Electrodynamics II : Autumn 2011

Midterm Examination

Saturday Oct 8, at 10:00 am, in AG66
(Duration: 3 hours, Total points: 100)

1. The threshold frequency for a rectangular waveguide of size $a \times 2a$ is ω_c. Calculate the group velocities v_g as functions of frequency ω (in units of ω_c) for the lowest five TE modes. Show their behaviour qualitatively on the same plot, indicating the relevant values of v_g (in units of c) and ω (in units of ω_c) along the axes.

   [10 points]

2. We want to make a microwave oven that will operate at 10 GHz. The walls of the cavity are to be coated with silver to ensure that not more than 10^{12}th fraction of the energy in EM waves leaks out. What is the minimum thickness of silver coating needed ? You may make any reasonable assumptions, but state them clearly.

   [Useful information: $1/(4\pi\epsilon_0) = 9{\times}10^9 N{\cdot}m^2/C^2$, $\mu_0/(4\pi) = 10^{-7} N/A^2$. The resistivity of silver at room temperature is $\sim 15\ n\Omega \cdot m$.]

   [10 points]

3. A train is moving with a constant large (relativistic) velocity $\vec{\mathbf{v}}$. A person sitting on the train is moving a pendulum in a vertical complete circle of radius R with a constant angular velocity ω. The axis of the circle is horizontal, and normal to the direction of motion of the train.

   To a stationary observer outside the train, it appears that the speed of the pendulum is the largest when it is at the bottom of its trajectory. At this point, what does this observer measure as
   (i) the velocity and acceleration of the pendulum bob ?
   (ii) the force on the pendulum bob ?

   [10 points]

4. A moving particle A is observed to decay into three almost massless particles that move in directions orthogonal to each other. If the energies of the decay products are measured to be E_1, E_2, E_3,
   (i) Determine the mass of the particle A.
   (ii) What was the speed of A ?

   [Keep track of all factors of c.]

   [10 points]

5. For a TM wave propagating through a rectangular waveguide,

$$E_z = A\sin(k_x x)\sin(k_y y)e^{i(k_z z-\omega t)} \ , \quad B_z = 0 \ .$$

Starting from Maxwell's equations for $\nabla \times \vec{\mathbf{E}}$ and $\nabla \times \vec{\mathbf{B}}$, calculate
(i) the transverse components of $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ fields.
(ii) the Poynting vector (Magnitude and direction)
(iii) the conductance of the waveguide
in terms of the parameters of E_z given above.

[20 points]

6. A charge $+q$ is moving with a frequency ω in a circle of radius a, centered at the origin and lying in the xy plane. At the locations $X = (d, 0, 0)$ and $Z = (0, 0, d)$, where $d \gg a$, calculate $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ up to their respective nonvanishing leading orders in $(1/d)$.

[20 points]

7. Polarized light ($\vec{\mathbf{E}}$ in the plane of incidence) of frequency ω is incident on an infinitely large dielectric surface (dielectric constant n) at an angle of incidence θ_I. It is partly reflected and partly transmitted.

As observed by an observer moving with a large (relativistic) velocity $\vec{\mathbf{v}}$ towards the dielectric surface, in a direction normal to the surface:
(i) Determine the angle of incidence θ'_I, the angle of reflection θ'_R, and the angle of transmission θ'_T.
(ii) Calculate the magnitude of the incident $\vec{\mathbf{E}}'$ as observed by this observer in terms of $|\vec{\mathbf{E}}|, \omega, \vec{\mathbf{v}}, \theta_I$.

[Lorentz transformations for $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ fields:

$$\begin{aligned} \vec{\mathbf{E}}'_{\parallel} &= \vec{\mathbf{E}}_{\parallel} \ , \qquad & \vec{\mathbf{E}}'_{\perp} &= \gamma(\vec{\mathbf{E}}_{\perp} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}_{\perp}) \ , \\ \vec{\mathbf{B}}'_{\parallel} &= \vec{\mathbf{B}}_{\parallel} \ , \qquad & \vec{\mathbf{B}}'_{\perp} &= \gamma(\vec{\mathbf{B}}_{\perp} - \vec{\mathbf{v}} \times \vec{\mathbf{E}}_{\perp}) \ . \qquad] \end{aligned}$$

[20 points]