



$$k \rightarrow \mu v_\mu$$

$$\quad \quad \quad \hookrightarrow e v_e v_\mu$$

double ratio

$$= \frac{(v_\mu/v_e)_{\text{detected}}}{(v_\mu/v_e)_{\text{expected}}} \frac{dN}{d \log E} = \frac{E}{E'} \frac{dN}{dE}$$

$$\nu_\alpha = \cos\theta \nu_1 + \sin\theta \nu_2$$

$$\begin{aligned} |\nu_\alpha(t)\rangle &= \cos\theta \cdot |\nu_1(t)\rangle + \sin\theta |\nu_2(t)\rangle \\ &= \cos\theta |\nu_1(0)\rangle e^{-ipt} e^{-\frac{im_1^2 t}{2E}} \\ &\quad + \sin\theta |\nu_2(0)\rangle e^{-ipt} e^{-\frac{im_2^2 t}{2E}} \end{aligned}$$

$$|\langle \nu_\alpha(0) | \nu_\alpha(t) \rangle|^2 \leftarrow P_{\nu_\alpha \rightarrow \nu_\alpha}$$

$$\nu_\alpha(0) = \cos\theta |\nu_1(0)\rangle + \sin\theta |\nu_2(0)\rangle$$

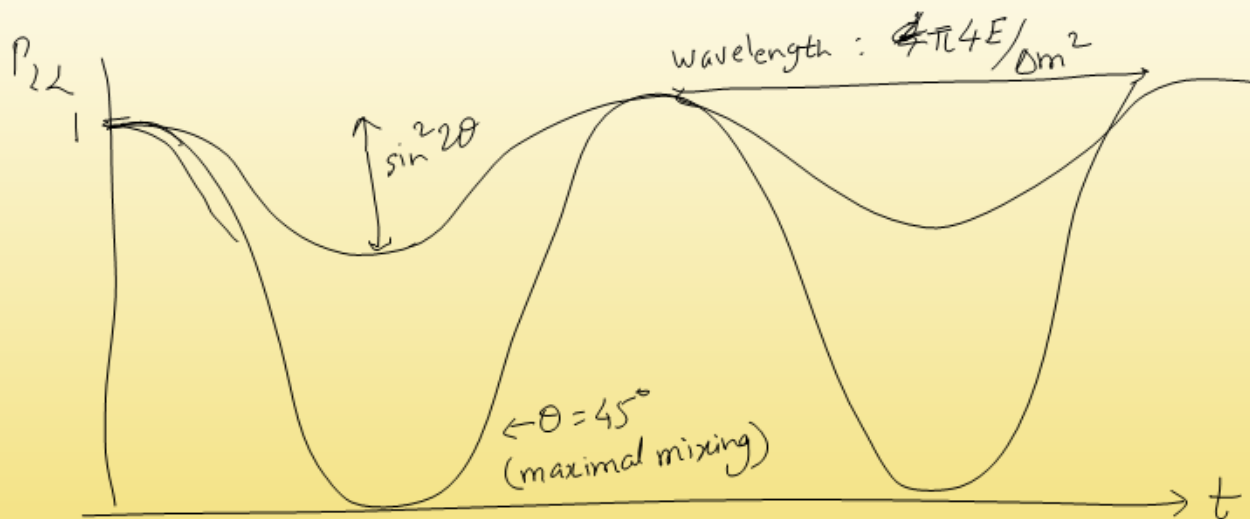
$$\langle \nu_\alpha(0) | \nu_\alpha(t) \rangle = \cos^2\theta e^{-ipt} e^{-\frac{im_1^2 t}{2E}} + \sin^2\theta e^{-ipt} e^{-\frac{im_2^2 t}{2E}}$$

$$\begin{aligned}
 |\langle 1 | \rangle|^2 &= \cos^4 \theta + \sin^4 \theta + 2 \cos^2 \theta \sin^2 \theta \cos\left(\frac{(m_2^2 - m_1^2)t}{2E}\right) \\
 &= \cos^4 \theta + \sin^4 \theta + 2 \sin^2 \theta \cos^2 \theta \cos\left(\frac{\Delta m^2 t}{2E}\right) \\
 &= \underbrace{\cos^4 \theta + \sin^4 \theta + 2 \sin^2 \theta \cos^2 \theta}_{=1} - 2 \sin^2 \theta \cos^2 \theta + 2 \sin^2 \theta \cos^2 \theta \cos\left(\frac{\Delta m^2 t}{2E}\right) \\
 &= 1 - \frac{\sin^2 2\theta}{2} \left[1 - \cos\left(\frac{\Delta m^2 t}{2E}\right)\right]
 \end{aligned}$$

$$P_{\nu_e \rightarrow \nu_e} = 1 - \sin^2 2\theta \cdot \sin^2\left(\frac{\Delta m^2 t}{4E}\right)$$

Survival probability

$$P_{\alpha\alpha} = 1 - \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 t}{4E} \right)$$



expt  $\longrightarrow \Delta m^2, \sin^2 2\theta$

$$\cos\theta = -1, \text{ low } E$$

$$P_{\alpha\alpha} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 t}{4E}\right)$$

↓

$$1 - \frac{1}{2} \sin^2 2\theta \sim \frac{1}{2} \text{ (observation)}$$