Electrodynamics II : Autumn 2018
DROP TEST

Sunday Aug 19, 2:00 pm

The total number of points in this test is 100. You need at least 65 points in order to have a choice of dropping the course.

Useful information:

Lorentz transformations for $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ fields:

$$\vec{\mathbf{E}}'_{\parallel} = \vec{\mathbf{E}}_{\parallel} \; , \qquad \vec{\mathbf{E}}'_{\perp} = \gamma(\vec{\mathbf{E}}_{\perp} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}_{\perp}) \; ,$$
$$\vec{\mathbf{B}}'_{\parallel} = \vec{\mathbf{B}}_{\parallel} \; , \qquad \vec{\mathbf{B}}'_{\perp} = \gamma(\vec{\mathbf{B}}_{\perp} - \vec{\mathbf{v}} \times \vec{\mathbf{E}}_{\perp}) \; .$$

where $\parallel$ and $\perp$ denote the components of the fields longitudinal and transverse to the direction of the boost.

1. Consider a lossless transmission line consisting of two long co-axial conducting cylinders of radii a and b with empty space in between ($a < b$). An AC voltage $V = V_0 e^{-i\omega t}$ is applied between the cables at one end.

   (a) Which are the EM traveling modes supported ?

   (b) Calculate the electric field $\vec{E}$, the magnetic field $\vec{B}$, and the current I flowing through the transmission line, as functions of time. What is the average power P transmitted ?

   [10 points]

2. In the frame of an observer A, an infinite straight wire carries a constant current I, and has zero charge density. Another observer C travels parallel to the wire with a constant large (relativistic) speed v with respect to A. In the frame of C, calculate

   (a) $\vec{E}$ and $\vec{B}$ as a function of the distance d from the wire.

   (b) the linear charge density, if any, measured by C. Interpret your result.

   [10 points]

3. A moving particle A is observed to decay into three almost massless particles that move in directions orthogonal to each other. If the energies of the decay products are measured to be E_1, E_2, E_3,

   (a) Determine the mass of the particle A.

   (b) What was the speed of A ?

   [Keep track of all factors of c.]

[10 points]

4. The electromagnetic field tensor is given by [$c = 1$ in this problem]

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

   The metric $g_{\alpha\beta}$ is a diagonal matrix with elements $(1, -1, -1, -1)$, which can be used to raise and lower the indices of tensors. Also, using the completely antisymmetric pseudotensor $\epsilon_{\alpha\beta\gamma\delta}$ with $\epsilon_{0123} = 1$, one can form the dual tensor $\widetilde{F}^{\gamma\delta} = \epsilon^{\alpha\beta\gamma\delta} F_{\alpha\beta}$.

   (a) Explicitly write down the matrices $F^{\gamma\delta}$ and $\widetilde{F}^{\gamma\delta}$ in terms of *components of* $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$.

   (b) *Using the above*, calculate the Lorentz-invariant quantities $Q_1 = F_{\mu\nu} F^{\mu\nu}$, and $Q_2 = F_{\mu\nu} \widetilde{F}^{\mu\nu}$.
   Give the answer in terms of $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ (*not* their components).

[10 points]

5. Polarized light ($\vec{\mathbf{E}}$ normal to the plane of incidence) of frequency ω is incident on an infinitely large dielectric surface (refractive index n) at an angle of incidence θ_I. It is partly reflected and partly transmitted.

   As observed by an observer moving with a large (relativistic) speed v towards the dielectric surface, in a direction normal to the surface:

   (a) Determine the angle of incidence θ'_I, the angle of reflection θ'_R, and the angle of transmission θ'_T, in terms of θ_I and v/c.

   (b) Calculate the magnitude of the incident $\vec{\mathbf{E}}'$ as observed by this observer in terms of $|\vec{\mathbf{E}}|, \omega, \vec{\mathbf{v}}, \theta_I$.

[20 points]

6. Two charges q_1 and q_2 are moving with uniform relativistic velocities along the x and y axis respectively. Their position vectors are given as

$$\vec{\mathbf{x}}_1 = v_1 t\ \hat{\mathbf{x}}\ , \quad \vec{\mathbf{x}}_2 = v_2 t\ \hat{\mathbf{y}}\ .$$

(a) Calculate the potentials $\phi(x, y, z, t)$ and $\vec{\mathbf{A}}(x, y, z, t)$ due to the charge q_1.

(b) Hence calculate the fields $\vec{\mathbf{E}}(x, y, z, t)$ and $\vec{\mathbf{B}}(x, y, z, t)$ due to the charge q_1.

(c) Draw a diagram showing the positions of q_1 and q_2 at an arbitrary time t. Qualitatively show the directions of $\vec{\mathbf{E}}(\vec{\mathbf{x}}_2, t)$ and $\vec{\mathbf{B}}(\vec{\mathbf{x}}_2, t)$ at the position of the second charge. Point out the important features.

(d) Calculate the force $\vec{\mathbf{F}}$ on the charge q_2 due to the charge q_1.

Your answers should be in terms of x, y, z, t, v_1, v_2 and other universal constants, but no other variables. There is no need to combine terms to simplify them.

[20 points]

7. Let a charge q be moving in the xy plane with a constant speed v, under the influence of an external constant magnetic field $\vec{\mathbf{B}} = B_0 \hat{\mathbf{z}}$. The instantaneous velocity of the charge is $\vec{\mathbf{v}}$.

(a) By explicitly calculating the RHS of the equation of motion

$$m(du^i/ds) = qF^{ik}u_k,$$

determine the components of the 4-acceleration $a^i = du^i/ds$ in terms of the components of $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$. Hence write down the 3-acceleration $\vec{\mathbf{a}}$.

(b) Neglecting the loss of energy due to radiation, the charge will keep on moving in a circle. Determine the radius of the circle and the frequency of revolution of the charge, in terms of $\vec{\mathbf{v}}$, $\vec{\mathbf{B}}$, and energy $\mathcal{E}$.

(c) The accelerating charge gives rise to the radiation fields

$$\vec{\mathbf{E}}_{\rm rad} = \frac{q}{4\pi\epsilon_0 c^2 s^3}\vec{\mathbf{r}} \times (\vec{\mathbf{r}}_{\vec{\mathbf{v}}} \times \vec{\mathbf{a}})\ , \quad \vec{\mathbf{B}}_{\rm rad} = \frac{1}{rc}(\vec{\mathbf{r}} \times \vec{\mathbf{E}}_{\rm rad})\ .$$

where $\vec{\mathbf{r}}_{\vec{\mathbf{v}}} = \vec{\mathbf{r}} - r(\vec{\mathbf{v}}/c)$. When $v \ll c$, calculate the magnitude and direction of the Poynting vector $\vec{\mathbf{N}}(\vec{\mathbf{r}})$ of the radiation, in terms of $\vec{\mathbf{v}}, \vec{\mathbf{B}}$, and $\mathcal{E}$. [Only keep the leading power in v/c)].

[20 points]