Electrodynamics II : Autumn 2011

Assignment 2

Given: Thursday Sep 29, Expected: Wednesday Oct 19

This assignment has two sections. You need to submit answers to only the questions from Section I. The questions from Section II are strongly recommended for practice / understanding of concepts, however you need not submit them and will not be graded on them.

Section I

1. Two charges, $+q$ and $-q$, are kept circulating about their common center of mass O (taken to be the origin), in a circle of radius a, with frequency ω. Calculate
   (i) the electric dipole, magnetic dipole and electric quadrupole components of $\vec{\mathbf{A}}, \vec{\mathbf{B}}, \vec{\mathbf{E}}$ at large distances
   (ii) the power radiated per solid angle in these three modes
   (iii) the total values of $\vec{\mathbf{A}}, \vec{\mathbf{B}}, \vec{\mathbf{E}}$ at large distances
   (iv) the total power radiated per solid angle.
   (v) What fraction of the total power radiated is accounted for by the combined power in ED, MD and EQ modes ? Comment.

2. A light source emitting light of wavelength λ isotropically (Intensity $I(\theta') = I'_0$) is mounted on a rocket moving with a large (relativistic) speed v along x direction.
   (i) Calculate an analytic expression for the intensity $I(\theta) \propto |\vec{\mathbf{E}}|^2$ of the emitted light, as observed in the stationary frame, as a function of θ. (Hint: You may separate $\vec{\mathbf{E}}$ into two components, one in the xy plane, one along the z axis.)
   (ii) Plot intensity as a function of θ for $v = 0.5c, v = 0.9c, v = 0.99c$.

3. A train is moving with a large (relativistic) speed v in the x direction. A ball is launched from the floor of the carriage at a speed u, making an angle θ' with the horizontal, in the xy plane. Seen from the frame of the train, it goes on a parabolic trajectory and returns to the floor. In the stationary frame, calculate
   (i) the trajectory $(x(t), y(t))$ of the ball
   (ii) the velocity $\vec{\mathbf{u}}(t)$
   (iii) the acceleration $\vec{\mathbf{a}}(t)$

4. (i) In Compton scattering, if the photon is scattered at an angle θ, what is its frequency after scattering ?
(ii) If a particle A of mass m_A decays into two particles B and C, of masses m_B and m_C, respectively, calculate the energy of B.

5. An infinite cylinder of radius R carries a constant current I, and has zero charge density as observed by an observer A. Another observer C travels parallel to the wire with a constant large (relativistic) speed v with respect to A.
(i) Find $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ observed by C, both inside and outside the cylinder.
(ii) Find the charge density measured by C. Comment on your answer.

Section II

1. Using the requirement of invariance of $\nabla \times \vec{\mathbf{E}} = -\partial \vec{\mathbf{B}}/\partial t$ and $\nabla \times \vec{\mathbf{B}} = (1/c^2)\partial \vec{\mathbf{E}}/\partial t$ under Lorentz transformations, determine the transformation properties of components of $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ under a boost along x direction with velocity v.

2. A frame is moving with a large (relativistic) velocity $\vec{\mathbf{v}} = v\hat{\mathbf{x}}$ If the velocity of a body in the relativistically moving frame is (u'_x, u'_y, u'_z), and its acceleration in that frame is (a'_x, a'_y, a'_z), calculate the velocity and acceleration in the stationary frame.

3. Show that
$$\vec{\mathbf{F}} = m\gamma\vec{\mathbf{a}} + \frac{\vec{\mathbf{u}}}{c^2}(\vec{\mathbf{F}} \cdot \vec{\mathbf{u}})$$
Hence, find the necessary and sufficient condition for $\vec{\mathbf{a}}$ to be in the same direction as $\vec{\mathbf{F}}$.

4. Given that $A \equiv (c\phi, \vec{\mathbf{A}})$ is a contravariant 4-vector, calculate how $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ fields change under Lorentz transformation.

5. Two Lorentz-invariant quantities can be formed from the electromagnetic field tensor F_{ij}: (i) $Q_1 = F_{ij}F^{ij}$, and (ii) $Q_2 = \epsilon_{ijkl}F^{ij}F^{kl}$. Here, ϵ_{ijkl} is the completely antisymmetric pseudotensor with $\epsilon_{0123} = 1$. Evaluate these two quantities in terms of $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$.