Kinetic

$$|\partial_\mu B_\nu|^2 \qquad \oplus \qquad U(1)$$

$$B_\mu \to B_\mu - \frac{g'}{2}(\partial_\mu \alpha)$$

$$\partial_\mu B_\nu - \partial_\nu B_\mu \longrightarrow \partial_\mu B_\nu - \cancel{\frac{g'}{2}\partial_\mu\partial_\nu\alpha}$$

$$- \left[\partial_\nu B_\mu - \cancel{\frac{g'}{2}\partial_\nu\partial_\mu\alpha}\right]$$

$$\downarrow$$

$$B_{\mu\nu}$$

$$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

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$$-\frac{g}{2}\,\bar{L}\,\gamma^\mu\,\vec{W}_\mu\cdot\vec{\tau}\,L \qquad L \to \begin{pmatrix}\nu_e\\ e\end{pmatrix}$$

$$-\frac{g}{2}\,(\bar{\nu}_e\ \ \bar{e})\,\underbrace{\vec{W}\cdot\vec{\tau}}_{W_1\tau_1 + W_2\tau_2 + W_3\tau_3}\begin{pmatrix}\nu_e\\ e\end{pmatrix}$$

$$-\frac{g}{2}\left[(\bar{\nu}_e\ \ \bar{e})\,W_1\tau_1\begin{pmatrix}\nu_e\\ e\end{pmatrix} + (\bar{\nu}_e\ \ \bar{e})\,W_2\tau_2\begin{pmatrix}\nu_e\\ e\end{pmatrix} + (\bar{\nu}_e\ \ \bar{e})\,W_3\tau_3\begin{pmatrix}\nu_e\\ e\end{pmatrix}\right]$$

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$$W_1 \tau_1 + W_2 \tau_2 + W_3 \tau_3$$

$$= W_1 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + W_2 \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + W_3 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & W_1 - iW_2 \\ W_1 + iW_2 & 0 \end{pmatrix} + \cancel{W_3} \begin{pmatrix} W_3 & 0 \\ 0 & -W_3 \end{pmatrix}$$

$$= \cancel{\frac{1}{\sqrt{2}}} \sqrt{2} \begin{pmatrix} & W^+ \\ W^- & \end{pmatrix} + W_3 \tau_3$$

$$= W^+ \tau^+ + W^- \tau^- + W_3 \tau_3$$

$$\tau^+ = (\tau^1 + i\tau^2)/\sqrt{2}$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

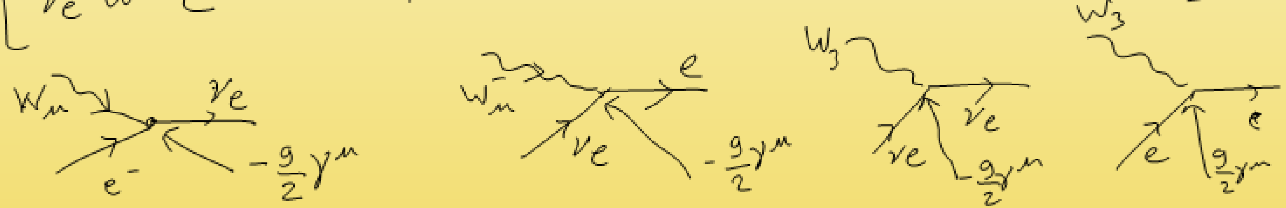
$$= \sqrt{2} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$-\frac{g}{2}(\bar{\nu}_e\ \bar{e})\left(W^+_\mu\tau^+ + W^-\tau^- + W_3\tau_3\right)\begin{pmatrix}\nu_e\\ e\end{pmatrix}$$

$$(\bar{\nu}_e\ \bar{e})W^+\begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}\nu_e\\ e\end{pmatrix} + (\bar{\nu}_e\ \bar{e})W^-\begin{pmatrix}0&0\\1&0\end{pmatrix}\begin{pmatrix}\nu_e\\ e\end{pmatrix} + (\bar{\nu}_e\ \bar{e})W_3\begin{pmatrix}1&\\&-1\end{pmatrix}\begin{pmatrix}\nu_e\\ e\end{pmatrix}$$

$$(\bar{\nu}_e\ \bar{e})W^+\begin{pmatrix}e\\ 0\end{pmatrix} + (\bar{\nu}_e\ \bar{e})W^-\begin{pmatrix}0\\ \nu_e\end{pmatrix} + (\bar{\nu}_e\ \bar{e})W_3\begin{pmatrix}\nu_e\\ -e\end{pmatrix}$$

$$-\frac{g}{2}\left[\bar{\nu}_e W^+ e + \bar{e}W^-\nu_e + \bar{\nu}_e W_3\nu_e - \bar{e}W_3 e\right]$$



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$\mathcal{L} \longrightarrow$ max dimension $\underline{4}$

fermion : $\frac{3}{2}$ bosons : 1

$\mathcal{L} \to \bar{\Psi} \gamma^\mu i \partial_\mu \Psi$ $\mathcal{L} \to \partial_\mu \varphi \partial^\mu \varphi$

$\int d^4x \, \mathcal{L}$

$$\mathcal{L} \to \frac{1}{M_\varphi^2} \varphi\varphi\varphi\varphi\varphi\varphi$$

$$\mu^2 \varphi\varphi$$

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Invariance under $U(1)_Y$

$\mathcal{L} \to \psi_1 \psi_2 \psi_3$

$\downarrow$

$e^{i\frac{g'}{2}\alpha Y}\psi_1 \, e^{i\frac{g'}{2}\alpha Y}\psi_2 \, e^{i\frac{g'}{2}\alpha Y}\psi_3$

$e^{i\frac{g'}{2}\alpha(y_1+y_2+y_3)}\psi_1\psi_2\psi_3 \xrightarrow{U(1)} y_1+y_2+y_3=0$

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Invariance under $SU(2)_L$

$\psi_\alpha \psi_\beta \psi_\gamma$

$\to \underbrace{e^{i\frac{g}{2}\overline{\alpha}\cdot\overline{\tau}}}\psi_\alpha \, e^{i\frac{g}{2}\overline{\alpha}\cdot\overline{\tau}}\psi_\beta \, e^{i\frac{g}{2}\overline{\alpha}\cdot\overline{\tau}}\psi_\gamma$

$e^{i\frac{g}{2}\overline{\alpha}\cdot(\overline{\tau}_\alpha+\overline{\tau}_\beta+\overline{\tau}_\gamma)}\psi_\alpha\psi_\beta\psi_\gamma$

$\frac{1}{2} \quad \frac{1}{2} \quad 0$

$0 \quad \frac{1}{2} \quad 0$ $\qquad 1 \quad \frac{1}{2} \quad \frac{1}{2}$

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$$\bar{e}_L \, \phi \, e_R$$

$(2, +1)$ $\quad (2, 1) \quad (1, -2)$

$\frac{1}{2} \quad \frac{1}{2}$

0

$$\bar{u}_L \, \phi^c \, d_R$$

$(2, -\frac{1}{3}) \quad (2, -1) \quad (1, 4/3)$

$$\bar{d}_L \, \phi \, d_R \quad \checkmark$$

$(2, -\frac{1}{3}) \quad (2, 1) \quad (1, -2/3)$

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Mass terms

$\bar{\psi}_L \psi_R \qquad \bar{\psi}_R \psi_L$

$$\psi = \psi_L + \psi_R$$

$$\psi_L \to \left(\frac{1-\gamma^5}{2}\right)\psi = L\psi \qquad \psi_R \to \left(\frac{1+\gamma^5}{2}\right)\psi = R\psi$$

$$m\bar{\psi}\psi = m(\bar{\psi}_L + \bar{\psi}_R)(\psi_L + \psi_R)$$

$$= m\left(\cancel{\bar{\psi}_L \psi_L} + \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L + \cancel{\bar{\psi}_R\psi_R}\right)$$

$$\psi^\dagger\left(\frac{1-\gamma^5}{2}\right)\gamma^0\left(\frac{1-\gamma^5}{2}\right)\psi \to \psi^\dagger\gamma^0\underbrace{\left(\frac{1+\gamma^5}{2}\right)\left(\frac{1-\gamma^5}{2}\right)}_{0}\psi$$

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Mass for electron

$$\lambda_e \bar{e}_L \varphi e_R + \lambda_e^* \bar{e}_R \varphi^* e_L$$

$$\lambda_e v \, \underline{(\bar{e}_L e_R)} + \lambda_e^* v \, \underline{(\bar{e}_R e_L)}$$

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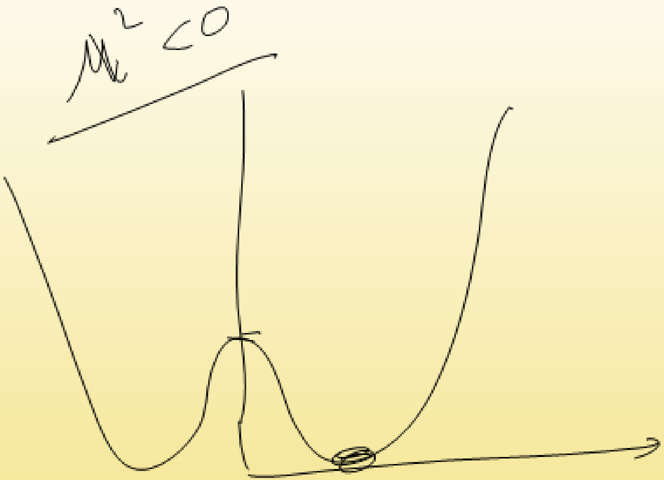
$V(\phi)$



$$V = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

$$V' \longrightarrow \mu^2 \phi + 2\lambda (\phi^\dagger \phi)\phi \Rightarrow$$

V_{min} at

$$\phi^\dagger \phi = -\frac{\mu^2}{2\lambda}$$

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$\mathcal{L} \to$ does not break $SU(2) \times U(1)$

Vacuum $\to$ breaks $SU(2) \times U(1)$

$\to$ does not break

$$\boxed{Q} \leftarrow T_3 + \frac{Y}{2}$$

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$U(1)_Q$

$$\psi \longrightarrow e^{i\alpha Q}\,\psi$$