Electrodynamics II : Autumn 2018
Assignment 1
Given: Monday Aug 27, Expected: Friday Sep 14

You need to submit answers to only the questions from Section I. You are strongly recommended to solve the questions from Section II, however you need not submit them and will not be graded on them.

Section I

1. The polarization of a light beam is defined as

$$P = \frac{I_1 - I_2}{I_1 + I_2}$$

where I_1 and I_2 are the intensities of the two orthogonal polarizations. If an unpolarized beam of light is incident on a dielectric with refractive index n and permeability μ_0, calculate the polarization of the reflected beam of light as a function of the angle of incidence θ_I. Plot this dependence for $n = 1.5$. You may use the results on reflection and transmission coefficients calculated in class and in Section II of this Assignment.

2. Current is slowly turned on in an infinite straight wire, such that

$$I(t) = \begin{cases} 0 & (t < 0) \\ I_0(t/\tau) & (0 \le t < \tau) \\ I_0 & (t \ge \tau) \end{cases}$$

(i) Calculate the resulting electric and magnetic fields as functions of time t and perpendicular distance d from the wire. (Note: the exact $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are needed, not just their radiative components.)

(ii) Show the answers in terms of plots of

- $|\vec{\mathbf{E}}|$ vs t (a few different values of d on the same plot), and
- $|\vec{\mathbf{B}}|$ vs t (a few different values of d on the same plot),

by choosing suitable values of parameters that bring out the important features. Comment on these features.

3. Consider a lossless transmission line consisting of two long co-axial conducting cylinders of radii a and b with empty space in between them $(a < b)$. An AC voltage $V = V_0 e^{-i\omega t}$, is applied between the cables at one end.

   (a) Calculate the electric field $\vec{E}$, the magnetic field $\vec{B}$, and the current I flowing through the transmission line, as functions of time. What is the average power P transmitted ?

   (b) For $a = 1$ cm and $b = 2$ cm, determine the capacitance, inductance and conductance of the transmission line for the transmission of TEM modes in SI units. (Some of these quantities may have to be defined per unit length, specify these clearly.)

   (c) For the above parameters, what are the smallest frequencies at which the TE and TM modes will be transmitted ?

4. The $x = 0$ plane is completely grounded, and a linear antenna of length L is kept at a distance $L/2$ away from it. [The two ends of the antenna are at the points $(L/2, 0, -L/2)$ and $(L/2, 0, L/2)$ respectively.] The antenna is fed a current

$$I = I_0 \sin\left[k\left(\frac{L}{2} - |z|\right)\right] \ .$$

Plot the radiation pattern (3-d plot as well as projections in yz and xy planes) for (i) $L = 0.1\lambda$, (ii) $L = \lambda$, (iv) $L = 10\lambda$.
Comment on the patterns.

5. Two charges, $+q$ and $-q$, are kept circulating about their common center of mass O (taken to be the origin), in a circle of radius a, with frequency ω.
(i) Calculate the electric dipole, magnetic dipole and electric quadrupole components of $\vec{\mathbf{A}}, \vec{\mathbf{B}}, \vec{\mathbf{E}}$ at large distances, and the power radiated per solid angle in these three modes
(ii) Calculate the total values of $\vec{\mathbf{A}}, \vec{\mathbf{B}}, \vec{\mathbf{E}}$ at large distances, and the total power radiated per solid angle.
(iii) Comment on the relative fractions of the powers radiated in ED, MD and EQ modes, and higher multipoles.

Section II

1. Let a plane EM wave with frequency ω travelling in free space be incident on the surface of a dielectric with refractive index n and permeability μ_0, at an angle of incidence θ_I. Let the electric field of the wave be normal to the plane of incidence, i.e. parallel to the surface of the dielectric. In terms of the parameter $\alpha = \cos\theta_T / \cos\theta_I$ and $\beta = n$, calculate the reflection coefficient R and the transmission coefficient T. Plot these quantities as functions of θ_I. Is there a "Brewster's angle" in this case ?

2. Given that the solutions to the Green's equation

$$\frac{1}{r}\frac{\partial}{\partial r}(rG) + k^2 G = -\delta(r)$$

   is of the form

$$G(r) = (A/r)e^{\pm ikr} \ ,$$

   determine the value of A by integrating the equation over a small sphere centered at the origin.

3. For a monochromatic source, using the continuity equation, show that

$$\vec{E}(\vec{x}, t) = \frac{1}{4\pi\epsilon_0} \int \left[\frac{[\dot{\rho}(\vec{x}')]}{cr}\hat{\mathbf{r}} - \frac{[\dot{\vec{J}}(\vec{x}')]}{c^2 r} \right] d^3\mathbf{x}'$$

   reduces to

$$\vec{E}(\vec{x}, t) = \frac{1}{4\pi\epsilon_0 c^2} \int \frac{\left(\left[\dot{\vec{J}}(\vec{x}')\right] \times \vec{\mathbf{r}}\right) \times \vec{\mathbf{r}}}{r^3} d^3\mathbf{x}'$$

   where $r = |\vec{x} - \vec{x}'|$.

4. Inside a conducting medium, an EM wave will propagate as well as decay. Let the form of the plane wave solution be

$$\vec{E} = \vec{E}_0 e^{-\kappa x} e^{ikx} e^{-i\omega t} \ .$$

   (a) Find $c\kappa/\omega$ and ck/ω as functions of $\omega\tau$, where τ is the relaxation time. Show your results in the form of a plot that brings out all the relevant features. Comment on the plot.

   (b) For an EM wave reflecting normally from the surface of a conductor with finite σ, calculate the surface current $\vec{K}$ and the time averaged value of the Poynting vector $\vec{N}$ into the surface. Hence determine the "surface resistance" R_s.

5. We want to make a microwave oven that will operate at 10 GHz. The walls of the cavity are to be coated with silver to ensure that not more than 10^{12}th fraction of the energy in EM waves leaks out. What is the minimum thickness of silver coating needed ? You may make any reasonable assumptions, but state them clearly.

[Useful information: $1/(4\pi\epsilon_0) = 9\times10^9 N\cdot m^2/C^2$, $\mu_0/(4\pi) = 10^{-7}N/A^2$. The resistivity of silver at room temperature is $\sim 15\ n\Omega \cdot m$.]