Relic Neutrino Density

The Cosmic Neutrino Background

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Outline of the talk

1. The Expanding Universe
2. A recap of Equilibrium Thermodynamics
3. Cosmic Backgrounds

The FRW metric

- We are familiar with the Minkowski metric from special relativity

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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- An expanding universe which is flat and therefore locally looks Euclidean can be described by the following metric, known as the Friedmann-Robertson-Walker metric

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a^2(t) & 0 & 0 \\ 0 & 0 & a^2(t) & 0 \\ 0 & 0 & 0 & a^2(t) \end{pmatrix}$$

Evolution of Energy in the FRW metric

- For the case of a perfect isotropic fluid

$$T^{\mu}_{\nu} = \begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

Evolution of Energy in the FRW metric

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- Since T^{μ}_{ν} is a conserved quantity, its covariant derivative must vanish:

$$T^{\mu}_{\nu;\mu} = \frac{\partial T^{\mu}_{\nu}}{\partial x^{\mu}} + \Gamma^{\mu}_{\alpha\mu} T^{\alpha}_{\nu} - \Gamma^{\alpha}_{\nu\mu} T^{\mu}_{\alpha} = 0$$

Evolution of Energy-2

- If we look at the $\nu = 0$ component, and use the fact that we are looking at an isotropic fluid, we get

$$-\frac{\partial \rho}{\partial t} - \Gamma^{\mu}_{0\mu}\rho - \Gamma^{\alpha}_{0\mu} T^{\mu}_{\alpha} = 0$$

- If we calculate the Christoffel symbols for the FRW metric we can rewrite the above equation as

$$\frac{\partial \rho}{\partial t} + \frac{\dot{a}}{a}(3\rho + 3P) = 0$$

Scaling of Radiation Density

- For radiation, pressure and density are related by $P = \rho/3$, and so we can write
$$\frac{\partial \rho_r}{\partial t} + \frac{\dot{a}}{a} 4\rho_r = a^{-4} \frac{\partial(\rho_r a^4)}{\partial t} = 0$$
- So the energy density of radiation scales as a^{-4}.

Outline of the talk

Number Density, Energy Density and Pressure

- We define the number density, energy density and the pressure of a gas in thermal equilibrium at temperature T in the following manner

$$
\begin{aligned}
n &= \frac{g}{(2\pi)^3}\int f(\vec{p})d^3\vec{p} \\
\rho &= \frac{g}{(2\pi)^3}\int E(\vec{p})f(\vec{p})d^3\vec{p} \\
P &= \frac{g}{(2\pi)^3}\int \frac{|\vec{p}|^2}{3E}f(\vec{p})d^3\vec{p}
\end{aligned}
$$

  where g gives the count of the internal degrees of freedom and $f(\vec{p})$ is the distrbution function which is either Bose-Einstein or Fermi Dirac.

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- Integrals of this form are well known and can be evaluated as

$$\int_0^\infty \frac{x^n dx}{e^x - 1} = \Gamma(n+1)\zeta(n+1)$$

where $\zeta(n)$ is the Riemann Zeta function.

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- These integrals can also be evaluted in terms of the Riemann Zeta functions.

- For bosons, we find

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\begin{aligned}
\rho &= \frac{g\pi^2}{30} T^4 \\
n &= \frac{g\zeta(3)}{\pi^2} T^3 \\
P &= \rho/3
\end{aligned}
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The Numbers

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$$\begin{aligned} \rho &= \frac{g\pi^2}{30} T^4 \\ n &= \frac{g\zeta(3)}{\pi^2} T^3 \\ P &= \rho/3 \end{aligned}$$

- For fermions,

$$\begin{aligned} \rho &= \frac{7}{8}\frac{g\pi^2}{30} T^4 \\ n &= \frac{3}{4}\frac{g\zeta(3)}{\pi^2} T^3 \\ P &= \rho/3 \end{aligned}$$

- From the above relations we see that $\frac{dP}{dT} = \frac{\rho+P}{T}$

Entropy density

- $$TdS = d(\rho V) + PdV = d[(\rho + P)V] - VdP$$

  Using the earlier relation we get

  $$dS = d\left[\frac{(\rho + P)V}{T} + constant\right]$$

Entropy density

- $$TdS = d(\rho V) + PdV = d[(\rho + P)V] - VdP$$
  Using the earlier relation we get
  $$dS = d\left[\frac{(\rho + P)V}{T} + constant\right]$$
- If we ignore this constant, which is dependent on the chemical potential μ, we can write the entropy density
  $$s = \frac{S}{V} = \frac{\rho + P}{T}$$

Outline of the talk

The Photon Background

- For photons, there are two degrees of internal freedom (the two spins). Hence putting it into the formula for number density and taking $T_\gamma = 2.7K = 2.34 \times 10^{-4} eV$, we get

$$n_\gamma = \frac{2\zeta(3)}{\pi^2} T_\gamma^4 \approx 410/cc$$

Decoupling of Neutrinos

- At early times, neutrinos were kept in equilibrium through charge current and neutral current interactions between themselves and the other species. These reactions typically have a cross section of $\sigma_F^2 s \approx G_F^2 T^2$. We have also seen that the number density goes at $n \approx T^3$. So the interaction rate per neutrino is

$$\Gamma_{int} \approx G_F^2 T^5$$

Therefore

$$\frac{\Gamma_{int}}{H} \approx \frac{G_F^2 T^5}{T^2/m_{pl}} \approx \left(\frac{T}{1MeV}\right)^3$$

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- So neutrinos decouple from the rest of the particles at about 1 MeV.

Annihilation Heating

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- This means that the present temperature of neutrinos is different from the temperature of photons.

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Before Annihilation

- Since we assume that the universe is expanding adiabatically, $S = sV = constant$, and so the entropy density scales as a^{-3}.
- For bosons, the contribution per degree of freedom to the entropy density is $2\pi^2 T^3/45$ while for fermions it is $7/8$ of this factor.
- There are two internal degrees for photons, 2 for electrons, 2 for positrons, 3 for neutrinos and 3 for antineutrinos. Therefore

$$s(a_1) = \frac{2\pi^2}{45} T_1^3 \left[2 + \frac{7}{8}(2 + 2 + 3 + 3)\right] = \frac{43\pi^2}{90} T_1^3$$

.

After Annihilation

- After annihilation, the electrons and positrons are no longer in equilibrium with the ambient radiation. So

$$s(a_2) = \frac{2\pi^2}{45}\left[2T_\gamma^3 + \frac{7}{8}6T_\nu^3\right]$$

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- Since $S_1 = S_2$ we get

$$\frac{43}{2}(a_1 T_1)^3 = 4\left[\left(\frac{T_\gamma}{T_\nu}\right)^3 + \frac{21}{8}\right](T_\nu(a_2)a_2)^3$$

Neutrino Temperature

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- Therefore, simplifying the equation on the last slide we find that

$$\frac{T_\nu}{T_\gamma} = \left(\frac{4}{11}\right)^{1/3}$$

The Background Neutrino Density

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- For neutrinos, we have

$$\begin{aligned} n_\nu &= \frac{3}{4}\frac{6\zeta(3)}{\pi^2}T_\nu^3 \\ &= \frac{9}{4}\left[\frac{2\zeta(3)}{\pi^2}T_\gamma^3\right]\left(\frac{T_\nu}{T_\gamma}\right)^3 \\ &\approx 410\left(\frac{9}{4}\right)\left(\frac{4}{11}\right) \approx 335/cc \end{aligned} \tag{1}$$

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The Background Neutrino Energy Density

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- While this works for the number density, the fact that neutrinos do have mass has to be taken into account while calculating the energy density when the universe has cooled sufficiently.
- The current neutrino temperature is of the order of $10^{-4} eV$, and the rest mass of the neutino is expected to be about one or two orders of magnitude above it.

The Background Neutrino Energy Density

- In this non-relativistic regime, the energy density of each species of neutrinos is given by $\rho_\nu = m_\nu n_\nu$. We have already calculated the number density of each species of neutrino to be about $112 cm^{-3}$.

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- It is conventional to quote the energy density of each species of matter or radiation in terms of the critical energy density $\rho_{cr} = 1.88 h^2 \times 10^{-29} g cm^{-3}$.
- Putting the numbers together, we get

$$\Omega_\nu = \frac{m_\nu}{94 h^2 eV}$$

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References

- *Modern Cosmology* Scott Dodelson
- *The Early Universe* Kolb and Turner