Electrodynamics II : Autumn 2018

Assignment 2

Given: Friday Sep 14, Expected: Wednesday Oct 3

You need to submit answers to only the questions from Section I. You are strongly recommended to solve the questions from Section II, however you need not submit them and will not be graded on them.

Section I

1. A train is moving with a constant large (relativistic) velocity $\vec{\mathbf{v}}$. A person sitting on the train is moving a pendulum in a vertical complete circle of radius R with a constant angular velocity ω. The axis of the circle is horizontal, and normal to the direction of motion of the train.

   To a stationary observer outside the train, it appears that the speed of the pendulum is the largest when it is at the bottom of its trajectory. At this point, what does this observer measure as
   (i) the velocity $\vec{\mathbf{v}}$ and acceleration $\vec{\mathbf{a}}$ of the pendulum bob ?
   (ii) the force $\vec{\mathbf{F}}$ on the pendulum bob ?

2. A light source emitting light of wavelength λ isotropically (Intensity $I(\theta') = I_0'$) is mounted on a rocket moving with a large (relativistic) speed v along x direction. The light is polarized such that the electric field $\vec{\mathbf{E}}$ is along the z axis.
   (i) Calculate an analytic expression for the intensity $I(\theta)$ of the emitted light, as observed in the stationary frame, as a function of θ.
   (ii) Plot intensity as a function of θ for $v = 0.5c, v = 0.9c, v = 0.99c$. Show polar plots in the x-z plane, with $r = \log(I/I_0')$. Use appropriate values of any parameters needed.

3. Relativistic force:
   (i) Show that

   $$\vec{\mathbf{F}} = m\gamma\vec{\mathbf{a}} + \frac{\vec{\mathbf{u}}}{c^2}(\vec{\mathbf{F}} \cdot \vec{\mathbf{u}})$$

   Hence, find the necessary and sufficient condition for $\vec{\mathbf{a}}$ to be in the same direction as $\vec{\mathbf{F}}$.
   (ii) Determine the 4-vector f^{k}, whose space components correspond to $d\vec{\mathbf{p}}/dt$, where $\vec{\mathbf{p}}$ is the relativistic momentum.

4. Compton scattering:
   (i) In the AB $\rightarrow$ CD scattering, write down the quantities

   $$\mathsf{p_A \cdot p_D} \;,\quad \mathsf{p_B \cdot p_C} \;,\quad \mathsf{p_B \cdot p_D} \;,\quad \mathsf{p_C \cdot p_D}$$

   in terms of $\mathsf{p_A \cdot p_B}$, $\mathsf{p_A \cdot p_C}$, and the masses of the particles.
   (ii) In Compton scattering (A and C photons, B and D electrons), write down the above six scalar products in terms of quantities measured in the lab frame.
   (iii) Hence, given the frequency ν of the incoming photon and the angle of scattering θ, determine the frequency of the outgoing photon.

5. An infinite cylinder of radius R has zero net charge density and a uniform current density J inside it, as observed by an observer A. Another observer C travels parallel to the wire with a constant large (relativistic) speed v with respect to A, in the same direction as the current.
   (i) Find $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ observed by C, both inside and outside the cylinder.
   (iii) Calculate the EM field tensor F as measured by A and C, both inside and outside the cylinder.
   (iii) Find the charge density measured by C. Comment on the conservation of charge.

Section II

1. Using the requirement of invariance of Maxwell's equations under Lorentz transformations, determine the transformation properties of components of $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ under a boost along x direction with velocity v. (Start by taking the transformation to be a general linear transformation.)

2. Given that $A \equiv (c\phi, \vec{\mathbf{A}})$ is a contravariant 4-vector, calculate how $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ fields change under Lorentz transformation.

3. A frame is moving with a large (relativistic) velocity $\vec{\mathbf{v}} = v\hat{\mathbf{x}}$. If the velocity of a body in the relativistically moving frame is (u'_x, u'_y, u'_z), and its acceleration in that frame is (a'_x, a'_y, a'_z), calculate the velocity and acceleration in the stationary frame.

4. Confirm the transformation properties of the completely antisymmetric fourth rank 4-tensor ϵ under Lorentz boost and space rotation.