Neutrino Physics: Assignment 2

(Given 16/03/2009, To be submitted 30/03/2009)

1. Let two neutrino flavours, ν_α and ν_β, mix to form two mass eigenstates ν_1 and ν_2, such that $\nu_\alpha = \cos\theta\ \nu_1 + \sin\theta\ \nu_2$, $\nu_\beta = -\sin\theta\ \nu_1 + \cos\theta\ \nu_2$. Let $\nu(0) = \nu_\alpha$. Find the probability of detecting a ν_β after the neutrino has travelled a distance L, when

   (a) The heavier neutrino ν_2 decays with a lifetime τ to particles that remain undetected.

   (b) The heavier neutrino ν_2 decays with a lifetime τ to ν_1 and other particles that remain undetected.

   In both cases, plot the conversion probability as a function of L, and find its value at L much greater than the coherence length. (Use any appropriate values for Δm^2 and θ.)

2. In the absence of neutrino mixing, the Kurie plot for nuclear beta decay has the form

$$K(p_e) \equiv \left[\frac{1}{p_e^2}\frac{d\Gamma}{dp_e}\right]^{1/2} = C\left[(E_0 - E_e)\sqrt{(E_0 - E_e)^2 - m_{\nu_e}^2}\right]^{1/2}$$

   Calculate $K(p_e)$ if ν_e mixes with ν_μ through $\nu_e = \cos\theta\ \nu_1 + \sin\theta\ \nu_2$, $\nu_\mu = -\sin\theta\ \nu_1 + \cos\theta\ \nu_2$. Assume that the detector is very close to the decaying nucleus. Show the new Kurie plot near the endpoint, using appropriate values of parameters that will bring out main features of the plot.

3. For atmospheric neutrinos, for ν_μ-ν_τ mixing,

   (a) Plot $P_{\mu\mu}$ as a function of the zenith angle Θ for three values of energy: $E = 0.2, 2, 20$ GeV (on the same plot, if possible). Show the numerical values on both the axes explicitly.

   (b) Plot the up-down asymmetry as a function of energy. The "up" events are defined as those with $\cos\Theta < -0.2$ and the "down" events are those with $\cos\Theta > 0.2$. You may do the integrals numerically.

   Use the paremeter values $\Delta m^2 = 2.4 \times 10^{-3}$ eV^2 and $\theta = 45°$.

4. Repeat the above problem for ν_μ-ν_s mixing. Use the same values of Δm^2 and θ. Take the density of the Earth to be constant, equal to 5 g/cc. Compare the results with those of the previous problem.

5. Consider two-neutrino mixing between ν_e and ν_μ, with $\nu_e = \cos\theta\ \nu_1 + \sin\theta\ \nu_2$, $\nu_\mu = -\sin\theta\ \nu_1 + \cos\theta\ \nu_2$. Let the neutrinos travel through matter, so that they experience a matter potential A. Plot the effective mixing angle in matter θ_m and the energy eigenvalues $m_1^2/(2E)$ and $m_2^2/(2E)$ (for $E = 1, 10$ MeV) as a function of A, when $m_1 = 0.10000$ eV, $m_2^2 = m_1^2 + \Delta m^2$, and $\theta = 10^\circ$, with

   (a) $\Delta m^2 = 10^{-4}\ \text{eV}^2$

   (b) $\Delta m^2 = -10^{-4}\ \text{eV}^2$

   Show the numerical values on both the axes explicitly.