Neutrino Physics: Assignment 1

(Given 16/02/2009, To be submitted 02/03/2009)

1. Show that the Hamiltonian $H = \alpha_i p_i + \beta m$ corresponds to a spin-1/2 particle. (You may use the Dirac-Pauli representation, but try without using it, use only the commutation relations among α_i and β.)

2. Find the solutions of the Dirac equation in the Weyl representation. Determine the quantum numbers that distinguish them from one another.

3. Show the following properties of γ matrices (if possible, without using the Dirac-Pauli representation)

   - $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$
   - $\{\gamma^\mu, \gamma^5\} = 0$
   - $(1 \pm \gamma^5)/2$ are orthogonal projection operators

4. Show that under Lorentz transformations, $\bar{\psi}\gamma^5\psi$ transforms as a pseudoscalar, $\bar{\psi}\gamma^\mu\psi$ trasforms as a vector, and $\bar{\psi}\gamma^\mu\gamma^5\psi$ as an axial vector.

5. Calculate the lifetime of muon, by computing the rate of $\mu^- \to e^-\bar{\nu}_e\nu_\mu$ from first principles. (Follow a procedure similar to the nuclear beta decay calculation).

6. The mass of ν_μ can in principle be determined by measuring the momentum of muon (in the rest frame of pion) produced in the decay $\pi^+ \to \mu^+\nu_\mu$. To what accuracy should the momentum of muon be measured, if the mass of ν_μ is to be measured to an accuracy of 1 eV ? Assume that the masses of π^+ and μ^+ are known to infinite precision.