Module III: Relativistic ED: applications

Lecture 19: Cherenkov radiation

Amol Dighe
TIFR, Mumbai

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Outline

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Uniformly moving charge in vacuum: no radiation

- In the last lecture, we calculated the EM fields due to a charge q moving with constant velocity $\vec{\mathbf{v}} = v\hat{\mathbf{x}}$ as

$$\vec{\mathbf{E}}(\vec{\mathbf{x}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{\gamma^2 s^3} (\tilde{x}, \tilde{y}, \tilde{z}) \ , \quad \vec{\mathbf{B}}(\vec{\mathbf{x}}, t) = \frac{1}{4\pi\epsilon_0 c^2} \frac{qv}{\gamma^2 s^3} (0, -\tilde{z}, \tilde{y}) \tag{1}$$

- Note that both of these fields go as $1/r^2$ at $r \to \infty$. Therefore, there is no radiative component and hence no power is radiated to infinity. That is, there is no power loss.
- This is not surprising, since in the frame where the charge is at rest, there is clearly no power loss, and simply going to another inertial frame where it is moving with constant velocity should not result in it losing any energy.

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Why is the situation different inside a medium

- Naively it would seem that the situation would be no different inside a medium, too. After all, a static charge inside a medium does not radiate, and we can just go to an appropriate frame to make the charge move with a constant velocity.
- However the naive argument above is incorrect. A simple counterexample is to notice that the speed of any particle is less than the speed of EM waves in vacuum, while it can exceed the speed of the EM waves in a medium.
- Let us see a formal treatment of this phenomenon, which will help distinguish between the two situations.

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Wave equation for EM potentials in vacuum

- The potentials due to a uniformly moving charge satisfy

$$\psi(\vec{\mathbf{x}}, t) = \psi(\vec{\mathbf{x}} + \vec{\mathbf{v}}\tau, t + \tau) \tag{2}$$

  Therefore,

$$\frac{\partial \psi}{\partial t} = -\vec{\mathbf{v}} \cdot \nabla \psi \tag{3}$$

  When $\vec{\mathbf{v}} = v\hat{\mathbf{x}}$, this becomes $\partial\psi/\partial t = -v\, \partial\psi/\partial x$.

- Now the potentials in vacuum satisfy the following wave equation:

$$\left(\frac{\partial^2}{c^2 \partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}\right) \psi(x, y, z, t) = g(x, y, z, t) \tag{4}$$

  where $g(x, y, z, t)$ is the source.

- Using the relationship between $\partial/\partial t$ and $\partial/\partial x$ above, we can go to the time-independent equation:

$$\left[\left(1 - \frac{v^2}{c^2}\right) \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right] \psi(x, y, z, t) = -g(x, y, z, t) \tag{5}$$

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Wave equation ≈ Poisson's equation

- The wave equation has now become

$$\left(\frac{1}{\gamma^2}\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\psi(x,y,z,t)=-g(x,y,z,t) \quad (6)$$

and with the substitution $x'=x\gamma, y'=y, z'=z$, it becomes

$$\left(\frac{\partial^2}{\partial x'^2}+\frac{\partial^2}{\partial y'^2}+\frac{\partial^2}{\partial z'^2}\right)\psi(\frac{x'}{\gamma},y',z')=-g(\frac{x'}{\gamma},y',z') \quad (7)$$

where ψ and g are to be calculated at the same time.

- But this is just the Poisson's equation, and our substitutions $(x,y,z)\rightarrow(x',y',z')$ were just the Lorentz contraction !
- This shows that the wave equation is equivalent to the Poisson equation, just with Lorentz transformations. The case of uniformly moving charge can thus be mapped to that of the stationary charge just by a Lorentz boost.

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Inside a medium

- Before jumping to the conclusion that the same argument should be valid even for the situation inside a medium, note that the argument crucially depends on the fact that $1 - v^2/c^2 > 0$. Only then do the coefficients of $\partial^2/\partial x^2$, $\partial^2/\partial y^2$, and $\partial^2/\partial z^2$ have the same sign, a requirement for Poisson's equation.
- The above condition is always satisfied in a vacuum, since here c, the speed of EM waves, is always greater than the speed v of the charge
- The condition may be violated inside a medium, which implies that a Lorentz transformation will not always be able to take a uniformly moving charge in a medium to a stationary charge. The two situations are not related by a frame transformation.
- Indeed when v exceeds the speed of EM waves in the medium, the charge radiates. This is Cherenkov radiation, which we shall study in detail now.

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An intuitive argument

- Since the speed of EM waves inside a medium of refractive index n is $c_n = c/n$, the retarded time satisfies

$$t_{r,n} = t - r(t_{r,n})/c_n = t - r(t_{r,n})n/c \tag{8}$$

where $r(t) = |\vec{\mathbf{x}} - \vec{\mathbf{x}}_0(t)|$. The vector potential is then given by

$$\vec{\mathbf{A}}(\vec{\mathbf{x}}, t) = \frac{\mu_0}{4\pi} \frac{q\,\vec{\mathbf{v}}}{s(t_{r,n})} \tag{9}$$

where

$$s(t_{r,n}) = r(t) - \frac{\vec{\mathbf{v}} \cdot \vec{\mathbf{r}}(t)}{c_n} = r(t) - \frac{n\,v\,r(t)\,\cos\theta}{c} \tag{10}$$

- Clearly when $\cos\theta = c/(nv)$, we have $s = 0$ and the potential $\vec{\mathbf{A}}(\vec{\mathbf{x}}, t)$ blows up. When this condition is satisfied, the potential does not simply behave as $1/r$ at infinity, and hence there is radiation.
- Note that the radiation takes place only at the angle where $\cos\theta = c/(nv)$, and hence, only when $v > c/n$, i.e. the charge is travelling faster than the speed of EM waves in the medium.

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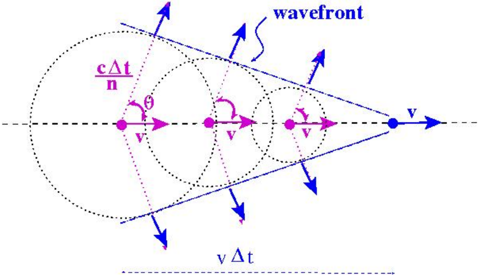
The motivation from a wavefront point-of-view

- An argument using Huygen's wavefront language can be given, which shows that if a charge is travelling faster than the speed of light in a medium, the wavefronts emenating from it interfere constructively at $\cos\theta = c/(nv)$. The argument may be constructed from the following figure.



- The argument shows that the radiation peaks at the particular angle, but is not enough to motivate why the radiation indeed falls slower that $1/r$, and hence carries power to infinity.

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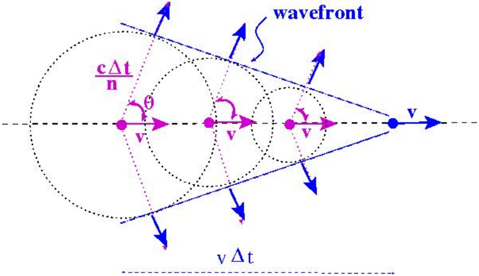

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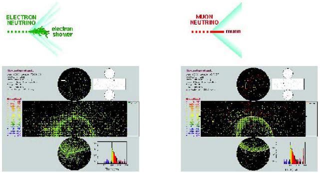
Cherenkov radiation: simulations

Fermilab simulation (BOONE: Booster Neutrino Experiment):

https://www.youtube.com/watch?v=x4Ir6E4IG64

Identifying neutrinos in a water Cherenkov detector

How to detect ν_e and ν_μ through Cherenkov cones



Diffused ring

Sharp ring

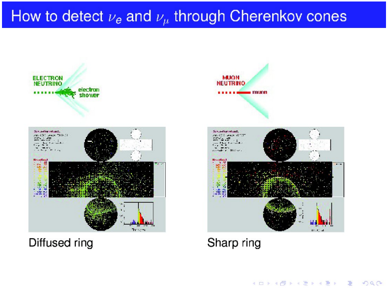
- When neutrinos interact in water, they may produce an electron, positron or muon (depending on whether they were $\nu_e, \bar{\nu}_e$ or $\nu_\mu/\bar{\nu}_\mu$) which is energetic enough to travel faster than the speed of light in water.

- These energetic charged particles produce "Cherenkov cones", which are detected by photomultiplier tubes.

- The Cherenkov cones of electron/positron and muon look different since the muon does not undergo much scattering. This helps distinguish between electron neutrinos and muon neutrinos.

- The first confirmed evidence of neutrino oscillations came from a water Cherenkov detector (Super-Kamiokande).

Identifying neutrinos in a water Cherenkov detector



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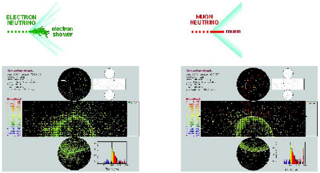

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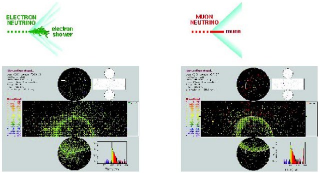

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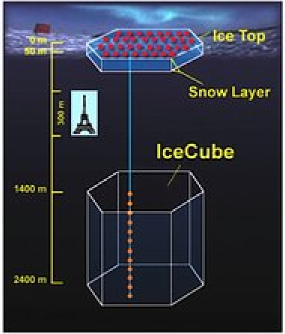
Large scale Cherenkov detectors

IceCube simulation:



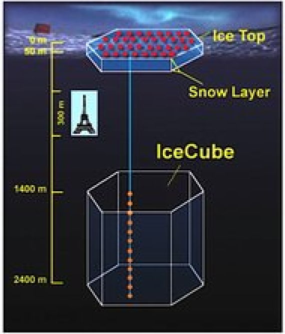
https://www.youtube.com/watch?v=3PZgfPHULHw

Gamma Ray simulation (Veritas):

https://www.youtube.com/watch?v=BItsAapGch0

Large scale Cherenkov detectors

IceCube simulation:

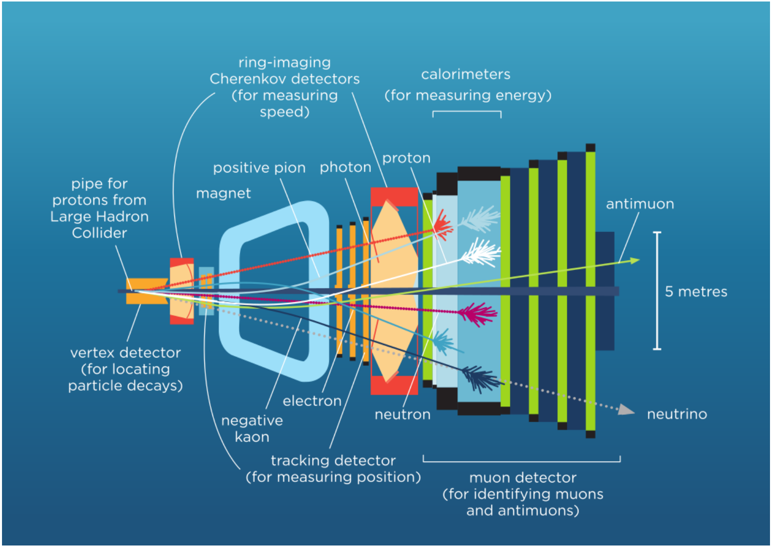


https://www.youtube.com/watch?v=3PZgfPHULHw

Gamma Ray simulation (Veritas):

https://www.youtube.com/watch?v=BItsAapGch0

Ring Imaging CHerenkov (RICH) detector



Outline

Fourier components of the current

- For a charge q moving with velocity $\vec{\mathbf{v}} = v\hat{\mathbf{x}}$ along the x axis,

$$\vec{\mathbf{J}}(\vec{\mathbf{x}}', t') = q\, v\, \hat{\mathbf{x}}\, \delta(x' - vt')\, \delta(y')\, \delta(z') \qquad (11)$$

- Its Fourier transform is given by

$$\begin{aligned} \vec{\mathbf{J}}_\omega(\vec{\mathbf{x}}') &= \frac{1}{2\pi} \int \vec{\mathbf{J}}(\vec{\mathbf{x}}', t') e^{i\omega t'} dt' \\ &= \frac{qv}{2\pi} \hat{\mathbf{x}}\, \delta(y')\, \delta(z') \int \delta(x' - vt') e^{i\omega t'} dt' \end{aligned} \qquad (12)$$

- The integral evaluates to $(1/v) e^{i\omega x'/v}$, so we have

$$\vec{\mathbf{J}}_\omega(\vec{\mathbf{x}}') = \frac{q}{2\pi}\, \delta(y')\, \delta(z') e^{i\omega x'/v}\, \hat{\mathbf{x}} \qquad (13)$$

We shall use this now to calculate the radiated power.

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Energy radiated by Cherenkov radiation

- In Module II, we have seen that

$$\frac{dU_\omega}{d\Omega} = \frac{1}{4\pi}\sqrt{\frac{\mu}{\epsilon}}\left|\int\left(\vec{\mathbf{J}}_\omega(\vec{\mathbf{x}}')\times\vec{\mathbf{k}}\right)e^{-i\vec{\mathbf{k}}\cdot\vec{\mathbf{x}}'}d^3x'\right|^2 . \qquad (14)$$

- Inside the medium with $\mu = \mu_0$, we have $\sqrt{\mu/\epsilon} = (1/n)\sqrt{\mu_0/\epsilon_0}$, and $|\vec{\mathbf{k}}| = n\omega/c$. Also, we take θ to be the angle between the velocity $\vec{\mathbf{v}}$ of the charge and the wave-vector $\vec{\mathbf{k}}$ of the radiation. The integration is over $d^3x' = dx'dy'dz'$, out of which the integrals over dy' and dz' are trivial because of the δ functions.
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Proper limits of integration

- If the limits of the integral in the expression for energy radiated are taken to be $-\infty$ to ∞, the integral would be a δ-function:

$$\int_{-\infty}^{\infty} e^{\frac{i\omega x'}{v} - ikx'\cos\theta} dx' = \frac{v}{\omega}\delta\left(1 - \frac{nv}{c}\cos\theta\right) \ , \qquad (16)$$

  which indicates that the energy is radiated only in the direction $\cos\theta = c/(nv)$.

- However if we take this result at the face value, we would get

$$\frac{dU_\omega}{d\Omega} = \frac{1}{16\pi^3}\frac{nv^2q^2}{\epsilon_0 c^3}\sin^2\theta\left[\delta\left(1 - \frac{nv}{c}\cos\theta\right)\right]^2 \ , \qquad (17)$$

  which involves the square of a delta function and is ill-defined.

- Even physically, a charge loses its speed via the radiated energy, so taking the range of x' to be throughout the x-axis is not physical. A reasonable question to ask is the energy lost by the charge while it travels a short distance: from $-\ell$ to ℓ, for example.

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Energy radiated during a short displacement

- For this finite interval, the speed of particle can be taken to be a constant. The relevant integral becomes

$$\int_{-\ell}^{+\ell} e^{i\frac{\omega x'}{v}\left(1-\frac{nv}{c}\cos\theta\right)} dx' = \int_{-\ell}^{+\ell} e^{i\frac{\omega x'}{v}\zeta} dx' = \frac{2\sin\left(\frac{\omega\ell}{v}\zeta\right)}{\frac{\omega}{v}\zeta} \,. \qquad (18)$$

where $\zeta = 1-(nv/c)\cos\theta$,

- The energy radiated during this time is

$$U_\omega = \int \frac{dU_\omega}{d\Omega}(2\pi)\, d\cos\theta$$

$$= \frac{1}{2\pi^2}\frac{n\omega^2 q^2}{\epsilon_0 c^3}\int_{-1}^{1} \sin^2\theta \frac{\sin^2\left(\frac{\omega\ell}{v}\zeta\right)}{\left(\frac{\omega}{v}\zeta\right)^2} d\cos\theta \,. \qquad (19)$$

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Energy radiated, after some approximations

- Since we know that the power will be radiated only along $\cos\theta \approx c/(nv)$, we can approximate $\sin^2\theta = 1 - c^2/(nv)^2$ and take it out of the integral in order to simplify calculations.
- The remaining integral is calculable analytically, when we change the limits from $(-1, 1)$ to $(-\infty, \infty)$. This is valid again because the power is radiated only at $\cos\theta \approx c/(nv)$ anyway. Since $\zeta = 1 - (nv/c)\cos\theta$,

$$\int_{-\infty}^{\infty} \frac{\sin^2\left(\frac{\omega X}{v}\zeta\right)}{\left(\frac{\omega}{v}\zeta\right)^2} d\cos\theta = \frac{c\ell}{n\omega}\pi \tag{20}$$

- Thus, finally we have

$$U_\omega = \frac{q^2\ell\omega}{2\pi\epsilon_0 c^2}\left(1 - \frac{c^2}{n^2 v^2}\right) \tag{21}$$

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Energy loss per unit length per frequency

- The calculation for radiated energy allows us to determine the energy loss of the particle per unit length due to Cherenkov radiation:

$$\frac{\Delta E}{\Delta x} = \frac{U_\omega d\omega}{2\ell} = \frac{q^2}{4\pi\epsilon_0 c^2}\left(1 - \frac{c^2}{n^2 v^2}\right)\omega d\omega \; . \tag{22}$$

- The number of photons radiated per unit length per frequency by a charge equal to the electron charge is

$$N_{\mathrm{ph}} = \frac{e^2}{4\pi\epsilon_0 \hbar c}\left(1 - \frac{c^2}{n^2 v^2}\right)\frac{d\omega}{c} \; . \tag{23}$$

- It would seem that photons of all frequencies get radiated at equal rates, and this would create problems with infinite energy radiation. However for photons, n is a function of frequency, and $n(\omega) \to 1$ for large ω, so the contribution to Cherenkov radiation from high frequencies is absent.

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Take-home message from this lecture

- Even though an uniformly moving charge in vacuum cannot radiate, a uniformly moving charge in matter can radiate, as long as its speed is more than that of the EM waves in that medium.
- This Cherenkov radiation takes place only in the direction making an angle $\cos\theta = c/(nv)$ with the direction of motion of the charge.