$$\nu_1 \longrightarrow e^{-iHt} \nu_1$$

$$\sim e^{-ipt} e^{-\frac{i m_1^2}{2E} t} \nu_1 \; e^{-i \frac{m_2^2 + m_1^2}{2E} t}$$

$$\nu_2 \sim e^{-ipt} e^{-\frac{i m_2^2}{2E} t} \nu_2 \; e^{-i \frac{m_2^2 + m_1^2}{2E} t}$$

$$\underline{P_{e\mu}} = \left| \langle \nu_\mu | \nu(t) \rangle \right|^2$$

$$\underline{\begin{pmatrix} 3 & 5 \\ 6 & 4 \end{pmatrix}} \sim \underline{\begin{pmatrix} 1 & 5 \\ 6 & 2 \end{pmatrix}} + 2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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$$i \frac{d}{dt} \Psi = H \Psi$$

$$H \text{ const} \Rightarrow \Psi(t) = e^{-iHt} \Psi(0)$$

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \exp\left[-i \begin{pmatrix} -\Delta & 0 \\ 0 & \Delta \end{pmatrix} t\right] \begin{pmatrix} \nu_1(0) \\ \nu_2(0) \end{pmatrix}$$

$$= \begin{pmatrix} e^{i\Delta t} & 0 \\ 0 & e^{-i\Delta t} \end{pmatrix} \begin{pmatrix} \nu_1(0) \\ \nu_2(0) \end{pmatrix}$$

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$$\nu_1(t) = e^{i\Delta t}\,\nu_1(0)$$

$$\nu_2(t) = e^{-i\Delta t}\,\nu_2(0)$$

$$\nu(0) = \nu_\mu = \cos\theta\,\nu_1(0) + \sin\theta\,\nu_2(0)$$

$$\nu(t) = \cos\theta\,\nu_1(t) + \sin\theta\,\nu_2(t)$$

$$= \cos\theta\,e^{i\Delta t}\,\nu_1(\cancel{t}) + \sin\theta\,e^{-i\Delta t}\,\nu_2(\cancel{t})$$

$$\langle \nu_\mu | \nu(t) \rangle = \cos^2\theta\,e^{i\Delta t} + \sin^2\theta\,e^{-i\Delta t}$$

$$P_{\mu\mu} = |\langle \nu_\mu | \nu(t) \rangle|^2 = \cos^4\theta + \sin^4\theta + 2\cos^2\theta\sin^2\theta\cos(2\Delta t)$$

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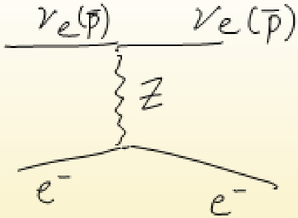
$$P_{\mu\mu} = 1 - \sin^2 2\theta \, \sin^2(\Delta t)$$

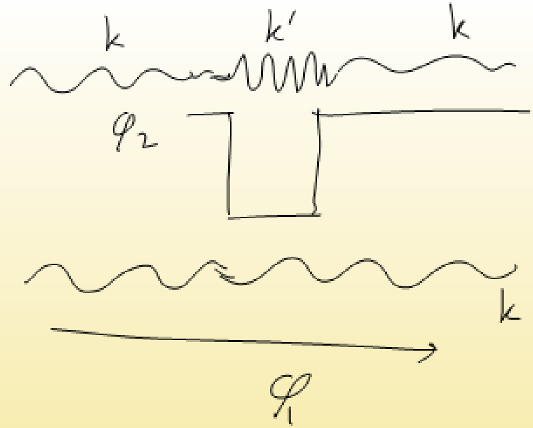
$$\begin{pmatrix} \nu_1(t) \\ \nu_2(t) \end{pmatrix} = \begin{pmatrix} e^{i\Delta t} & 0 \\ 0 & e^{-i\Delta t} \end{pmatrix} \begin{pmatrix} \nu_1(0) \\ \nu_2(0) \end{pmatrix}$$

$$U^+ \begin{pmatrix} \nu_\alpha(t) \\ \nu_\beta(t) \end{pmatrix} = \begin{pmatrix} e^{i\Delta t} & 0 \\ 0 & e^{-i\Delta t} \end{pmatrix} U^+ \begin{pmatrix} \nu_\alpha(0) \\ \nu_\beta(0) \end{pmatrix}$$

$$\begin{pmatrix} \nu_\alpha(t) \\ \nu_\beta(t) \end{pmatrix} = \underbrace{U \begin{pmatrix} e^{i\Delta t} & 0 \\ 0 & e^{-i\Delta t} \end{pmatrix} U^+}_{H_f} \begin{pmatrix} \nu_\alpha(0) \\ \nu_\beta(0) \end{pmatrix}$$

Hamiltonian in flavour basis $\longrightarrow H_f$ $\qquad e^{-iH_f t}$





① Some interactions

② no change in momentum

(forward scattering)

$\downarrow$

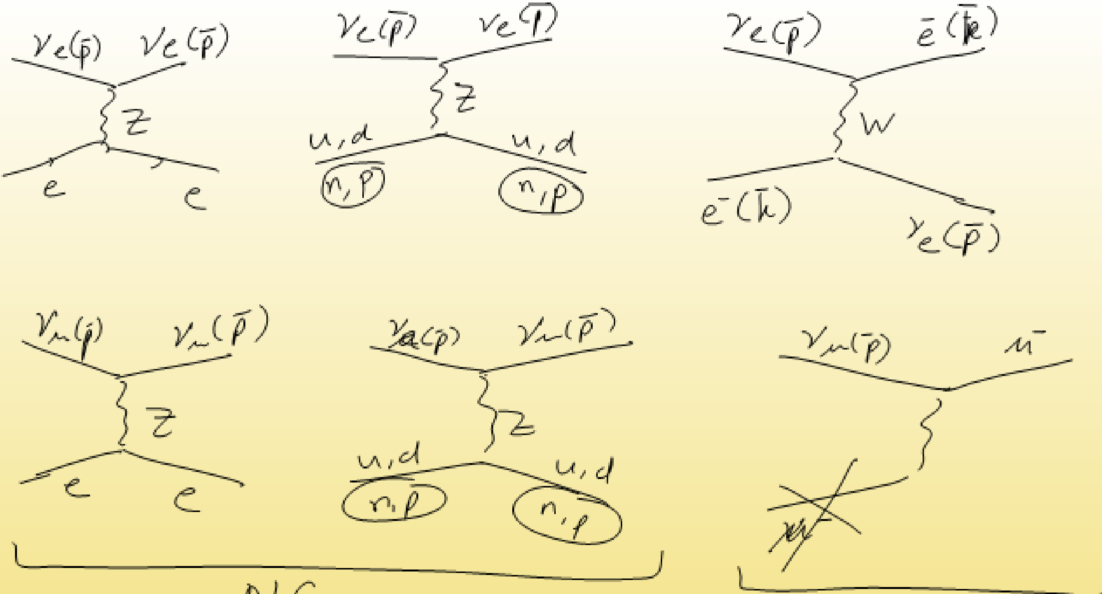
ν experience a "potential"

$$\varphi_1 \neq \varphi_2$$

$$e^{i\frac{m^2}{2E}t}$$

ν_e :

ν_μ :



NC

ν_e , ν_μ, ν_τ

$$\boxed{V_N = -\frac{G_F}{\sqrt{2}} N_n}$$

CC

ν_e

$$\boxed{V_c = \sqrt{2}\, G_F N_e}$$

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$$\begin{pmatrix} a & b \\ b & c \end{pmatrix} \qquad \tan 2\theta = \frac{2b}{c-a}$$

$$\tan 2\theta_m = \frac{2\Delta \sin 2\theta}{2\Delta \cos 2\theta - V_C}$$

$$\Delta_m = \sqrt{\left(\Delta \cos 2\theta - \frac{V_c}{2}\right)^2 + \left(\Delta \sin 2\theta\right)^2}$$

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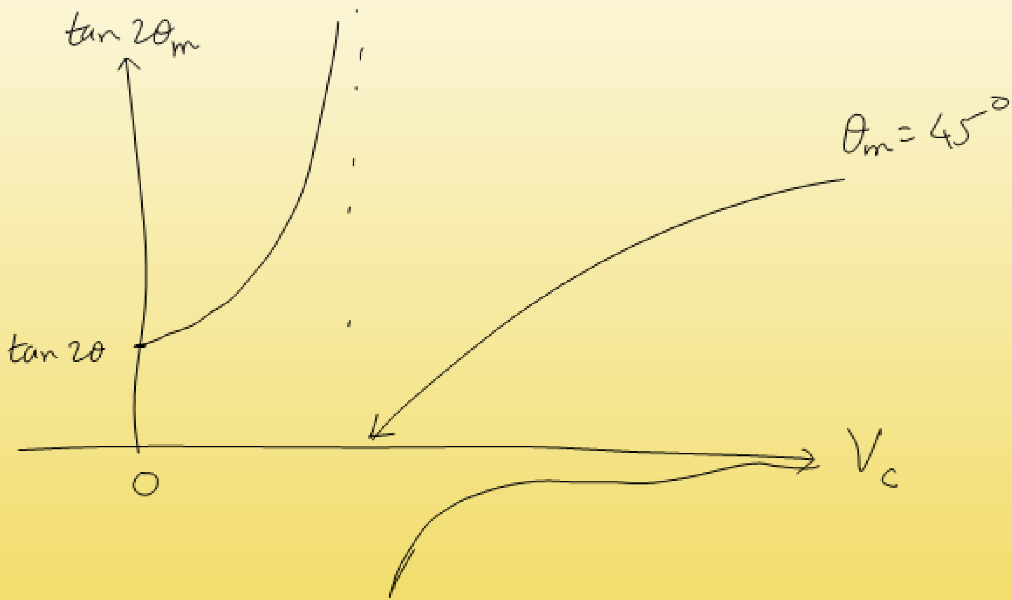
$$\tan 2\theta_m = \frac{2\Delta \sin 2\theta}{2\Delta \cos 2\theta - V_c}$$

$V_c = \sqrt{2} G_F N_e$



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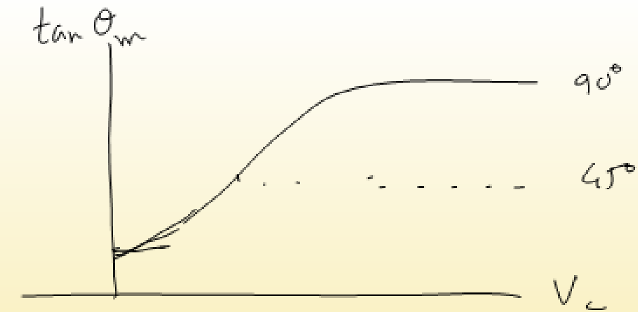
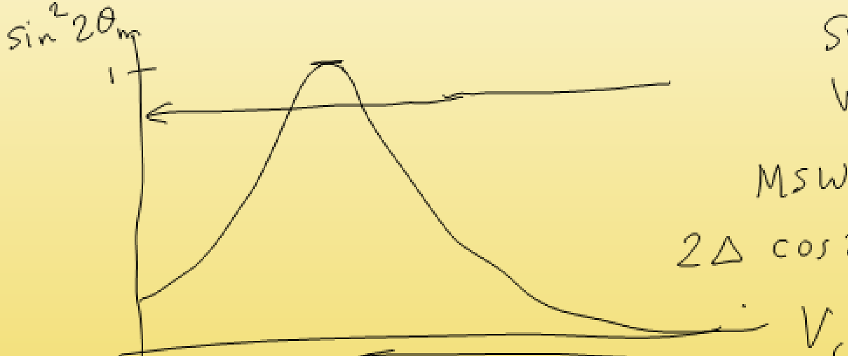


Mikheyev
Smirnov
Wolfenstein



MSW resonance

$$2\Delta \cos 2\theta = V_c$$

$$\frac{\Delta m^2}{2E} \cos 2\theta = \sqrt{2}\, G_F N_e$$

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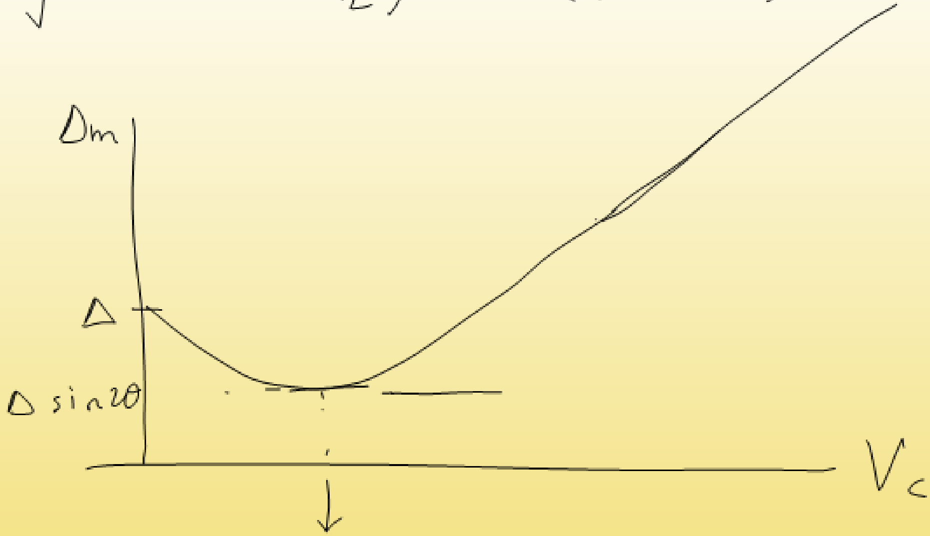
$$\Delta_m = \sqrt{\left(\Delta\cos 2\theta - \frac{V_c}{2}\right)^2 + \left(\Delta\sin 2\theta\right)^2}$$



$$\Delta\cos 2\theta = \frac{V_c}{2}$$ ← MSW resonance

$$\boxed{2\Delta\cos 2\theta = V_c}$$

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Atm:

$$P_{\mu\mu} = 1 - \sin^2 2\theta \ \sin^2(\Delta t)$$

$P_{\mu\mu}$ (sterile)

θ_m

Δ_m