Flavour Physics, Spring 2014

Assignment 1

(Given: 10/03/2014, To be submitted: 24/03/2014)

1. Use the PDG data to determine the ratio of invisible width of the Z boson to its total width (including the uncertainty on it). Determine the number of neutrinos from this data (including uncertainty). If the mass of the top quark were 10 GeV, what would the central value of this ratio become ? If the masses of the top and bottom quarks were interchanged, what would the central value of the ratio become ?

2. Assume only two generations for the sake of this exercise. Determine the Cabibbo angle (and the uncertainty on it) by using the *current* measurements of neutron decay, pion decay and muon decay through their semileptonic modes. Perform the same exercise by using the measurements of leptonic modes of π^+ and K^+. How consistent are these two measurements ?

3. Calculate the expression for the semileptonic decay $\Gamma(K^+_{\mu 3})$. Using this result and employing isospin arguments, determine $\Gamma(K^0_{\mu 3})$. Hence calculate the numerical value of $\mathrm{BR}(K_L \to \pi^+ e^- \bar{\nu}_e)$. Neglect CP violation.

4. Ignoring any direct CP violation, calculate the semileptonic asymmetry in K_L decays:

$$A_{SL}(t) \equiv \frac{\frac{d\Gamma}{dt}(K_L \to \pi^+ e^- \bar{\nu}_e) - \frac{d\Gamma}{dt}(K_L \to \pi^- e^+ \nu_e)}{\frac{d\Gamma}{dt}(K_L \to \pi^+ e^- \bar{\nu}_e) + \frac{d\Gamma}{dt}(K_L \to \pi^- e^+ \nu_e)}$$

   in terms of ϵ, as a function of time.

5. The phase of $|K^0\rangle$ and $|\bar{K}^0\rangle$ can be arbitrarily defined. The usual convention ensures $CP|K^0\rangle = |\bar{K}^0\rangle$. Let us go to a convention where $CP|K^0\rangle_{\text{new}} = e^{i\phi}|\bar{K}^0\rangle_{\text{new}}$. In this convention, determine

   - The most general, normalized CP eigenstates $|K_1\rangle_{\text{new}}, |K_2\rangle_{\text{new}}$ in terms of the flavour eigenstates $|K^0\rangle_{\text{new}}, |\bar{K}^0\rangle_{\text{new}}$
   - The elements of M_{new} and Γ_{new} matrices in terms of M_{ij}'s and Γ_{ij}'s with the usual convention.

6. Calculate $|\eta_{00}|$ and $|\eta_{+-}|$ up to first order in the small quantities ϵ and $|A_2/A_0|$. Use the notation $A_I = |A_I|e^{i\phi_I}e^{i\delta_I}$ where ϕ_I is the weak phase and δ_I is the strong phase for the amplitude with isospin I.

Not to be submitted

1. Calculate the semileptonic decay rates

$$\Gamma(\mu^+ \to e^+\nu_e\bar{\nu}_\mu)\ , \quad \Gamma(K^+ \to \pi^0 e^+\nu_\mu)\ , \quad \Gamma(n \to pe^-\bar{\nu}_e)$$

and the leptonic decay rates

$$\Gamma(\pi^+ \to \mu^+\nu_\mu)\ , \quad \Gamma(\pi^+ \to e^+\nu_e)\ , \quad \Gamma(K^+ \to \mu^+\nu_\mu)\ , \quad \Gamma(K^+ \to \mu^+\nu_\mu)$$

2. Draw all Feynman diagrams contributing to $K^0 \to \mu^+\mu^-$. Check the gauge invariance of individual diagrams and that of their sum. Calculate $\Gamma(K_L \to \mu^+\mu^-)$