Flavour Physics, Spring 2014

Assignment 2

(Given: 16/04/2014, To be submitted: 30/04/2014)

1. Let M_U and M_D be the mass matrices of the up- and down-type quarks, respectively. They can be diagonalized through $M_U^d = U_L^\dagger M_U U_R$ and $M_U^d = D_L^\dagger M_D D_R$, respectively. Calculate the commutators

$$iK_L = [M_U M_U^\dagger, M_D M_D^\dagger] \ , \quad \text{and} \quad iK_R = [M_U^\dagger M_U, M_D^\dagger M_D]$$

   in terms of the quark masses and the CKM elements. Relate $Det(K_L)$ and $Det(K_R)$ to the Jarlskog invariant. (This is what Jarlskog did.)

2. Using the definitions $\sin\theta_{12} = \lambda$, $\sin\theta_{23} = A\lambda^2$. $\sin\theta_{13} = A\lambda^3 C$, determine the CKM matrix elements accurate up to $\mathcal{O}(\lambda^6)$, in terms of (λ, A, C, δ). Denote $C = \bar{\rho} - i\bar{\eta}$, and write down the elements in terms of $(\lambda, A, \bar{\rho}, \bar{\eta})$. (This is "extended" Wolfenstein parametrization.)

3. Determine the smallest angles of the two "squashed" unitarity triangles, β_s and β_K, in terms of these four quantities to leading order in λ. Prove the "unitarity relation" between the angles of two of the unitarity triangles:

$$\sin\beta_s = \left|\frac{V_{us}}{V_{ud}}\right|^2 \frac{\sin\beta \ \sin(\gamma + \beta_s)}{\sin(\beta+\gamma)} \ [1 + \mathcal{O}(\lambda^4)] \ . \qquad (1)$$

   (This is the "true" test of unitarity.)

4. Assume that the CPT conservation is broken to a small extent in the mixing of two pseudoscalars, so that $M_{11} \neq M_{22}$ and $\Gamma_{11} \neq \Gamma_{22}$. In terms of the two CPT-violating quantities $\delta_M \equiv M_{22} - M_{11}$ and $\delta_\Gamma \equiv \Gamma_{22} - \Gamma_{11}$, calculate
   (a) the mass difference Δm,
   (b) the lifetime difference $\Delta\Gamma$, and
   (c) the quantity q/p.

5. In a collider, a neutral pseudoscalar meson P and its antiparticle $\bar{P}$ are produced in a coherent state. Let one of the particles decay to f_1 and the other to f_2. Show that the "double time distribution" of this decay rate may be written in the form

$$\begin{aligned}\frac{\frac{d\Gamma}{dt}(P\bar{P} \to f_1 f_2)}{e^{-\Gamma|\Delta t|}N_{f_1 f_2}} &= C_\Gamma \cosh(\Delta\Gamma t/2) + C_m \cos(\Delta m t) \\ &+ S_\Gamma \sinh(\Delta\Gamma t/2) + S_m \sin(\Delta m t)\ .\end{aligned}$$

Write the coefficients $C_\Gamma, C_m, S_\Gamma, S_m$ in terms of the quantities

$$\begin{aligned} a_+ &\equiv \bar{A}_{f_1} A_{f_2} - A_{f_1}\bar{A}_{f_2} \\ a_- &\equiv \frac{p}{q} A_{f_1} A_{f_2} - \frac{q}{p}\bar{A}_{f_1}\bar{A}_{f_2}\ . \end{aligned}$$

Calculate the time distribution of $P \to f_1$ if $\bar{P} \to f_2$ is a flavour-specific mode. (This is "tagging").

6. Calculate the maximum energy that an electron can have, when it is a product of (i) $b \to c e^- \nu$, and (ii) $b \to u e^- \nu$. Argue how the difference in these energies can be used to identify a pure sample of $b \to u$ decays.

7. Draw **all** Feynman diagrams (up to one electroweak loop, no gluon loops) that contribute to the following amplitudes, and write the CKM factors involved next to each diagram.
(a) $B^0 \to \pi^0\pi^0$
(b) $B^0 \to K^0\bar{K}$
(c) $B^+ \to K^+\pi^0$
(d) $B^+ \to \pi^+\pi^0$

8. Find the leading power of λ present in the "direct" CP asymmetry

$$\mathcal{A}_{dir} = \frac{\Gamma(P^+ \to f^+) - \Gamma(P^- \to f^-)}{\Gamma(P^+ \to f^+) + \Gamma(P^- \to f^-)}$$

for (a) $K \to \pi\pi$, (b) $D \to K\pi$, (c) $B \to D\pi$ decays. Argue why B decays should typically show more CP asymmetry than D or K decays.

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