Electrodynamics II : Autumn 2011
DROP TEST
Saturday Aug 13, 09:45 am

- This test has two sections. Section I has four questions of 10 points each. Section II has three questions of 20 points each. The total is 100.
- You need at least 65 points in order to have a choice of dropping the course.

Section I

1. Consider a lossless transmission line consisting of two long co-axial conducting cylinders of radii a and b with empty space in between ($a < b$). An AC voltage $V = V_0 e^{-i\omega t}$ is applied between the cables at one end.
   (a) Which are the EM traveling modes supported ?
   (b) Calculate the electric field $\vec{E}$, the magnetic field $\vec{B}$, and the current I flowing through the transmission line, as functions of time. What is the average power P transmitted ?

   [10 points]

2. A particle A at rest decays into two lighter particles B and C. The masses of A and B are known, the mass of C is unknown.
   (a) How many independent Lorentz invariants can be formed from the 4-momenta of A, B, C and their combinations ? Give one such complete set.
   (b) Using these invariants, show how the mass of C can be determined from the measurement of the energy of B.
   (c) Before performing the experiment, what can we say about the range of possible values for the energy of B ?

   [10 points]

3. The electromagnetic field tensor is given by

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

The metric $g_{\alpha\beta}$ is a diagonal matrix with elements $(1, -1, -1, -1)$, which can be used to raise and lower the indices of tensors. Two Lorentz-invariant quantities can be formed from the field tensor:

(a) $Q_1 = F_{\mu\nu}F^{\mu\nu}$, and

(b) $Q_2 = \epsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta}F^{\gamma\delta}$.

Here, $\epsilon_{\alpha\beta\gamma\delta}$ is the completely antisymmetric pseudotensor with $\epsilon_{0123} = 1$. Evaluate these two quantities in terms of $\vec{E}$ and $\vec{B}$. Show your work in sufficient detail.

[10 points]

4. In a water Cherenkov neutrino detector, a high energy muon neutrino interacts with a nucleus, producing a highly relativistic muon that gives out Cherenkov radiation (refractive index of water: 4/3). This radiation is detected by a photosensitive plane (made of photomultiplier tubes).

Two muons (mass = 100 MeV) of energies 1 GeV and 10 GeV, respectively, are produced at the same point and travel in a direction normal to the photosensitive plane at a distance 40 m from the point of production. Sketch the pattern of light seen on the photosensitive plane, for both the muons (in the same figure). Show all the relevant distances.

[Assume that the muons, once produced, are absorbed in water within a relatively short distance (as compared to 40 m).]

[10 points]

Section II

5. An infinite grounded conductor in the x-y plane has an oscillating electric dipole $\vec{p} = p_0 \hat{z} e^{i\omega t}$ placed at a distance a from it.

   (a) Find the total power radiated.

   (b) Qualitatively sketch the radiation pattern in the x-z plane. (If you can give arguments motivating the pattern, there is no need to calculate it explicitly.)

   [20 points]

   [Hint: For an electric dipole,

$$\frac{dP}{d\Omega} = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \sin^2\theta \ .$$

   For an electric quadrupole,

$$\frac{dP}{d\Omega} = \frac{c^2}{1152\pi^2}\sqrt{\frac{\mu_0}{\epsilon_0}}\,|(\hat{n} \times Q(\hat{n})) \times \hat{n}|^2 \ .$$

   Here $Q_\alpha(\hat{n}) = \sum_\beta Q_{\alpha\beta} n_\beta$ where $Q_{\alpha\beta}$ is the quadrupole moment tensor.]

6. An infinite cylinder of radius R carries a constant current I, and has zero charge density as observed by an observer A. Another observer C travels parallel to the wire with a constant large (relativistic) speed v with respect to A.

   (a) Find $\vec{E}$ and $\vec{B}$ observed by C, both inside and outside the cylinder.

   (b) Find the charge density measured by C. Comment on your answer.

   [20 points]

7. The power radiated from an accelerated electron is

$$P = \frac{1}{4\pi\epsilon_0}\frac{2}{3}\frac{e^2}{c^3}|\vec{a}|^2$$

where $\vec{a}$ is the relativistic acceleration of the charge, defined as $\vec{a} = (1/m)d\vec{p}/d\tau$. At the particle colliders, electrons are accelerated to a relativistic speed $v \approx c$ and kept circulating in a ring with radius R.

(a) Calculate $|\vec{a}|$ in terms of R and $\gamma = E/(mc^2)$ for the electron. Neglect the change in γ due to the energy loss.

(b) At the LEP accelerator at CERN, electrons of E=100 GeV were kept circulating in a ring of R=4 km. Calculate the energy lost by an electron (in GeV) due to Synchrotron radiation while completing one circle in the ring.

(c) If protons are accelerated instead of electrons to the same energy, by what factor will the Synchrotron loss increase / decrease ?

[20 points]