

# Mathematical Physics, Autumn 2008

## Assignment 1 (due Monday 1/9/2008)

1. If  $u, v$  are members of an inner product space, show that the “norm”, defined as  $||u|| \equiv \sqrt{\langle u, u \rangle}$  satisfies the following properties:
  - (a) Triangle inequality:  $||u + v|| \leq ||u|| + ||v||$ .
  - (b) Cauchy-Schwartz inequality:  $|\langle u, v \rangle| \leq ||u|| \cdot ||v||$ .  
(Hint: find  $||u - \lambda v||^2$ , where  $\lambda \equiv \langle u, v \rangle / \langle v, v \rangle$ .)
  - (c) Pythagoras theorem: If  $\langle u, v \rangle = 0$ , then  $||u||^2 + ||v||^2 = ||u + v||^2$ .
2. Show that:
  - (a) The completely antisymmetric  $2 \times 2$  matrix  $\epsilon_{ij} \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$  is a second rank tensor.
  - (b) The matrix  $\tilde{\epsilon}_{ij} \equiv \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}$  is not a tensor.
3. Estimate the area of the Golden Quadrilateral (whose four corners are Delhi, Mumbai, Kolkata and Chennai), by using the latitudes and longitudes of these metros. (The curvature of the Earth within the quadrilateral may be neglected.) What is the solid angle subtended by this quadrilateral at the center of the Earth ?
4. Arfken 1.9.11
5. Arfken 2.2.4 (Use results from 2.2.3)
6. Arfken 2.4.11

**Not to be submitted:**

1. The Levi-Civita tensor in three dimensions,  $\epsilon_{ijk}$ , is completely anti-symmetric, i.e. its elements change sign when any two indices are interchanged. Moreover,  $\epsilon_{123} = 1$ .
  - (a) Write down all the nonzero elements of  $\epsilon_{ijk}$ .
  - (b) Show that  $\epsilon_{ijk}$  is indeed a tensor.
  - (c) Give an argument to show that  $\sum_{i,j} \epsilon_{ijk} \epsilon_{ijl} = 2\delta_{kl}$ .
2. Arfken 1.8.15
3. Arfken 1.8.18
4. Arfken 1.11.10
5. Arfken 2.2.3
6. Arfken 2.4.13
7. Arfken 2.5.7
8. Arfken 2.5.17
9. Arfken 3.1.3