time t and perpendicular distance d from the wire. (Note: the exact $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are needed, not just their radiative components.)

6. A linear antenna of length L (at $-L/2 < x < L/2$) is fed a current

$$I = I_0 \frac{\sin[k(L/2 - |x|)]}{\sin(kL/2)} \ .$$

   Plot the radiation pattern (3-d plot as well as projections in yz and xy planes) for (i) $L = 0.01\lambda$, (ii) $L = 0.5\lambda$, (iii) $L = \lambda$, (iv) $L = 10\lambda$.

Section II

1. Let a plane EM wave with frequency ω travelling in free space be incident on the surface of a dielectric with refractive index n and permeability μ_0, at an angle of incidence θ_I. Let the electric field of the wave be normal to the plane of incidence, i.e. parallel to the surface of the dielectric. In terms of the parameter $\alpha = \cos\theta_T / \cos\theta_I$ and $\beta = n$, calculate the reflection coefficient R and the transmission coefficient T. Plot these quantities as functions of θ_I. Is there a "Brewster's angle" in this case ?

2. Consider the waves propagating in a cylindrical waveguide, with the form
$$\vec{\mathbf{E}} = \vec{\mathbf{E}}(x,y)e^{i(k_z z - \omega t)} \; , \vec{\mathbf{B}} = \vec{\mathbf{B}}(x,y)e^{i(k_z z - \omega t)} \; .$$
Using Maxwell's equations, calculate E_x, E_y, B_x, B_y in terms of E_z and B_z. Determine the second-order differential equations that E_z and B_z should separately satisfy.

3. Given that the solutions to the Green's equation
$$\frac{1}{r}\frac{\partial}{\partial r}(rG) + k^2 G = -\delta(r)$$
is of the form
$$G(r) = (A/r)e^{\pm ikr} \; ,$$
determine the value of A by integrating the equation over a small sphere centered at the origin.

4. Using the continuity equation, show that
$$\vec{\mathbf{E}}(\vec{\mathbf{x}}, t) = \frac{1}{4\pi\epsilon_0}\int \left[\frac{[\dot{\rho}]}{cr}\hat{\mathbf{r}} - \frac{[\dot{\vec{\mathbf{J}}}]}{c^2 r} \right] d^3\mathbf{x}'$$
reduces to
$$\vec{\mathbf{E}}(\vec{\mathbf{x}}, t) = \frac{1}{4\pi\epsilon_0 c^2}\int \frac{(\left[\dot{\vec{\mathbf{J}}}(\tilde{\mathbf{x}}')\right] \times \vec{\mathbf{r}}) \times \vec{\mathbf{r}}}{r^3} d^3\mathbf{x}'$$
where $r = |\vec{\mathbf{x}} - \vec{\mathbf{x}'}|$.

5. Starting from the "retarded potential" solution for $\vec{\mathbf{A}}(x,t)$, calculate its Fourier transform $\vec{\mathbf{A}}_\omega(\vec{\mathbf{x}})$, and through it, calculate $\vec{\mathbf{B}}_\omega(\vec{\mathbf{x}})$.