Neutrino Physics: Assignment 4

(Given 20/05/2009, To be submitted 27/05/2009)

1. Consider the seesaw mechanism with three left handed neutrinos and n right handed neutrinos. Let the neutrino Dirac matrix be $[m_D]_{3\times n}$ and the Majorana matrix for the right-handed neutrinos be $[M_M]_{n\times n}$, where the subscripts give the dimensions of the matrix. The net neutrino mass Lagrangian is then

$$-\mathcal{L}_m \;=\; \left(\begin{array}{cc} \left[\overline{\nu_L^C}\right]_{1\times 3} & \left[\overline{\nu_R}\right]_{1\times n} \end{array} \right) [\mathcal{M}]_{(3+n)\times(3+n)} \left(\begin{array}{c} [\nu_L]_{3\times 1} \\ \left[\nu_R^C\right]_{n\times 1} \end{array} \right) .$$

   (a) Write the effective neutrino mass matrix $\mathcal{M}$ explicitly in terms of m_D and M_M (block form). For all the matrices, denote their dimensions explicitly.

   (b) Show that the matrix

$$U \equiv \left(\begin{array}{cc} \left[1 - \frac{W^\dagger W}{2}\right]_{3\times 3} & [W]_{3\times n} \\ \left[-W^\dagger\right]_{n\times 3} & \left[1 - \frac{WW^\dagger}{2}\right]_{n\times n} \end{array} \right)$$

   is unitary, as long as all the elements of $[W]_{3\times n}$ are small (so that elements of the order $(W_{ij})^3$ can be neglected).

   (c) Let U be used to block-diagonalize $\mathcal{M}$, such that

$$U^T \mathcal{M} U = \left(\begin{array}{cc} [m_{LL}]_{3\times 3} & [0]_{3\times n} \\ [0]_{n\times 3} & [m_{RR}]_{n\times n} \end{array} \right) .$$

   Neglecting terms of the order of $[W_{ij}]^3$ and higher, determine the "mixing angle matrix" W, the "light neutrino mass matrix" m_{LL}, and the "heavy neutrino mass matrix" m_{RR}, in terms of the matrices m_D and M_M. Note that matrices do not commute, so the order of multiplication needs to be taken care of.

   (d) Write down the light neutrino mass eigenstates in terms of the flavour eigenstates $[\nu_L], [\nu_R^C]$, and the mass matrices m_D, M_M.