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Standard Model

$$SU(3)_C \times \underbrace{SU(2)_L \times U(1)_Y}$$

$$\boxed{\begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}}$$

L

u_R, d_R

e_R

R

L : 12 doublets of fermions

R : 21 fermions

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

Electromagnetism $U(1)_Q$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\mathcal{L} = \underbrace{-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}}_{\text{kinetic for gauge field } A} + \cancel{\bar{\Psi} \gamma \partial_\mu \Psi}$$

$$\underbrace{\bar{\Psi} \gamma^\mu \partial_\mu \Psi}_{\text{kinetic term for fermion}}$$

$$\underbrace{+ (-Q) \bar{\Psi} \gamma^\mu A_\mu \Psi}$$

$\rightarrow$ interaction

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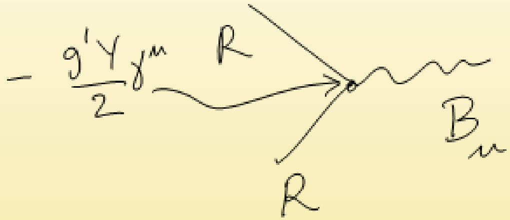
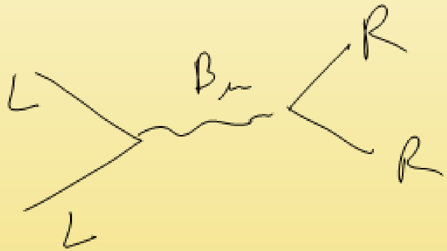
Interaction terms

$$-\frac{g'}{2}\,\bar{R}\,\gamma^{\mu} B_{\mu}\, Y\, R$$





Kinetic term:

Fermion. $\bar{L}\gamma^\mu i\partial_\mu L$

Higgs: $|i\partial_\mu\phi|^2 - V(\phi)$

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Weak hypercharge Y

$$Y\psi = \#\psi$$

$$\psi_e \to e^{i\frac{g'}{2}\alpha(x)Y}\psi_e$$

$Y_{e_L} \simeq -1$

$Y_{e_R} \simeq -2$

$$\psi_{e_L} \to e^{-i\frac{g'}{2}\alpha(x)}\psi_{e_L}$$

$$\psi_{e_R} \to e^{-i\frac{g'}{2}\alpha(x)\cdot 2}\psi_{e_R}$$

Electromagnetism

$$\psi \rightarrow e^{ie\alpha(x)Q}\psi$$

$$\psi_{e^-} \rightarrow e^{-ie\alpha(x)}\psi_{e^-}$$

$$\psi_{e^+} \rightarrow e^{+ie\alpha(x)}\psi_{e^+}$$

$$\mathcal{L}_{kinetic}: \bar{\psi}\gamma^\mu i\partial_\mu\psi$$

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$\mathcal{L}_{kinetic}$: $\quad \bar{\Psi}\gamma^\mu i D_\mu \Psi$ ✓

$$\cancel{D_\mu = \partial_\mu - e(\partial_\mu \alpha)}$$

$$D_\mu = \partial_\mu + ieA_\mu Q$$

$$A_\mu \rightarrow A_\mu - e(\partial_\mu \alpha)$$ ✓

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Gauge boson mass

$$(\Box - m^2) B = 0$$

$$\mathcal{L} = \frac{1}{2}\left[(\partial_\mu B)^*(\partial^\mu B) - m^2 B^2\right]$$

$m^2 B_\mu B^\mu$ not invariant under $U(1)$

$\Downarrow$

$m^2 = 0$

$$-m^2 B^2$$

$$-m^2 B_\mu B^\mu$$

$$\hookrightarrow -m^2\left(B_\mu - \frac{g'}{2}(\partial_\mu \alpha)\right)\left(B^\mu - \frac{g'}{2}(\partial^\mu \alpha)\right)$$

Kinetic term for B_μ

$$\underline{(\partial_\mu B_\nu)^*(\partial_\mu B_\nu)}$$

↳ involves derivatives (2 derivatives)

↳ invariant under $B_\mu \to B_\mu - \frac{g'}{2}(\partial_\mu \alpha)$

$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \Rightarrow B_{\mu\nu} B^{\mu\nu} \to$ gauge invariant

$$\boxed{(\partial_\mu B_\nu - \partial_\nu B_\mu)(\partial^\mu B^\nu - \partial^\nu B^\mu)}$$

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$$\begin{pmatrix} & \end{pmatrix} = a_0 I + \underbrace{\vec{a} \cdot \vec{\sigma}}$$

SU(2):

$$\begin{pmatrix} & \\ & \end{pmatrix} \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix}$$

$$e^{i \frac{g}{2} \vec{\alpha} \cdot \vec{\tau}}$$

$$e^{i \frac{g'}{2} \alpha Y}$$

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Kinetic term for $\vec{W}$

$$\boxed{\vec{W}_{\mu\nu} = \partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu - \underbrace{g(\vec{W}_\mu \times \vec{W}_\nu)}_{g\,\varepsilon_{ijk} W^j_\mu W^k_\nu}}$$

$W_{\mu\nu} \cdot W^{\mu\nu}$ invariant ? ✓