

## Discussion on Faraday wheel

# Faraday's law of induced emf

## Maxwell's equations without external sources

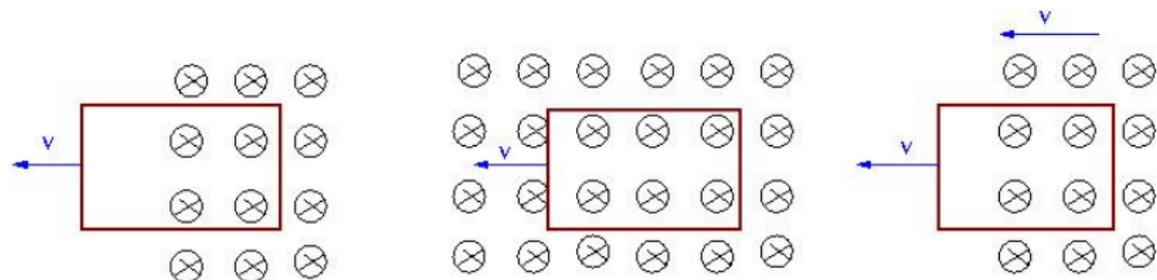
$$\begin{aligned}\nabla \cdot \vec{\mathbf{E}} &= 0 & \nabla \times \vec{\mathbf{E}} &= -\partial \vec{\mathbf{B}} / \partial t \\ \nabla \cdot \vec{\mathbf{B}} &= 0 & \nabla \times \vec{\mathbf{B}} &= \mu_0 \epsilon_0 \partial \vec{\mathbf{E}} / \partial t\end{aligned}\quad (1)$$

## Faraday's law

$$\nabla \times \vec{\mathbf{E}} = -\partial \vec{\mathbf{B}} / \partial t \quad \Rightarrow \quad \oint \vec{\mathbf{E}} \cdot d\vec{\ell} = -\partial \Phi / \partial t \quad (2)$$

- The total emf induced in a closed loop is equal to the rate of change of magnetic flux through the loop.
- Is this valid in all situations ?

# Wire loop cutting through magnetic field lines



- In the first case, the magnetic flux through the loop clearly changes, so emf is induced in the wire loop, and current flows.
- In the second case, the wire cuts through the magnetic lines of force, but the total magnetic flux through the loop never changes. **But current still flows !** (momentarily, when the loop starts moving, till the charge buildup cancels the EMF)
- In the third case, both the source of magnetic flux and the wire loop move together. The flux lines are not cut by the wire. **But current still flows !** (and now, continues to flow)
- Maxwell's equations must be incomplete then...

# Lorentz force

Force on charge  $q$  in the presence of  $\vec{\mathbf{E}}$  and  $\vec{\mathbf{B}}$

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}) \quad (3)$$

- This is an experimental result.
- It may be interpreted as an effective electric field:

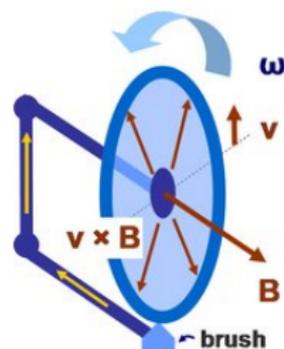
$$\vec{\mathbf{E}}' = \vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}} . \quad (4)$$

- This relation is not contained in the Maxwell's equations (*at this stage*)
- The combination of two relations

$$\nabla \times \vec{\mathbf{E}} = -\partial\vec{\mathbf{B}}/\partial t , \quad \text{and} \quad \vec{\mathbf{E}}' = \vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}} \quad (5)$$

is sufficient to take care of all situations. (See Feynman lectures for a detailed discussion).

# Faraday Disc: a problem



- Bar magnet along the axis of the conducting disc
- Conducting loop as shown in the figure

Does the current flow through the loop when:

- The magnet is stationary and the disc is spinning ?
- The disc is stationary and the magnet is spinning about its axis ?
- Both the disc and the magnet are spinning with the same angular speed ?