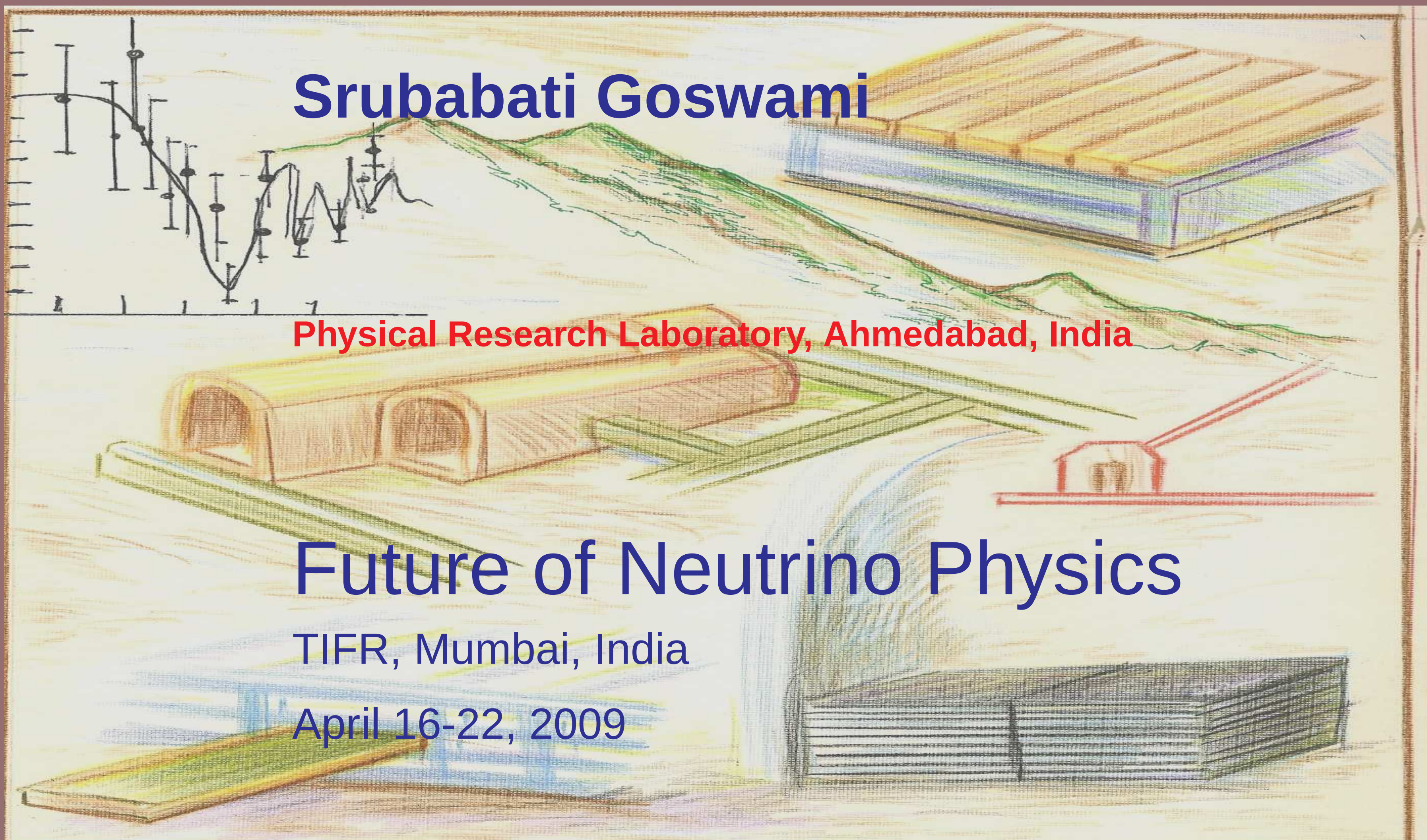
INO/2005/01
Interim Project Report
Volume I

Atmospheric Neutrinos at the INDIA-BASED NEUTRINO OBSERVATORY



Srubabati Goswami

Physical Research Laboratory, Ahmedabad, India

Future of Neutrino Physics

TIFR, Mumbai, India

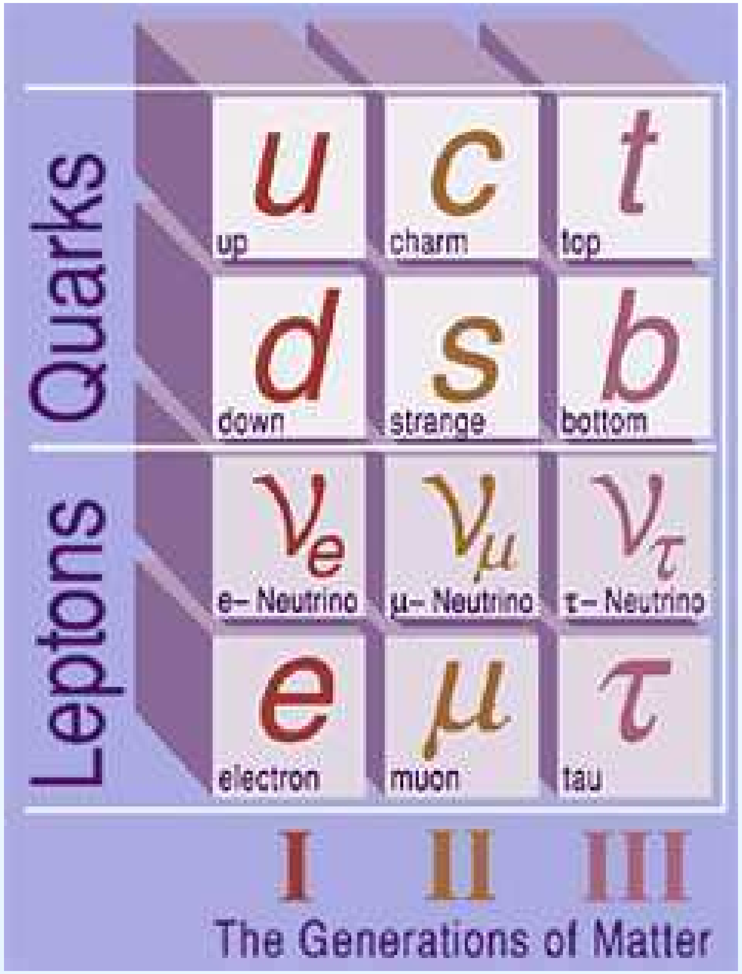
April 16-22, 2009

INO

Plan of Talk

- Introduction and Overview of Neutrino Oscillation
- Physics Goals of INO
- Atmospheric Neutrinos
- Matter effects at large baselines
  - Hiearchy sensitvity
  - Octant sensitvity
- Probing CPT violation

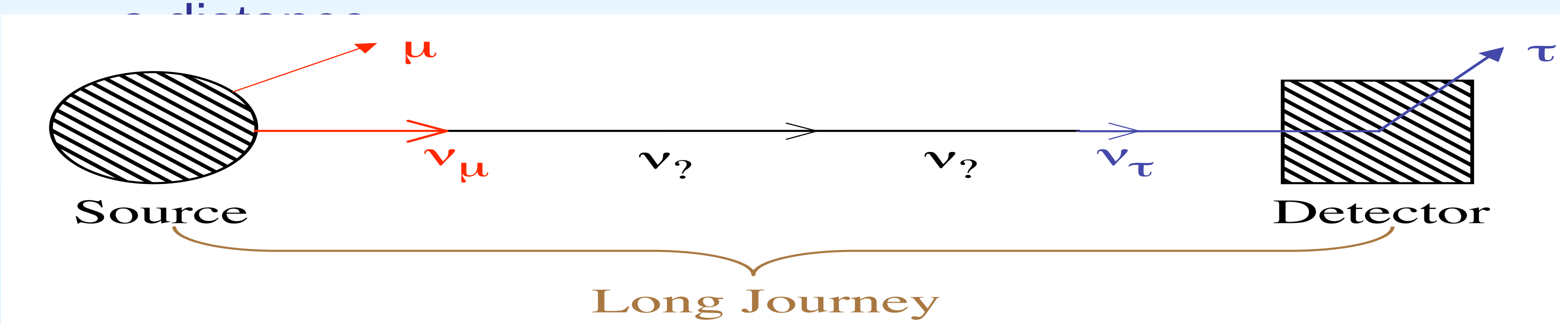
Neutrinos in the Standard Model



- There are three Flavours of Neutrinos: ν_e, ν_μ, ν_τ
- Massless
- Neutral
- Weakly interacting
- Neutrino Mass $\longrightarrow$ beyond Standard Model

Bounds on Neutrino Masses

- From Direct Measurements
  - $m_{\nu_e} < 2.2$ eV $(^3H \rightarrow ^3 He + e^- + \bar{\nu}_e)$
  - $m_{\nu_\mu} < 190KeV$ $(\pi \rightarrow \mu\nu_\mu)$
  - $m_{\nu_\tau} < 18.2MeV$ $(\tau \rightarrow n\pi\nu_\tau)$
- Cosmological Mass bound $\Sigma m_i < 1eV$
- Very small neutrino masses can be probed by Neutrino Oscillation
- Quantum Mechanical Interference phenomena in which one flavour of neutrino convert to another flavour after passing through a distance



Snapshot of ν Oscillation experiments

- Atmospheric Neutrinos
  SuperKamiokande ($> 20\sigma$)
- Solar Neutrinos
  Homestake,SAGE, Gallex, GNO, SuperKamiokande, SNO (7σ) ,
  Borexino
- Long baseline reactor experiment
  KamLAND (5σ)
- Long baseline accelerator based experiment K2K ($\sim 3\sigma$), MINOS ($\sim 5\sigma$)
- Short Basline Accelerator based experiment
  LSND evidence for $\bar{\nu}_\mu \rightarrow \bar{\nu_e}$, not corroborated by KARMEN, MiniBooNE
- Shortbaseline accelerator and Greenreactor experiments
  E776,KARMEN, CDHS, NOMAD and
  GreenCHOOZ, Buegey have not observed any oscillation.

Neutrino Oscillation in Vacuum

- If neutrinos have mass then the flavour states ν_α produced in $l_\alpha + N \to \nu_\alpha + N'$ are linear combinations of mass states ν_i

$$\nu_\alpha = U_{\alpha i}\nu_i$$

- The Probability of flavour conversion after traveling a distance L is

$$\begin{aligned}\mathcal{P}(\nu_\alpha \longrightarrow \nu_\beta) &= |<\nu_\beta|\nu_\alpha(t)>|^2 \\ &= \delta_{\alpha\beta} - 4\,\Sigma_{i>j}\,\mathrm{R}e(U_{\alpha i}U^\star_{\beta i}U^\star_{\alpha j}U_{\beta j})\,\sin^2(\Delta_{ij}) \\ &\quad +2\,\Sigma_{i>j}\,\mathrm{I}m(U_{\alpha i}U^\star_{\beta i}U^\star_{\alpha j}U_{\beta j})\,\sin(2\Delta_{ij})\end{aligned}$$

- $\Delta_{ij} = \frac{1.27\ \Delta m^2_{ij}(eV^2)\ L(Km)}{E(GeV)}, \qquad \Delta m^2_{ij} = m^2_j - m^2_i$

Neutrino Mixing Matrix

- 2 generations : 1 mixing angle

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

- 3 generations : 3 mixing angles, 1 phase

$$U_{PMNS} = R_{23}(\theta_{23}) R_{13}(\theta_{13}, \delta) R_{12}(\theta_{12})$$

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} \\ -s_{12} & c_{12} \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- Majorana phases unobservable in neutrino oscillation

2 Flavour Oscillation Probabilites

- Survival and Conversion Probabilites (in vacuum)

  $P_{\nu_e \nu_e} = 1 - \sin^2 2\theta \sin^2(1.27 \Delta m^2 L/E)$; $\Delta m^2 = m_2^2 - m_1^2$

  $P_{\nu_e \nu_e} = 1 - \sin^2 2\theta \sin^2(\pi L/\lambda)$

- Oscillation Wavelength (in vacuum)

  $\lambda = 2.5 m (E/MeV)(eV^2/\Delta m^2)$

  - $\lambda >> L, \sin^2(\pi L/\lambda) \to 0$
  - $\lambda << L, \sin^2(\pi L/\lambda) \to 1/2$
  - $\lambda \sim 2L, \sin^2(\pi L/\lambda) \sim 1 \to \Delta m^2 \sim E/L$

- Neutrino Oscillation probabilities depend on
  - Two fundamental parameters
    - At least one non-zero neurino mass
    - Non-zero mixing angles
  - Two experimental parameters
    - Neutrino Energy E
    - Source to detector distance L
- Oscillations experiments are not sensitive to absolute masses
- Solar Neutrinos : $E \sim 10$ MeV, $L \sim 10^8$km, $\Delta m^2 \sim 10^{-10}$ eV^2
- Matter effects need to be considered

Matter effect

- Matter has free electrons but no free muons or tau particles.
- ν_e has both CC & NC interaction with electrons, while ν_μ & ν_τ have only NC interaction with electrons.
- This changes the effective potential acting on ν_e but not on ν_μ & ν_τ, since the potential common to all 3 flavors cancels from oscillation probabilities.
- The effective masses and mixing angles in matter are different.
- No matter effect for 2 flavour $\nu_\mu - \nu_\tau$ oscillation

Matter effects: Two Flavours

- The evolution equation in flavor basis is

$$i\frac{d}{dt}\begin{pmatrix}\nu_\alpha(t)\\ \nu_\beta(t)\end{pmatrix} = \tilde{H}\begin{pmatrix}\nu_\alpha(t)\\ \nu_\beta(t)\end{pmatrix}$$

- Hamiltonian in flavor basis is:

$$\tilde{H} = E + \frac{m_1^2+m_2^2}{4E} - \frac{G_F n_n}{\sqrt{2}} + \frac{\Delta m_{21}^2}{4E}\begin{pmatrix}-\cos 2\theta + \frac{2A}{\Delta m_{21}^2} & \sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{pmatrix}$$

$A = 2\sqrt{2}G_F n_e E$. For anti-neutrinos, $A \to -A$.

- Effective mixing angle $\tilde{\theta}$ in matter : $\tan 2\tilde{\theta} = \frac{\Delta m_{21}^2 \sin 2\theta}{\Delta m_{21}^2 \cos 2\theta - A}$
- Maximal mixing or resonance:

  When $A = \Delta m_{21}^2 \cos 2\theta$, flavor states mix maximally, i.e. $\tilde{\theta} = \pi/4$ even if vacuum mixing angle is small.

Global Analysis — Ingredients . . .

- Experimental Data
  –statistical error
  –systematic errors and their correlations
- Theoretical Predictions
  –the fluxes and their uncertainties
  –the interaction cross-sections and their uncertainties
  –the oscillation probabilities (depends on the density profile of the propagating medium Δm^2 , θ, E_ν ....)
- $\mathrm{rate} = \mathrm{flux} \times \mathrm{cross-section} \times \mathrm{probability}$
- Minimisation of χ^2_{global}
  — covariance method
  — pull method

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- Best-fit values of parameters Δm^2, $\sin^2\theta \ldots$

Definition of χ^2

⋆ The Covariance Approach

$$\chi^2 = \sum_{i,j} (R_i^{data} - R_i^{theory})(\sigma_{ij}^2)^{-1}(R_j^{data} - R_j^{theory}) \ ,$$

$$\sigma_{ij}^2 = \delta_{ij}\sigma_i\sigma_j + \sum_{k=1}^{K} \sigma_i^k \sigma_j^k \rho_{ij}$$

⋆ The “Pull” Approach

$$\chi^2 = \min_{\xi_k} \left[\sum_{i=1}^{N} \left(\frac{R_i^{data} - (R_i^{theory} + \sum_{k=1}^{K} \xi_k \sigma_k)}{\sigma_i^{uncorr}} \right)^2 + \sum_{k=1}^{K} \xi_k^2 \right]$$

$$\xi_i \longrightarrow \text{free parameters}$$

Summary : Three Flavour Oscillation Parameters

- The best-fit and 1σ ranges

$\Delta m^2_{21} = 7.7^{+0.22}_{-0.21} \times 10^{-5}$ eV^2 $\quad |\Delta m^2_{31}| = 2.4 \pm 0.12 \times 10^{-3}$ eV^2

$\theta_{12} = (34.5 \pm 1.4)^o \quad \theta_{23} = (43.1^{+4.4}_{-3.5})^o \quad \theta_{13} = (7.2 \pm 6)^o$

Schwetz, Maltoni arXiv:0812.3161

Fogli, Lisi Maronne, Rotunno, Palazo arXiv:0805:2517

Gonzalez-Garcia, Maltoni arXiv:0704:1800

$$U_{PMNS}(3\sigma) = \begin{pmatrix} 0.77-0.86 & 0.50-0.63 & <0.22 \\ 0.22-0.56 & 0.44-0.73 & 0.57-0.80 \\ 0.21-0.55 & 0.40-0.71 & 0.59-0.82 \end{pmatrix}$$

- Very different from quark sector
- Considerable progress but precision not as good as V_{CKM}

Three Neutrino Oscillation Parameters

Δm^2_{31} Δm^2_{21}

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta} s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta} s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

“atmospheric” “solar”

known params.	bounded params.	unknown params.
$\|\Delta m^2_{31}\|$ $\sin^2\theta_{23}$ Δm^2_{21} $\sin^2\theta_{12}$	$\sin^2\theta_{13}$ $D_{23} \equiv \sin^2\theta_{23} - 0.5$	δ sign(Δm^2_{31})

Important Future Goals

- Improve the errors in $\delta(\sin^2\theta_{12})$
- Improve the errors in $\delta(\Delta m^2_{31})$
- Improve the errors in $\delta(\sin^2\theta_{23})$
- Determine the octant of θ_{23}
- Ascertaining if θ_{13} is different from zero and improve sensitivity
- Determination of $sgn(\Delta m^2_{31})$
- Discovering the leptonic CP phase δ
- Search for non-standard interactions and new physics
- Sterile neutrinos
- Are neutrinos Dirac or Majorana ?
- Absolute scale of neutrino masses
- ......

Physics Goals for INO

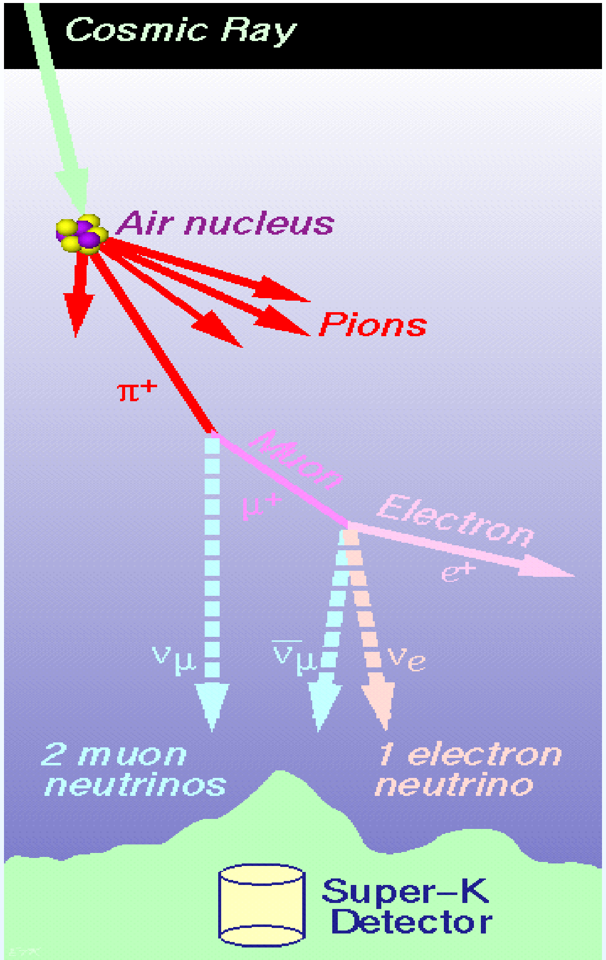
- First phase – measurement of atmospheric neutrino flux
  - Reconfirmation of the first oscillation dip as a function of L/E
  - Improved precision of oscillation parameters
  - Determination of the octant of θ_{23}
  - Matter effects and determination of sign of Δm^2_{31}
  - Probing CPT violation, Lorentz violation
  - Discrimination between $\nu_\mu - \nu_\tau$ and $\nu_\mu - \nu_s$
  - ......
- Second Phase – end detector for beta beams, neutrino factory
  - hierarchy, θ_{13}, CP violation
  - CERN to INO baseline $\sim$ 7000 km, the magic baseline

Atmospheric neutrinos . . .

Cosmic Ray + $A_{air} \longrightarrow \pi^+ + \ldots$

$\pi^+ \longrightarrow \mu^+ + \nu_\mu$

$\mu^+ \longrightarrow e^+ + \bar{\nu}_\mu + \nu_e$



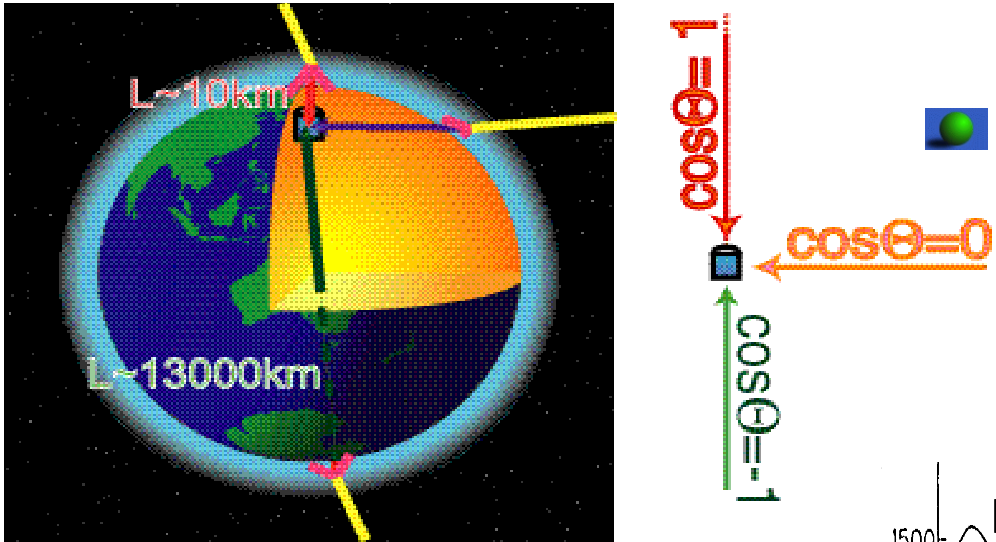
- Energy: 100 MeV - TeV
- Pathlength: 15 -13,000 km
- Provides broad L/E band

$\nu_\mu : \nu_e$ = 2: 1 (expected)

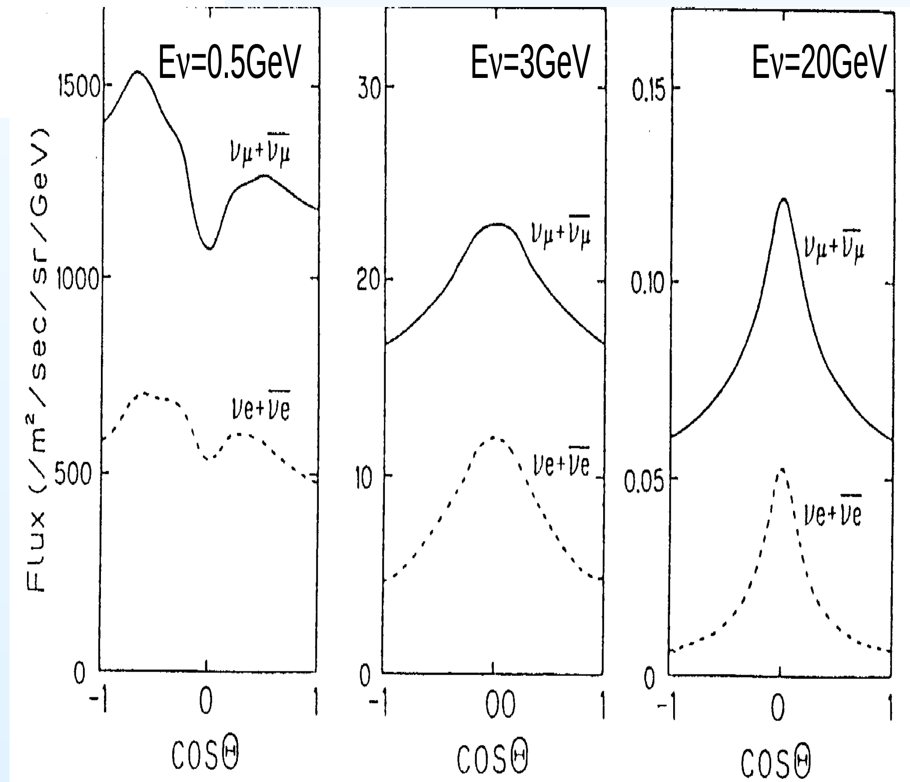
$\nu_\mu/\nu_e \sim$ 0.9 – 1 (observed)

$\Longrightarrow \nu_\mu$ conversion

Atmospheric neutrino Flux



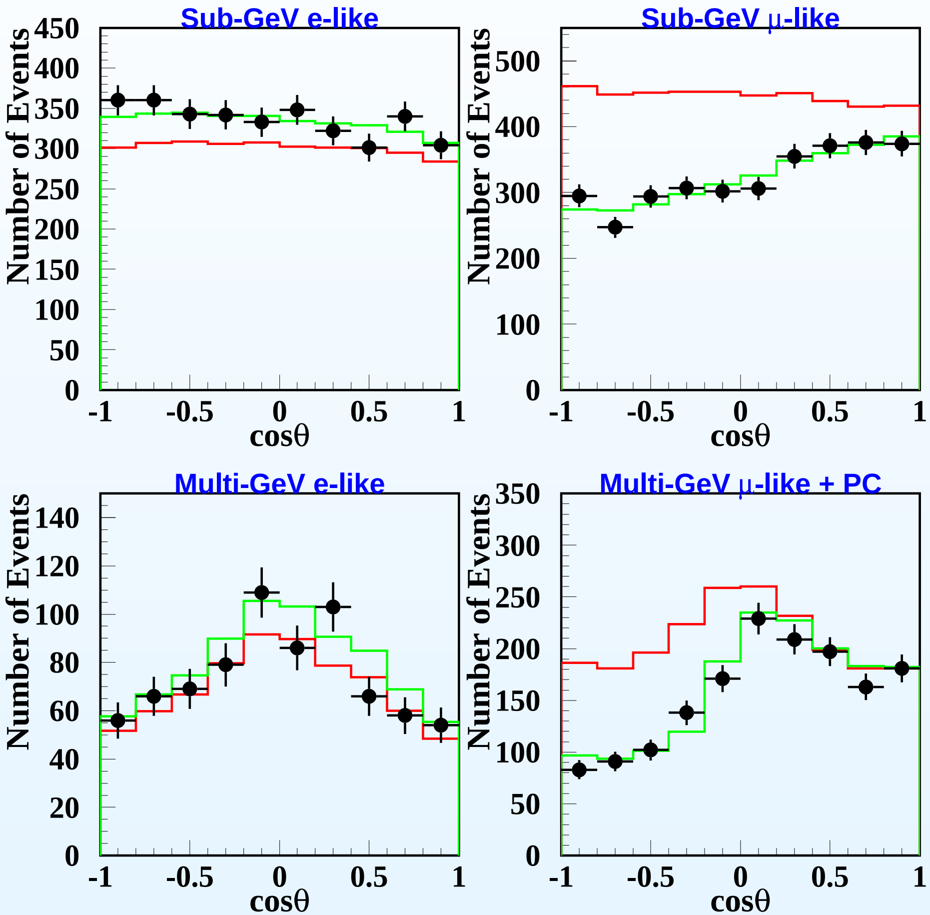
- The flux is Up-down symmetric for E $>$ a few GeV



- Up-down asymmetry $\Longrightarrow$ neutrino oscillations

Detection of Atmospheric neutrinos at SK

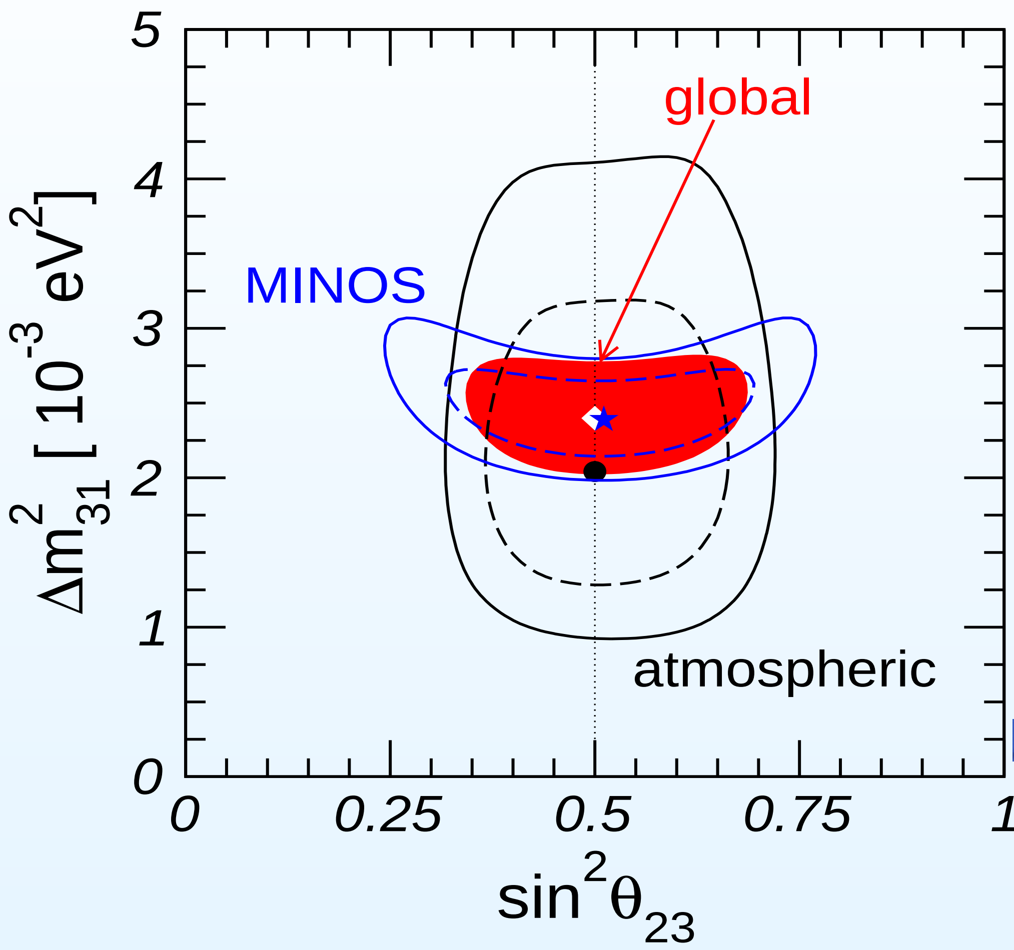
- SK-I 1489 days zenith angle spectrum



- Two generation $\nu_\mu - \nu_\tau$ oscillation
- Matter Effect not relevant
- $N_\mu(up)/N_\mu(down) =$ P$_{\mu\mu}$
- $P_{\mu\mu} = 1 - \sin^2 2\theta_{atm} \sin^2 \left(\frac{\Delta m^2_{atm} L}{4E} \right)$

  $(\theta_{atm} \equiv \theta_{23}, \Delta m^2_{atm} \equiv \Delta m^2{}_{31})$
- $\theta_{23} - (\pi/2 - \theta_{23})$ symmetry
- No information on $sgn(\Delta m^2_{atm})$.
- Earth Matter effect important for upward going neutrinos and $\theta_{13} \neq$

Atmospheric Neutrino Oscillation parameters



Best-fit (ATM+MINOS)

$|\Delta m^2_{atm}|$=2.4$\times 10^{-3}$eV^2 $\sin^2\theta_{23}$ = 0.5

3σ range (ATM+MINOS)

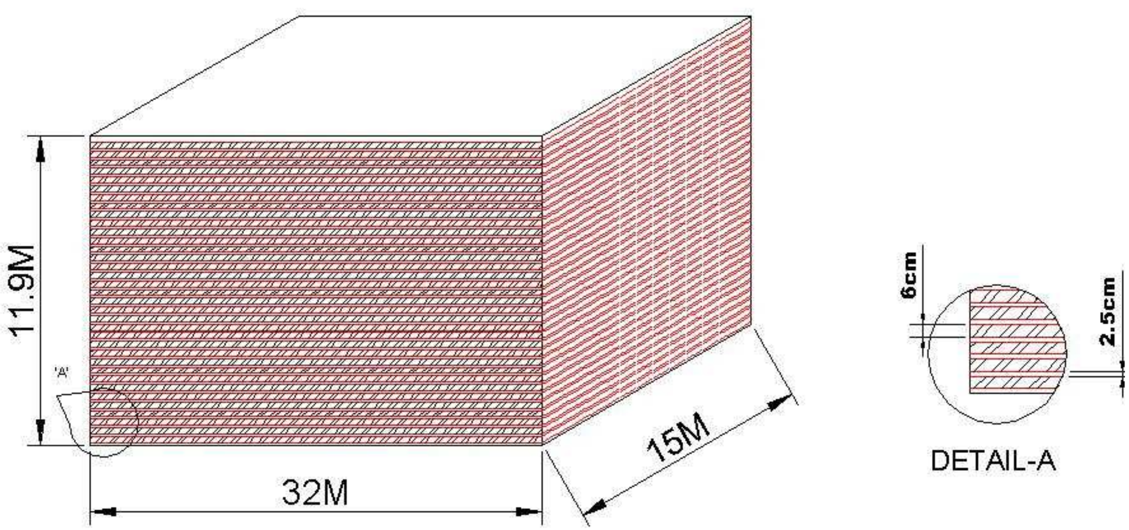
$|\Delta m^2_{atm}|$=(2.1-2.8) $\times 10^{-5}$eV^2
$\sin^2\theta_{23}$ = 0.36 -0.67

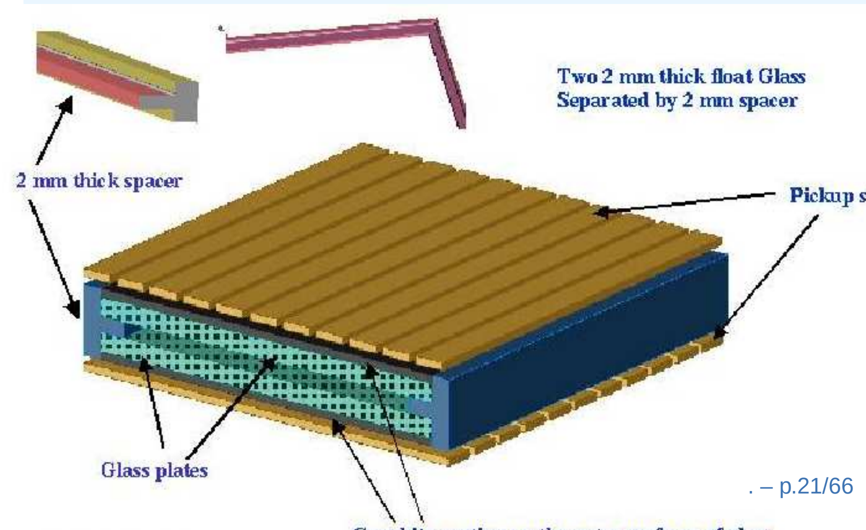
- 3σ Precision:
  $|\Delta m^2_{atm}| \sim$ 14% mainly by MINOS
  $\sin^2\theta_{23} \sim$ 30 % mainly by Atmosphe
- No informatio on $sgn(\Delta m^2_{atm})$
- No information on octant of θ_{23}

The detector

- Magnetised iron calorimeter
- Modular structure - 3 modules
- Module dimension 16m $\times$ 16m $\times$ 12m
- Detector: 48 m $\times$ 16m $\times$ 12m
- 140 horizontal iron layers interspersed with Glass RPC
- Iron Plate thickness 6 cm
- Gap for RPC trays 2.5 cm

- Sensitive to muons
- Energy determination from
  - Track length
  - Track curvature in a magnetic field
- Direction of parent neutrino from the track
- Charge identification from track curvature in magentic field



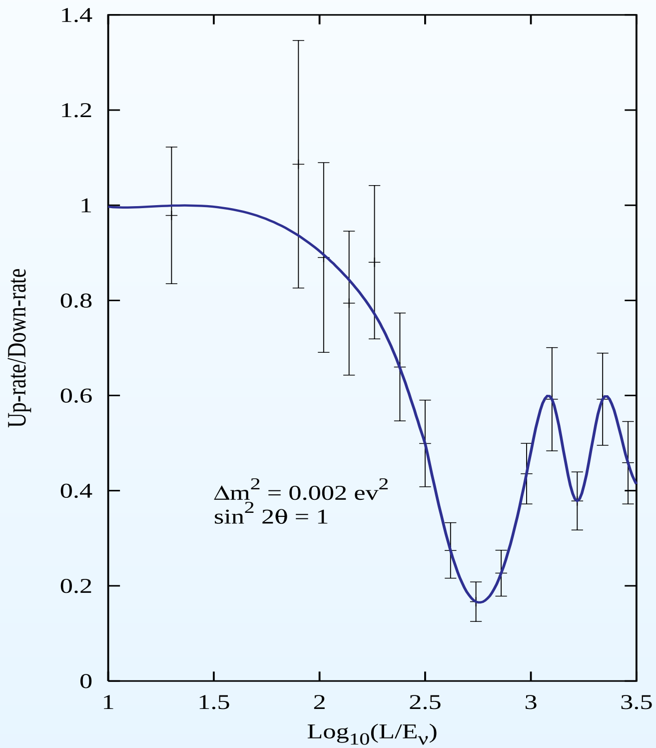


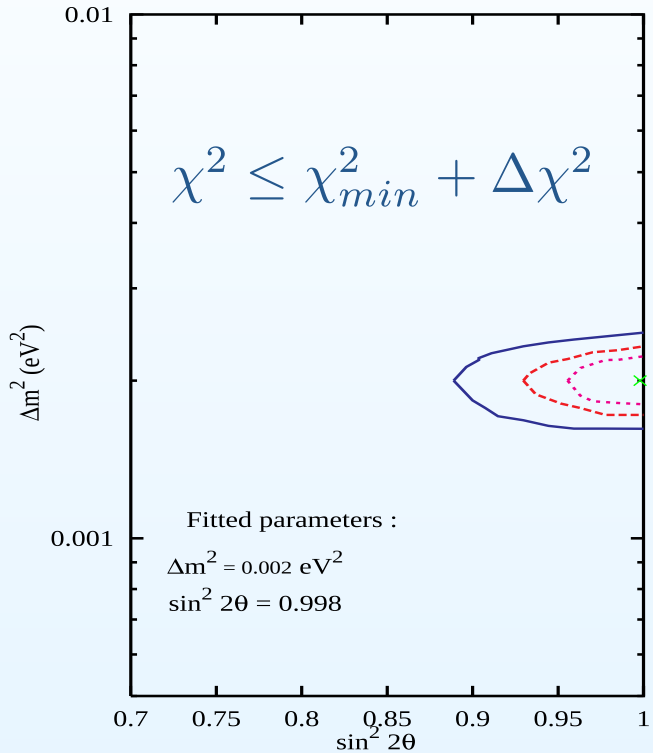
Atmospheric Neutrinos in INO

- Sensitive to Muons
  - $\nu_\mu + N \rightarrow \mu^- + N'$ (QE)
    $\bar{\nu}_\mu + N \rightarrow \mu^+ + N'$ (QE)
  - 1 Pion
  - DIS
- Muon event number: $(\phi_\mu \times P_{\mu\mu}$ + $\phi_e \times P_{e\mu}) \times \sigma_{CC} \times \epsilon$
- $$\frac{\mathrm{d^2N_\mu}}{\mathrm{d\Omega_m dE_m}} = \frac{1}{2\pi}\int_1^{100} \mathrm{dE_t} \int \mathrm{d\Omega_t R(E_t, E_m) R(\Omega_t, \Omega_m)} [\Phi_\mu^{\mathrm{d}} \mathrm{P}_{\mu\mu} + \Phi_{\mathrm{e}}^{\mathrm{d}} \mathrm{P}_{\mathrm{e}\mu}]\sigma\epsilon \quad (1)$$
- $$\mathrm{R(\Omega_t, \Omega_m) = N \exp}\left[-\frac{(\theta_\mathrm{t} - \theta_\mathrm{m})^2 + \sin^2\theta_\mathrm{t}(\phi_\mathrm{t} - \phi_\mathrm{m})^2}{2(\Delta\theta)^2}\right]. \quad (2)$$
  $$\mathrm{R(E_m, E_t)} = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(\mathrm{E_m - E_t})^2}{2\sigma^2}\right]. \quad (3)$$

Atmospheric Neutrinos and INO

- Observation of fall and rise of up/down ν_μ events





- Increased precision of Δm^2_{atm}

Comparison with Long Baseline Experiments

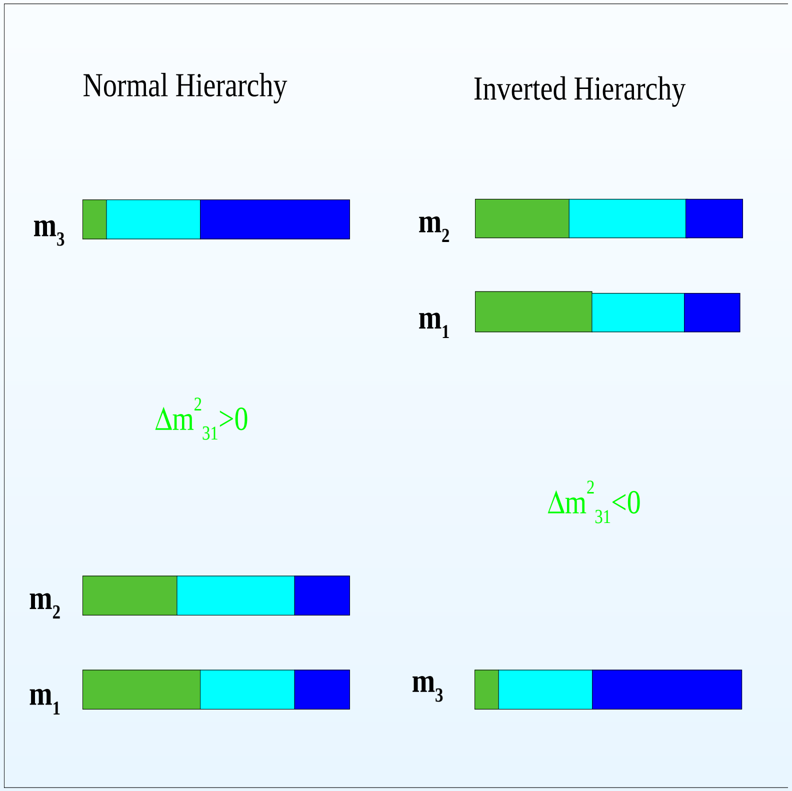
- 3σ spread ($|\Delta m^2{}_{31}|$ = 2×10^{-3} eV^2, $\sin^2\theta_{23} = 0.5$).

	$\lvert\Delta m^2{}_{31}\rvert$	$\sin^2\theta_{23}$
current	29%	33%
MINOS+CNGS	13%	39%
T2K	6%	23%
Nova	13%	43%
INO, 50 kton, 5 years	10%	30%

M. Lindner, hep-ph/0503101

Table refers to the older NOνA proposal; the revised March 2005 NOνA proposal is expected to be competitive with T2K.

Ambiguity in Mass Hierarchy



- Normal Hierarchy :

$$m_3^2 \approx \Delta m_{atm}^2 >> m_2^2 \approx \Delta m_{\odot}^2 >> m_1^2$$

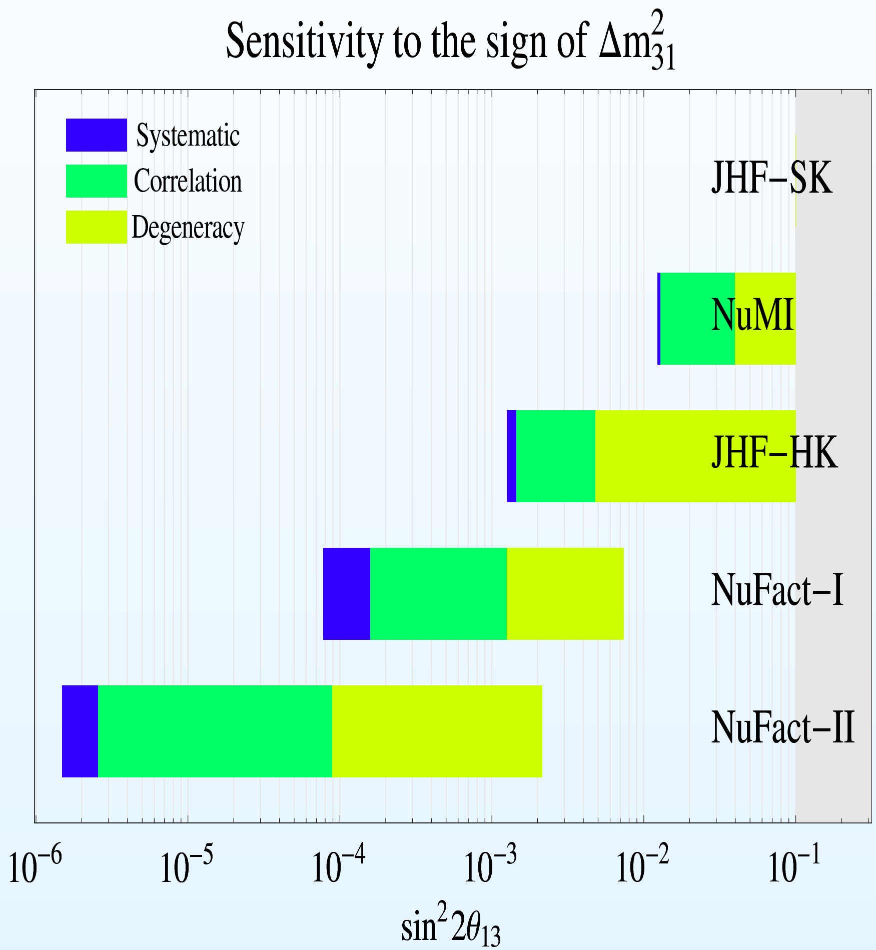
- Inverted Hierarchy :

$$m_1^2 \approx \Delta m_{atm}^2 \approx m_2^2 >> m_3^2$$

- Quasi-Degenerate

$$m_3 \approx m_2 \approx m_1 >> \sqrt{\Delta m_{atm}^2}$$

Ambiguity in Mass Hierarchy



M. Lindner, hep-ph/0503101

- Hierarchy difficult to determine in superbeams
- Sensitivity limited by correlation and degeneracies
- Synergistic use of experiments
- Use of Matter effects
- Use of Magic baseline

Matter effects: Three Flavours

- The effective Hamiltonian is

$$\tilde{H} = \frac{1}{2E}[U \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U^\dagger + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}]$$

excluding terms $\propto$ identity matrix. $U = R_{23} R_{13} R_{12}$

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  excluding terms $\propto$ identity matrix. $U = R_{23}R_{13}R_{12}$

- Subtracting m_1^2 from the first part,

$$\tilde{H} = \frac{1}{2E}[U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} U^\dagger + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}]$$

Matter effects: Three Flavours

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$$\tilde{H} = \frac{1}{2E}[U \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U^\dagger + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}]$$

excluding terms $\propto$ identity matrix. $U = R_{23}R_{13}R_{12}$

- 1 MSD ($\Delta m^2_{21} = 0$) limit

$$\tilde{H} = \frac{1}{2E}[U \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Delta m^2_{31} \end{pmatrix} U^\dagger + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}]$$

$\longrightarrow$ Resonance in the 1-3 sector

Matter effects : Three flavours

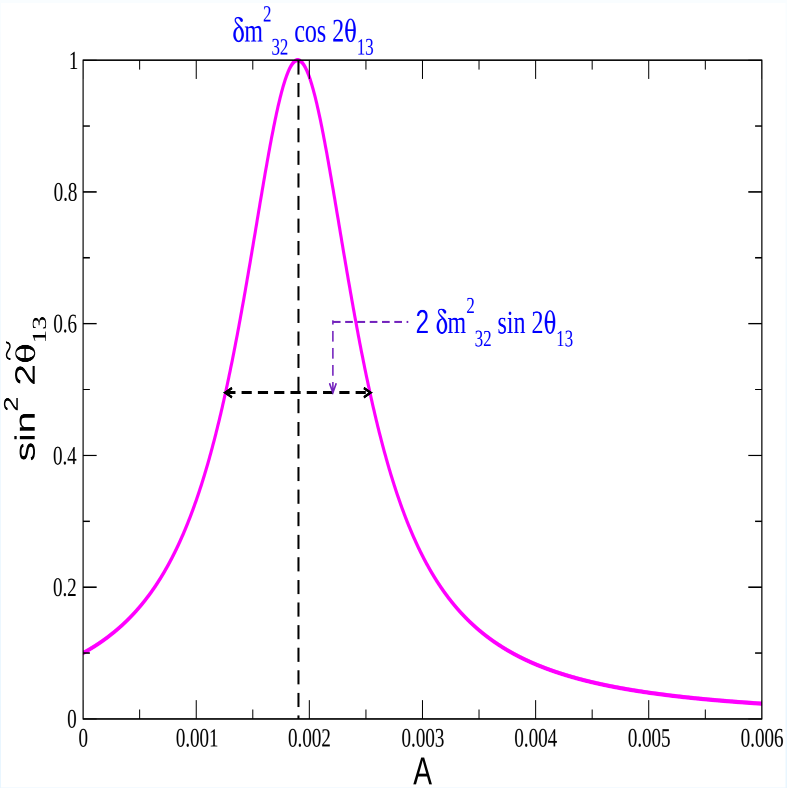
- Resonance in the 1-3 sector

$$\sin^2 2\tilde{\theta}_{13} = \frac{\sin^2 2\theta_{13}}{(\frac{A}{\delta m^2_{31}} - \cos 2\theta_{13})^2 + \sin^2 2\theta_{13}}$$

$$\tilde{\Delta}_{31} = \Delta_{31}\sqrt{(\frac{A}{\delta m^2_{31}} - \cos 2\theta_{13})^2 + \sin^2 2\theta_{13}}$$

- Maximal mixing or resonance: $\pm\frac{A}{\delta m^2_{31}} = \cos 2\theta_{13}$
  Lower sign denotes anti-neutrino case, where $A \to -A$
- At resonance $\sin^2 2\tilde{\theta}_{13} = 1$ or $\tilde{\theta}_{13} = \pi/4$.
- ν: resonant enhancement in $\sin^2 2\tilde{\theta}_{13}$ for $\Delta m^2_{31} > 0$, anti-ν: $A \to -A$, so resonance for $\Delta m^2_{31} < 0$.
- Experiments sensitive to matter effects can probe mass hierarchy
- Matter effects for Δm^2_{31} channel depend crucially on θ_{13}

Matter resonance



- The resonance condition:

$$\frac{2\sqrt{2}G_F n_e E}{\Delta m^2_{31}} = \cos 2\theta_{13}$$

- This gives:

$$E_{res} = \frac{\Delta m^2_{31} \cos 2\theta_{13}}{2\sqrt{2}G_F n_e}$$

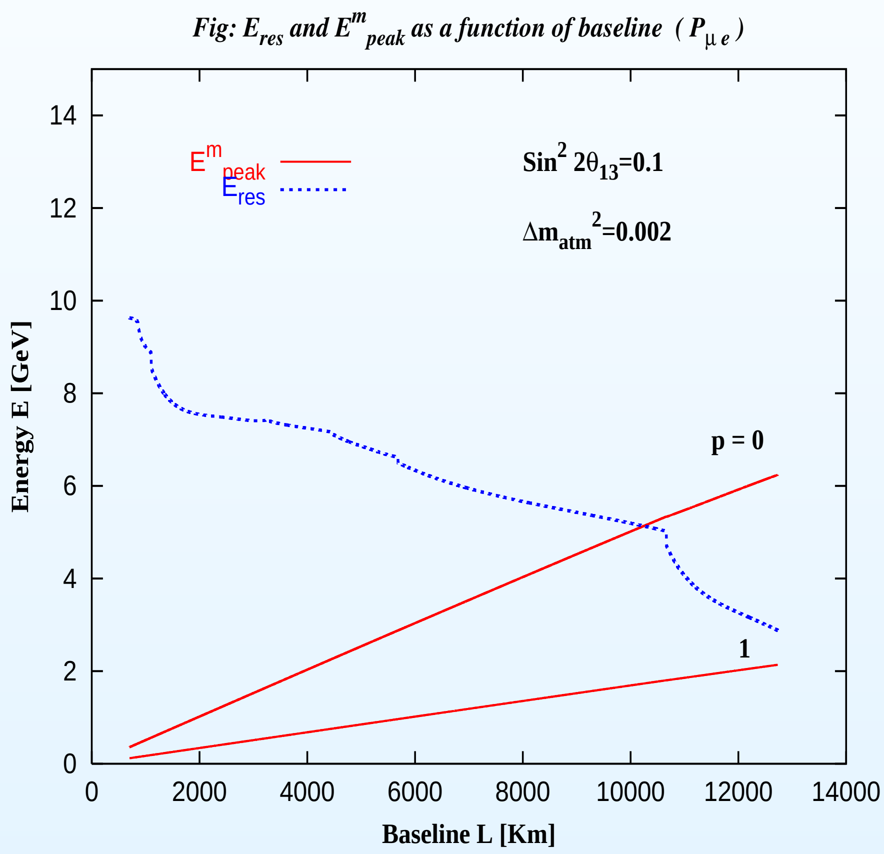
- For $\Delta_{31} = 0.002$ eV^2, $E_{res} \sim 5$ GeV
- E_{res} depends on Δm^2_{31} and n_e
- For a fixed Δm^2_{31} it is different at different baselin

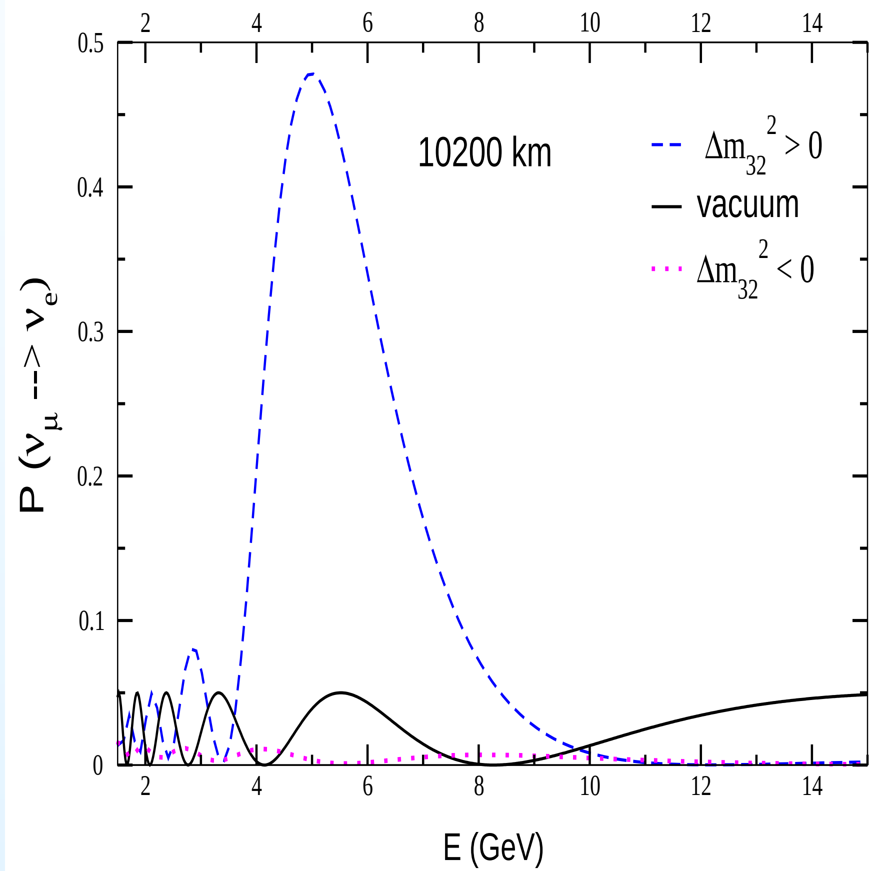
Conditions For Maximum Matter effect

- $\mathcal{P}^m_{\nu_\mu \to \nu_e} = s^2_{23} \sin^2 2\theta_{13}{}^m \sin^2 [\Delta^m_{31} L/E]$
  $\mathrm{P}^{\mathrm{m}}_{\nu_{\mathrm{e}} \to \nu_{\mathrm{e}}} = 1 - \sin^2 2\theta^{\mathrm{m}}_{13} \; \sin^2 \left[\, \Delta^{\mathrm{m}}_{31} \; \mathrm{L/E} \right]$
- Matter effect is observed near $E \sim E_{res}$, where the amplitude is large, but we also require large phase.
- $\mathcal{P}^m_{\nu_\mu \to \nu_e}$ and $\mathcal{P}^m_{\nu_\mu \to \nu_e}$ is maximum when simultaneously
  $\sin^2 (2\theta_{13})^m = 1$
  $\sin^2 \Delta^m_{31} = 1 = \sin^2((2p+1)\pi/2)$
- This implies: $E = E_{res} = E^m_{peak}$.
- This gives the maximum matter effect condition for L:

$$[\rho L]^{max}_{\mu e} = \frac{(2p+1)\pi \times 5.18 \times 10^3}{\tan 2\theta_{13}} \;\; km \; gm/cc$$

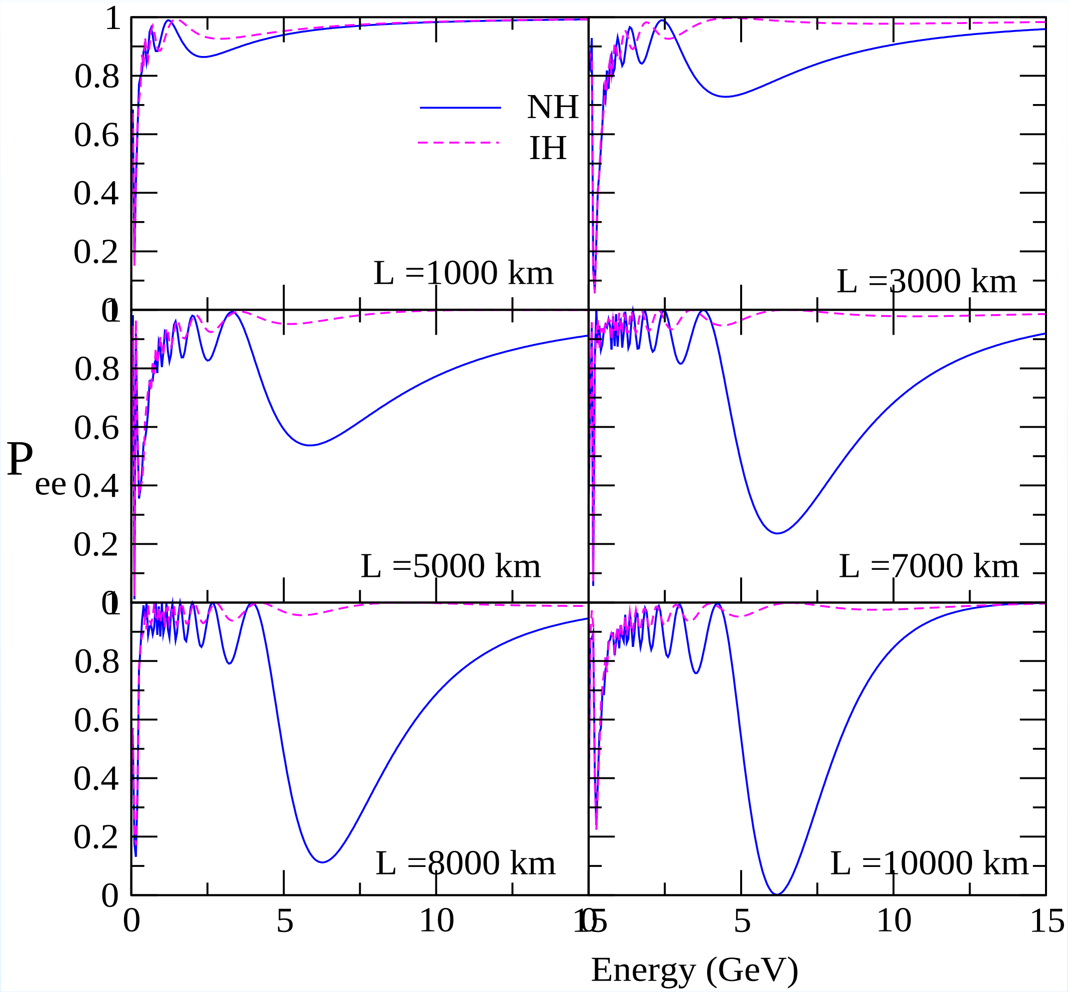
E_{res} & E^m_{peak} vs L for $P(\nu_\mu \to \nu_e)$





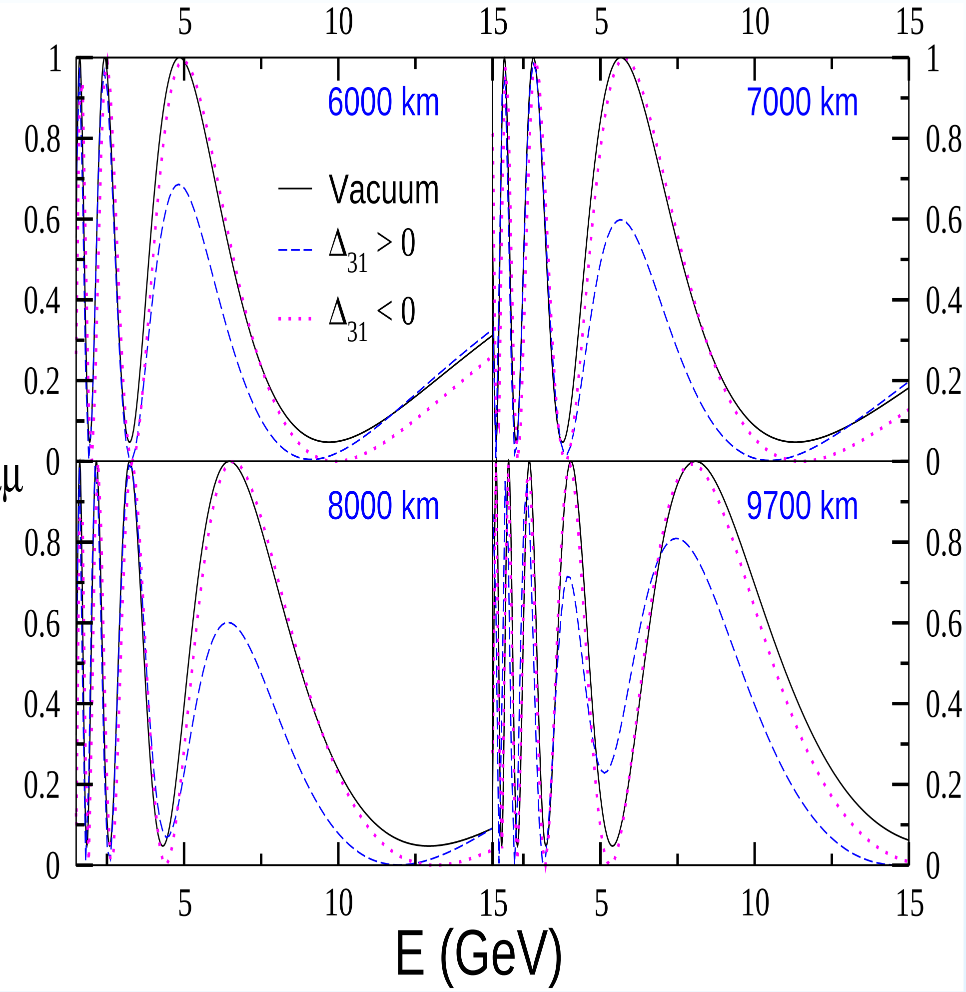
- For $\sin^2 2\theta_{13} = 0.1$, p=0, the maximum matter effect comes at $L \sim$ 10,000 km

Matter effect in P_{ee} channel



- Matter effect maximum around $\sim$ 10,000 km.

Condition for Maximum Matter effect in $P_{\nu_\mu \to \nu_\mu}$



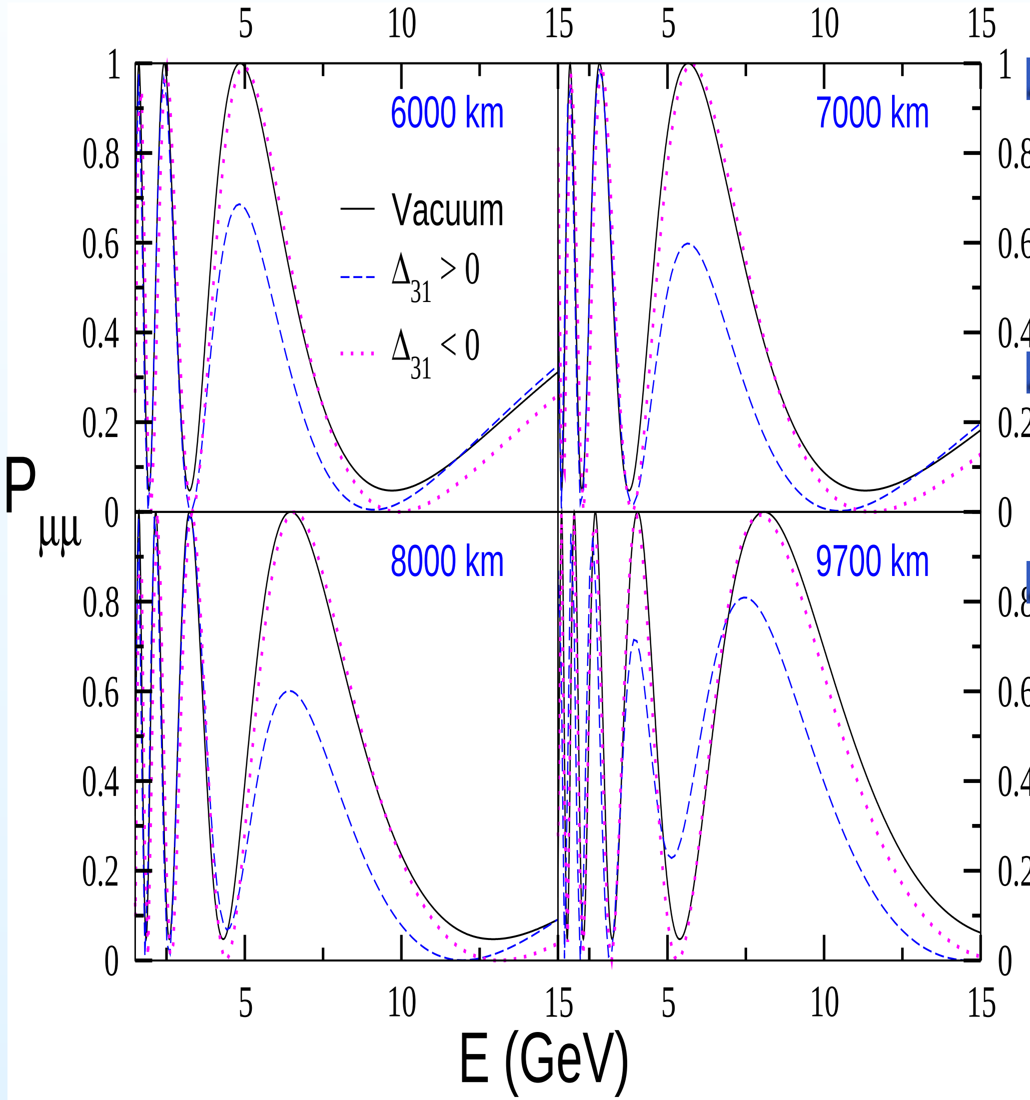
$$\mathrm{P}^{\mathrm{m}}_{\mu\mu} = 1 - \mathrm{P}^{\mathrm{m}}_{\mu\mu}{}^{1} - \mathrm{P}^{\mathrm{m}}_{\mu\mu}{}^{2} - \mathrm{P}^{\mathrm{m}}_{\mu\mu}{}^{3}$$

$$\mathrm{P}^{\mathrm{m}}_{\mu\mu}{}^{1} = \mathrm{c}^{2}_{13}{}^{\mathrm{m}}\sin^2 2\theta_{23} \sin^2 \left[\frac{1.27(\Delta_{31} + \mathrm{A} + \Delta^{\mathrm{m}}_{31})\mathrm{L}}{2\mathrm{E}}\right.$$

$$\mathrm{P}^{\mathrm{m}}_{\mu\mu}{}^{2} = \mathrm{s}^{2}_{13}{}^{\mathrm{m}}\sin^2 2\theta_{23} \sin^2 \left[\frac{1.27(\Delta_{31} + \mathrm{A} - \Delta^{\mathrm{m}}_{31})\mathrm{L}}{2\mathrm{E}}\right.$$

$$\mathrm{P}^{\mathrm{m}}_{\mu\mu}{}^{3} = \sin^4\theta_{23}\sin^2 2\theta^{\mathrm{m}}_{13} \sin^2 (1.27\Delta^{\mathrm{m}}_{31}\mathrm{L/E})$$

Condition for Maximum Matter effect in $P_{\nu_\mu \to \nu_\mu}$

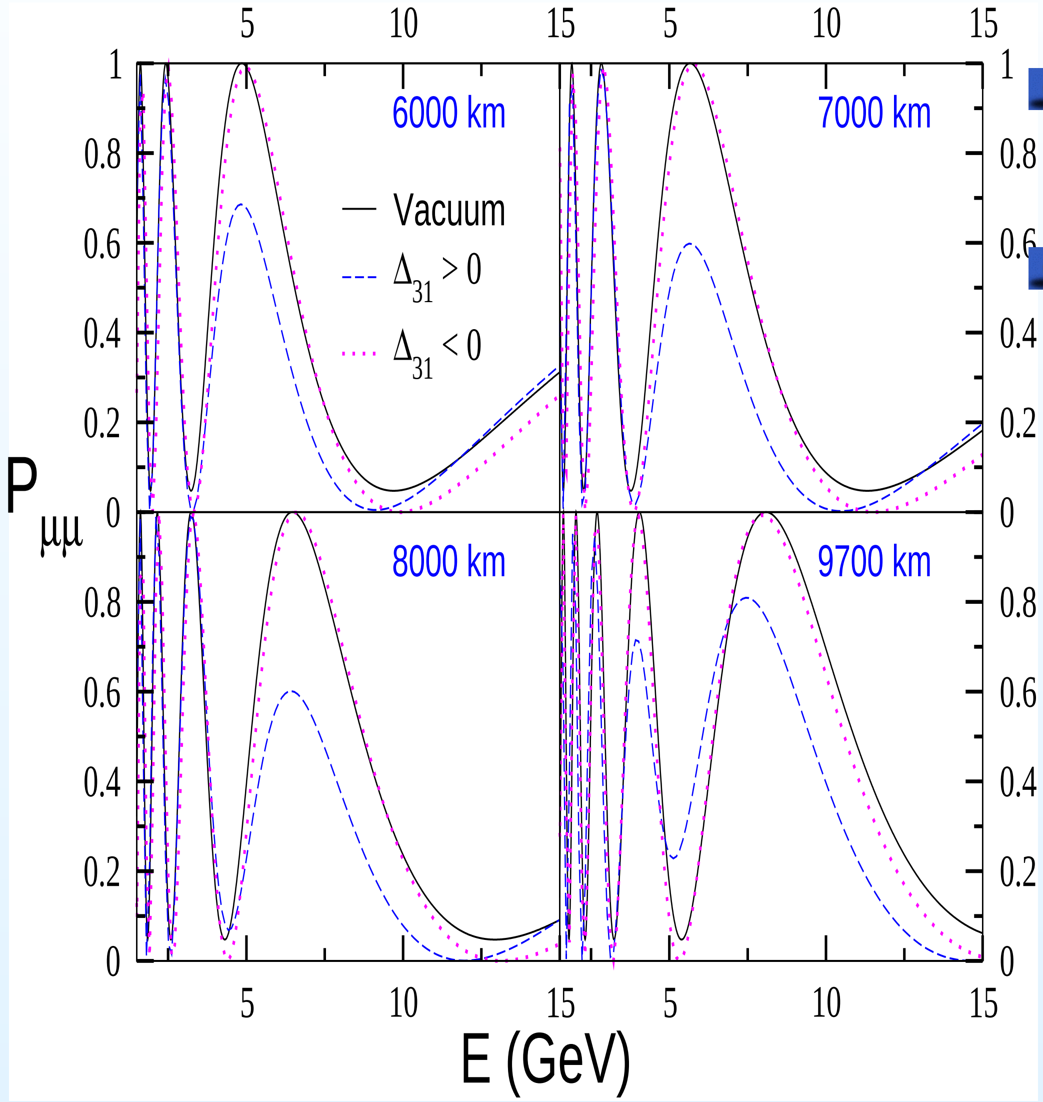


- Condition for maximum matter effect in $P_{\mu\mu}$ is
  - $E^{v}_{peak} = E_{res}$
  - $1.27 \frac{L}{E^{v}_{peak}} = p\,\pi$
- This gives

  $[\rho L]^{max,peak}_{\mu\mu} \simeq p\,\pi \times 10^4 \times \cos 2\theta_{13}$ km

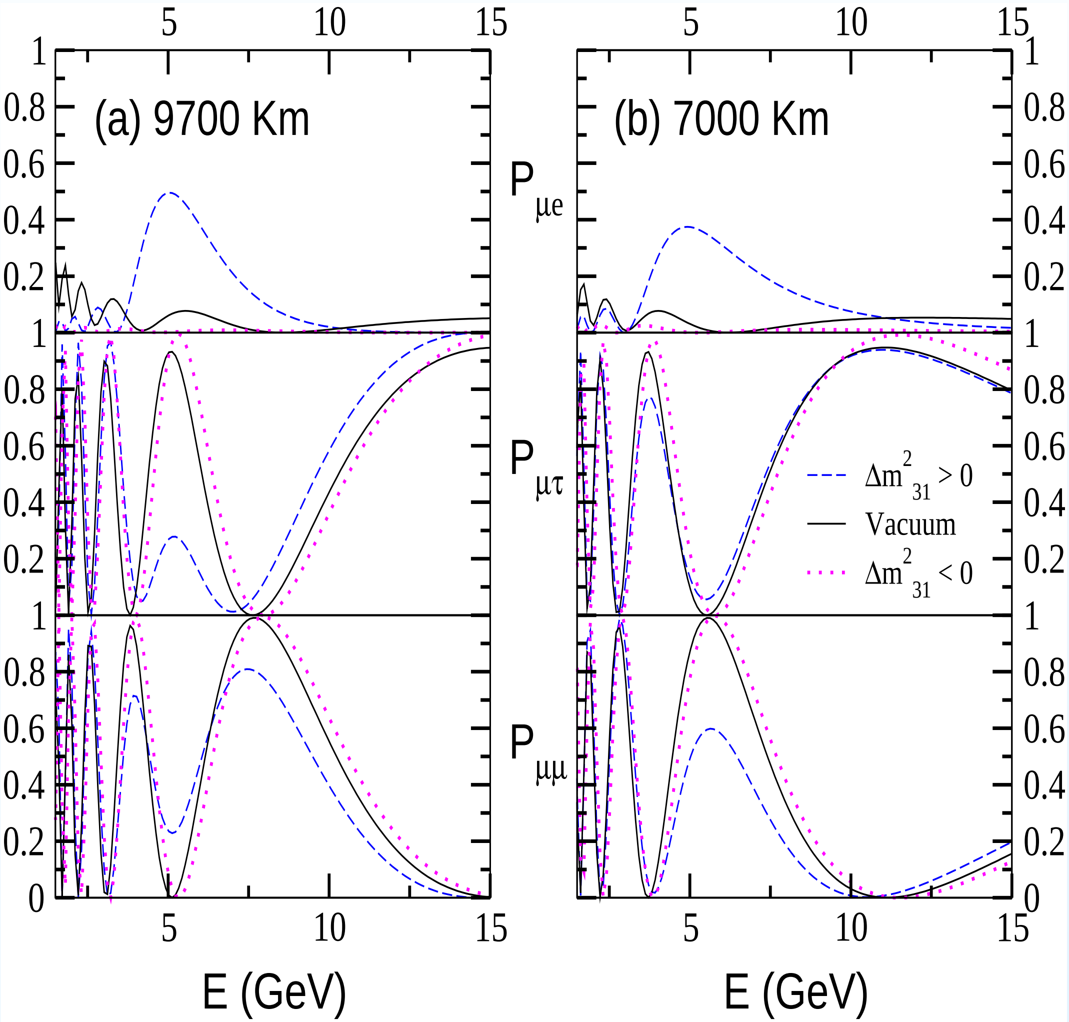
  for p =1, L $\simeq$ 7000 km
- Fall in $P_{\mu\mu}$ in matter

Condition for Maximum Matter effect in $P_{\nu_\mu \to \nu_\mu}$



- At 9700 km rise in $P_{\mu\mu}$ in matter near a vacuum dip
- Condition for a dip in vacuum in $P_{\mu\mu}$ is
  - $1.27 \frac{\mathrm{L}}{\mathrm{E^v_{dip}}} = (2\mathrm{p}+1)\,\pi/2$
  - for p=1, $E^v_{dip} \sim 6.6$ GeV at L = 10,000 km
  - At 10,000 km $E_{res} \sim 6.6$ GeV
  - Thus we have the condition $\mathrm{E^v_{dip}} = \mathrm{E_{res}}$

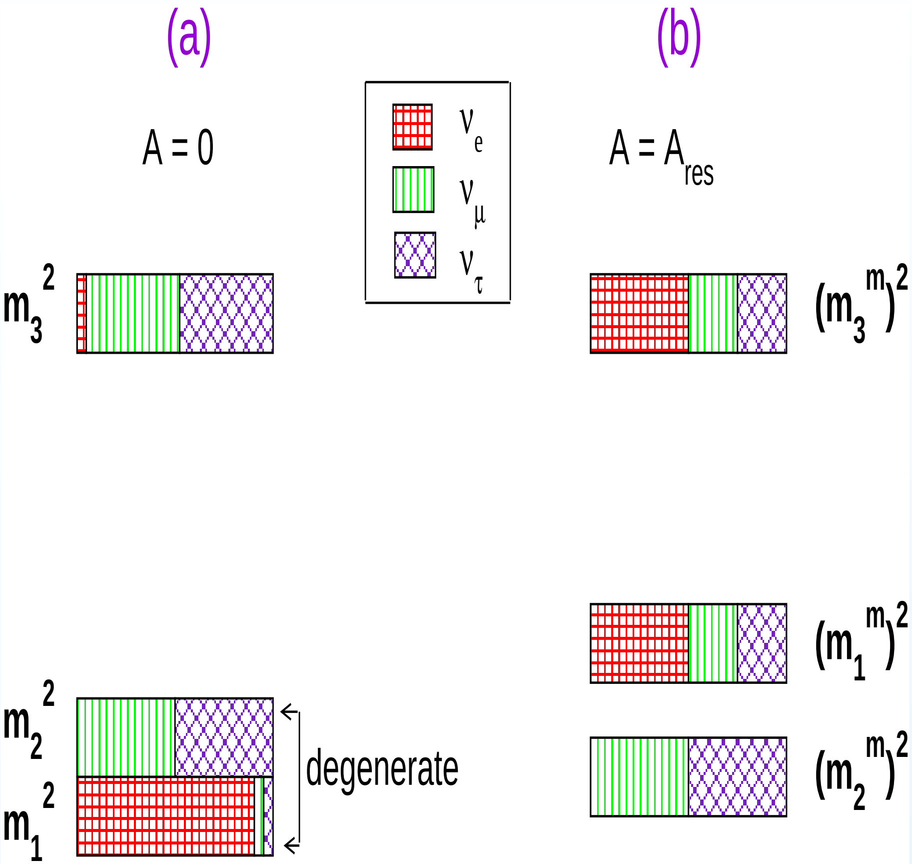
Matter effect in $P_{\mu\tau}$ at large baselines



- No matter effect in two flavor $\nu_\mu - \nu_\tau$ oscillation since both interact via neutral current
- At 9700 km significant matter effect in $P_{\mu\tau}$
- 50% rise in $P_{\mu e}$, 20% rise in $P_{\mu\mu}$
- $P_{\mu\tau} = 1 - P_{\mu e} - P_{\mu\mu}$
- $\Delta P_{\mu\tau} = -(\Delta P_{\mu e} + \Delta P_{\mu\mu})$
- 70% matter induced fall in $P_{\mu\tau}$
- Genuine three flavour effect

R. Gandhi, P. Ghoshal, S.G., P. Mehta, S. UmaShanakar , PRL, 2005

Flavour composition of mass states



- The vacuum mass eigenstate ν_1 is largely ν_e, ν_3 is largely ν_μ & ν_τ and ν_2 has no ν_e component.
- Matter effect (increasing A) causes ν_e in ν_1^m to decrease & ν_μ, ν_τ to increase. At $A = A_{res}$ they are 50%. Similarly, ν_e in ν_3^m increases to 50%.
- At resonance, all matter-dependent mass eigenstates ν_1^m, ν_2^m & ν_3^m have significant ν_μ & ν_τ components.
- $P(\nu_\mu \rightarrow \nu_\tau)$ depends on all 3 mass-squared differences.

Effect of δ_{CP}

- For OMSD – effective two generation – no CP phase
- For L $<$ 1000 km (matter effect negligible)

$$\begin{aligned} P(\nu_\mu \to \nu_e) \;&\approx \sin^2 2\theta_{13} \sin^2 \theta_{23} \sin^2 \Delta_{31} \\ &\mp \alpha \sin 2\theta_{13} \sin \delta_{CP} \sin 2\theta_{12} \sin 2\theta_{23} \Delta_{31} \sin^2 \Delta_{31} \\ &+ \alpha \sin 2\theta_{13} \cos \delta_{CP} \sin 2\theta_{12} \sin 2\theta_{23} \Delta_{31} \cos \Delta_{31} \sin \Delta_{31} \\ &+ \alpha^2 \cos^2 \theta_{23} \sin^2 2\theta_{12} \Delta_{31}^2 \end{aligned}$$

- $\alpha = \Delta_{21}/\Delta_{31}$ (Best-fit α = 0.03, $\alpha \sin 2\theta_{12}$ = 0.028)
- Problem of Eightfold degeneracy
  - $\delta_{CP} - \theta_{13}$, $sgn(\Delta {m^2}_{13})$, $\theta_{23} - (\pi/2 - \theta_{23})$

Burguet-Castell et al, 2001

Minakata and Nunokawa, 2001

Fogli and Lisi,1996

Barger, Marfatia,Whisnanat,2002

The Magic Baseline

The appearance probability ($\nu_e \to \nu_\mu$) in matter, upto second order in the small parameters $\alpha \equiv \Delta m^2_{12}/\Delta m^2_{13}$ and $\sin 2\theta_{13}$,

$$
\begin{aligned}
P_{e\mu} &\simeq \sin^2 2\theta_{13} \, \sin^2\theta_{23} \frac{\sin^2[(1-\hat{A})\Delta]}{(1-\hat{A})^2} \\
&\pm \alpha \sin 2\theta_{13} \, \xi \sin\delta \sin(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1-\hat{A})\Delta]}{(1-\hat{A})} \\
&+ \alpha \sin 2\theta_{13} \, \xi \cos\delta \cos(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1-\hat{A})\Delta]}{(1-\hat{A})} \\
&+ \alpha^2 \, \cos^2\theta_{23} \sin^2 2\theta_{12} \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2};
\end{aligned}
$$

where $\Delta \equiv \Delta m^2_{13} L/(4E)$, $\xi \equiv \cos\theta_{13} \, \sin 2\theta_{12} \, \sin 2\theta_{23}$,

and $\hat{A} \equiv \pm(2\sqrt{2} G_F n_e E)/\Delta m^2_{13}$.

The Magic Baseline

The appearance probability ($\nu_e \to \nu_\mu$) in matter, upto second order in the small parameters $\alpha \equiv \Delta m^2_{12}/\Delta m^2_{13}$ and $\sin 2\theta_{13}$,

$$
\begin{aligned}
P_{e\mu} \quad &\simeq \quad \sin^2 2\theta_{13}\, \sin^2\theta_{23} \frac{\sin^2[(1-\hat{A})\Delta]}{(1-\hat{A})^2} \\
&\pm \quad \alpha \sin 2\theta_{13}\, \xi \sin\delta \sin(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1-\hat{A})\Delta]}{(1-\hat{A})} \\
&+ \quad \alpha \sin 2\theta_{13}\, \xi \cos\delta \cos(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1-\hat{A})\Delta]}{(1-\hat{A})} \\
&+ \quad \alpha^2\, \cos^2\theta_{23} \sin^2 2\theta_{12} \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2};
\end{aligned}
$$

The δ dependence disappears for $\to \sin(\hat{A}\Delta) = 0$

The Magic Baseline

- For $\sin(\hat{A}\Delta) = 0$
  - The δ dependence disappears from $P(\nu_e \rightarrow \nu_\mu)$.
  - A clean measurement of the hierarchy and θ_{13} is possible without any degeneracy with δ.
- The first non-trivial solution: $\sqrt{2}G_F n_e L = 2\pi$
  - Assuming a medium of constant density ρ : $L_{\rm magic}[{\rm km}] \approx 32726/\rho[{\rm gm/cm}^3]$.
  - Taking averaged density $\rho \approx$ 4.5 gm/cc $L_{\rm magic} \approx$ 7000 km. (CERN-INO) baseline

The Magical Reach of INO

- CERN to INO distance = 7152 km

The Magical Reach of INO

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- JPARC to INO distance = 6556 km

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- RAL to INO distance = 7653 km

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INO is wonderfully close to magic baseline

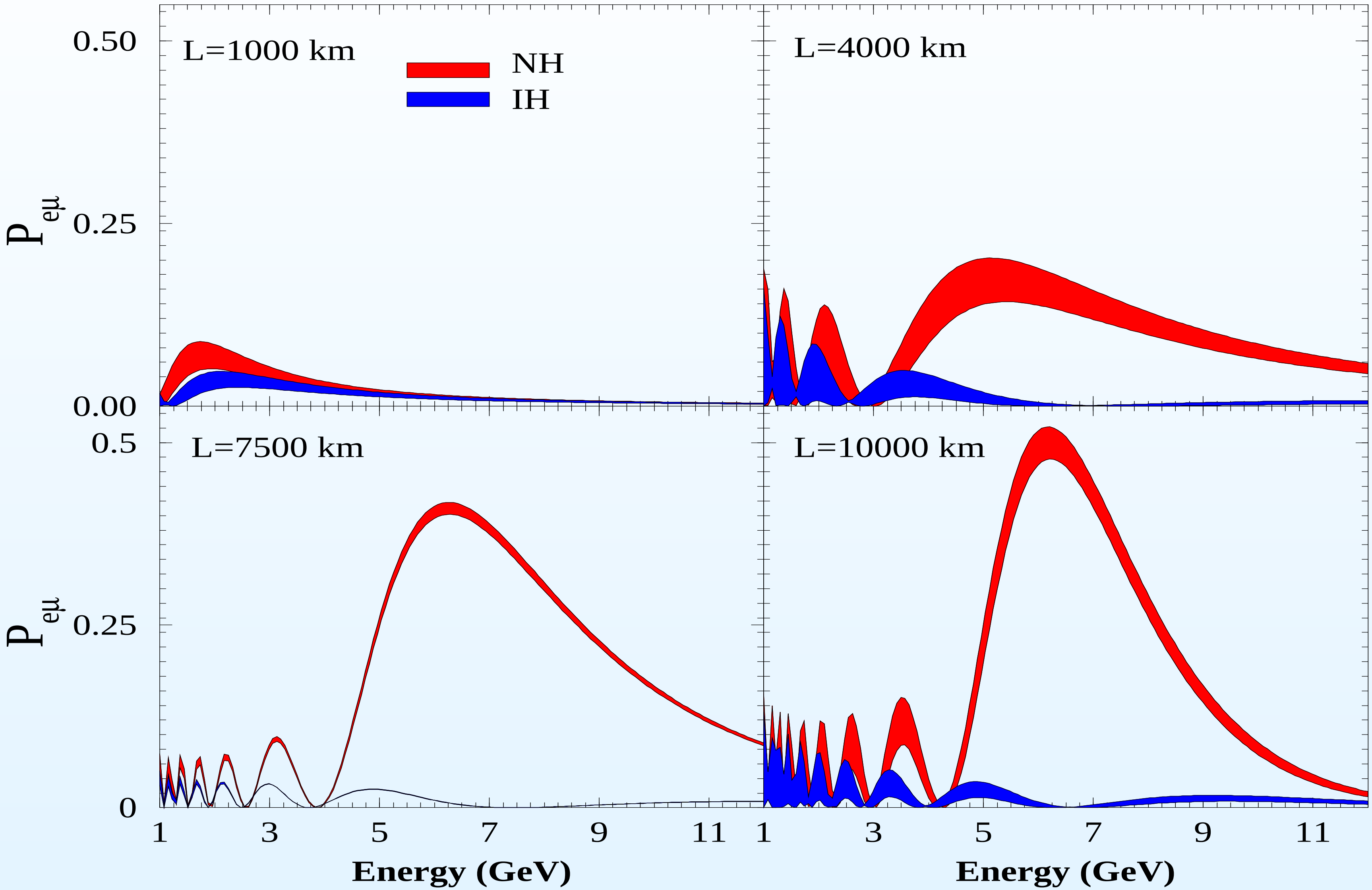
The Magical Reach of INO

- CERN to INO distance = 7152 km
- JPARC to INO distance = 6556 km
- RAL to INO distance = 7653 km

INO is wonderfully close to magic baseline

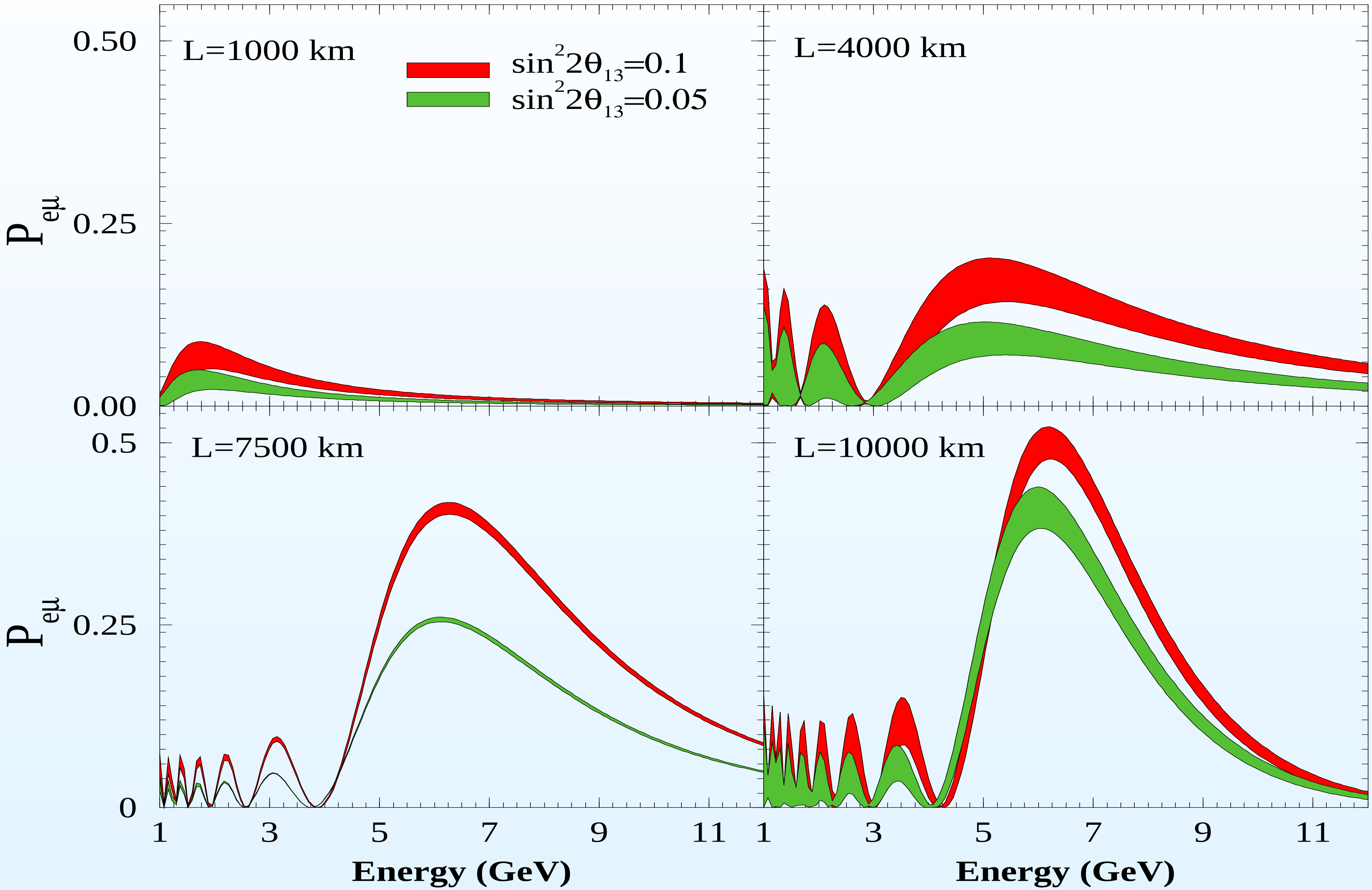
- Can be important for beam based experiments
- Atmospheric neutrinos cover a large range in L and E

$P_{e\mu}$ for NH and IH at different baselines



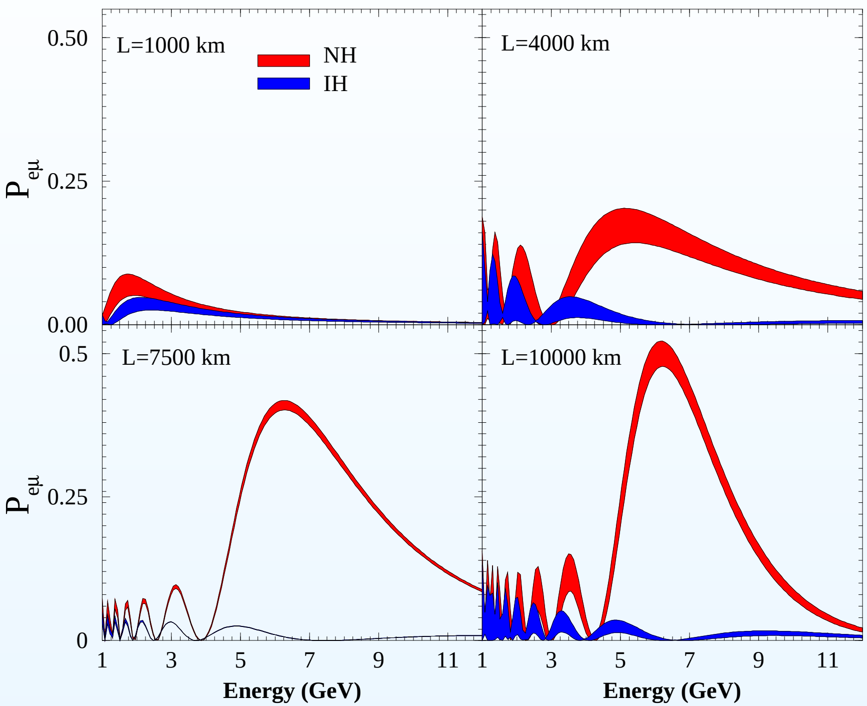
Agarwalla, Choubey, Raychaudhuri, hep-ph/0610333

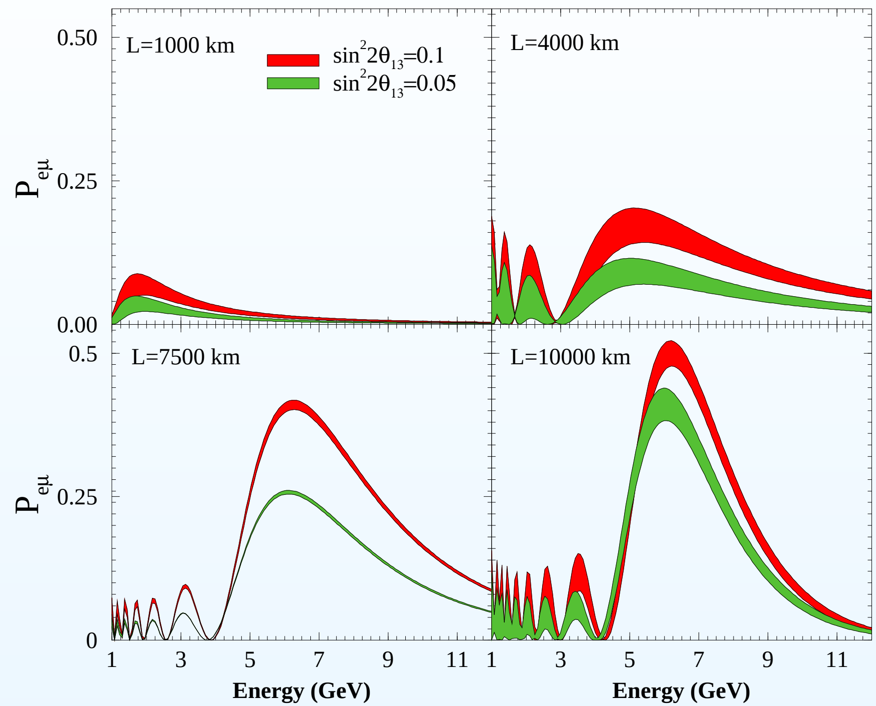
$P_{e\mu}$ for two values of θ_{13} at different baselines



Agarwalla, Choubey, Raychaudhuri, hep-ph/0610333

The Magic baseline

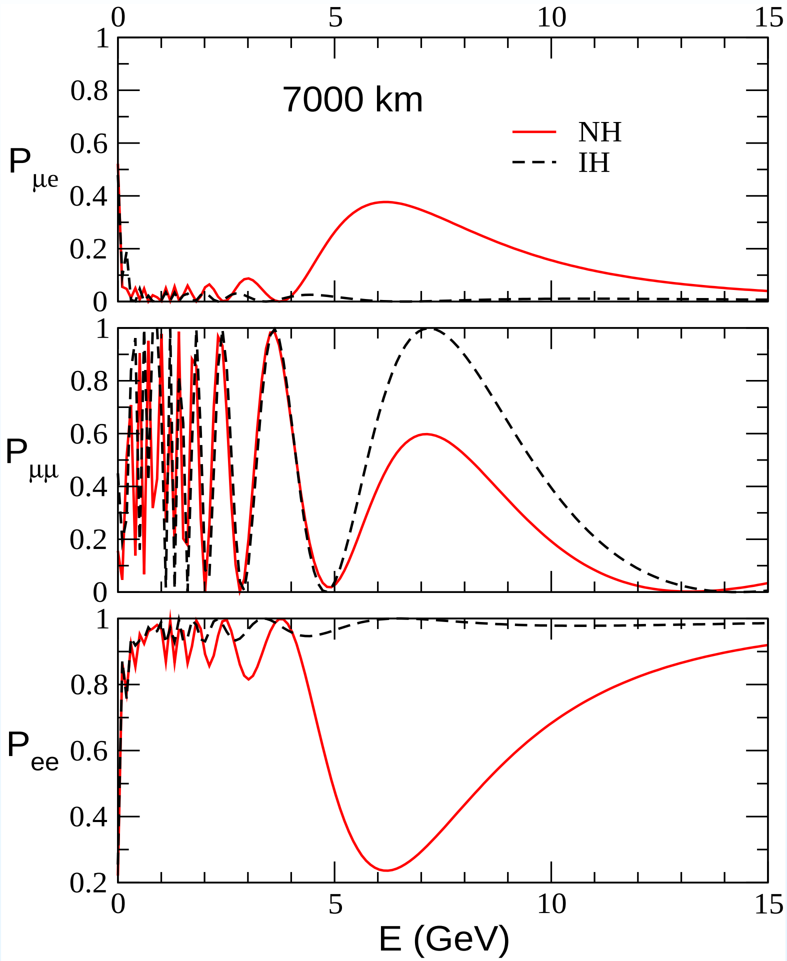


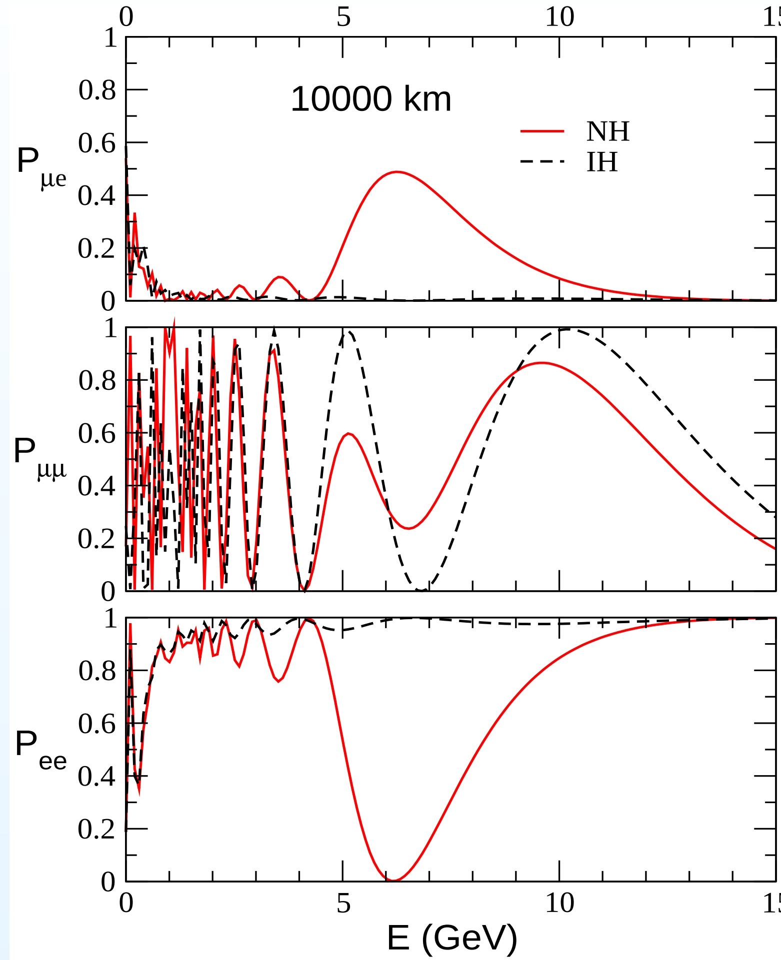


Agarwalla, Choubey, Raychaudhuri, hep-ph/0610333

- At $\sim$ 7500 km δ_{CP} dependence negligible
- $(\delta_{CP}, \theta_{13})$ and $(\delta_{CP}, sgn(\Delta m^2_{\mathrm{atm}}))$ degeneracies vanish
- Clean measurement of $sgn(\Delta m^2_{\mathrm{atm}})$ θ_{13}

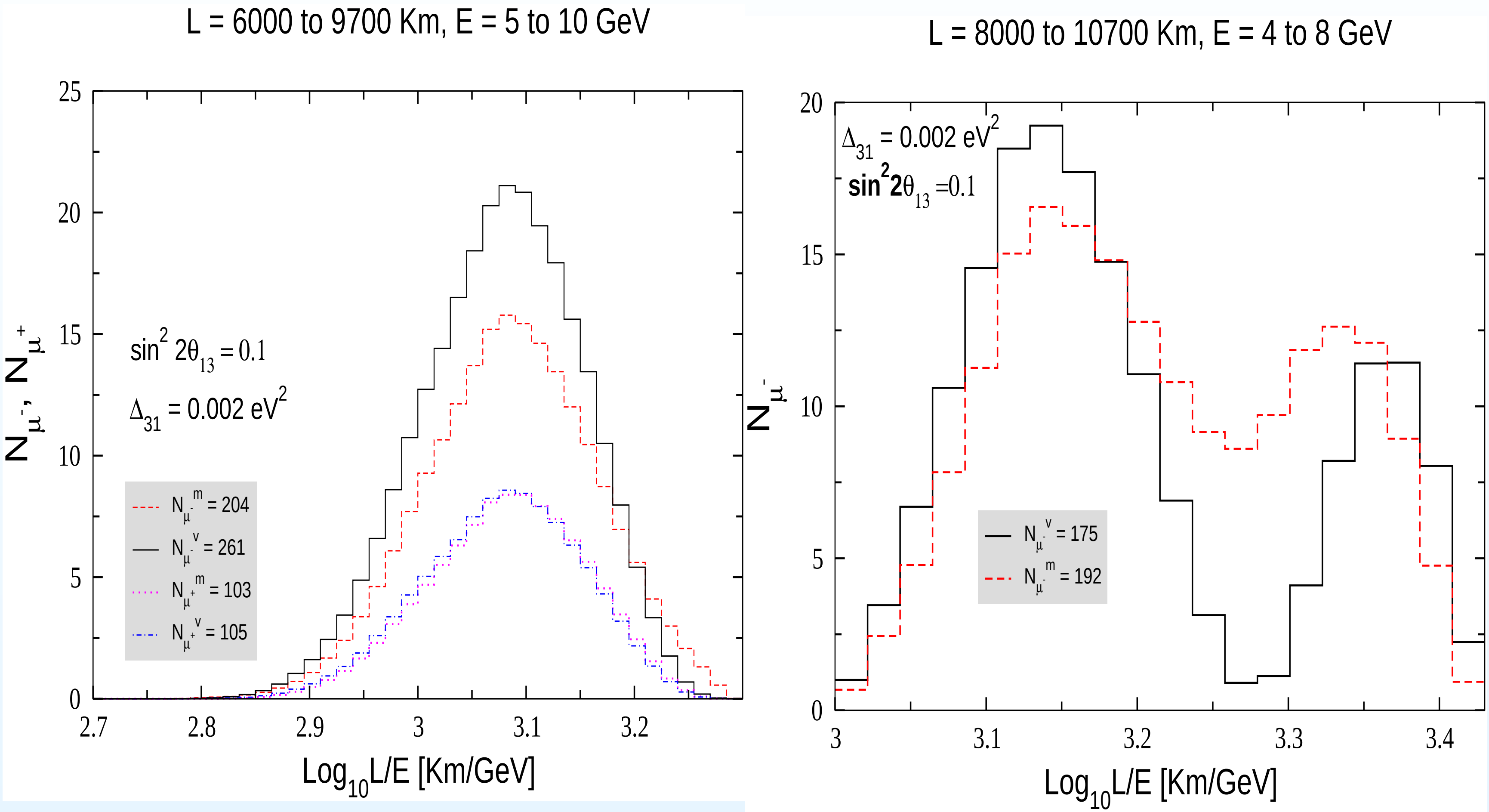
Matter effect and hierarchy at large baselines





- Large matter effects at long baselines
- All three probabilites sensitive to hierarchy
- Problem of δ_{CP} degeneracy less

Hierarchy Sensitivity in Atmospheric ν events



- For $\Delta m^2_{31} > 0$ matter effect in ν_ν and $(\mathrm{N}^{\mathrm{mat}}_{\mu^-} \neq \mathrm{N}^{\mathrm{vac}}_{\mu^-})$
- $(\mathrm{N}^{\mathrm{mat}}_{\mu^+} \approx \mathrm{N}^{\mathrm{vac}}_{\mu^+})$

R. Gandhi, P. Ghoshal, S.G., P. Mehta, S. Umashanakr, PRD, 2005

Atmospheric Neutrinos in INO

- Exposure:
  100 Kt $\times$ 10 yr $=$ 1000 Kt yr
- Detection efficiency:
  87%
- Charge i.d. of muons
  100%
- 3-dimensional Honda fluxes
- Range studied for matter effects:
  E $=$ 2 to 10 GeV, $\cos\theta_z =$ -0.1 to -1.0
- Muon threshold:
  1 GeV
- Detector resolution of
  $10^o, 15\%$

Statistical analysis

- Energy and $\cos\theta_z$ range divided into 8 $\times$ 18 = 144 bins
- INO: sensitive to both muons and antimuons

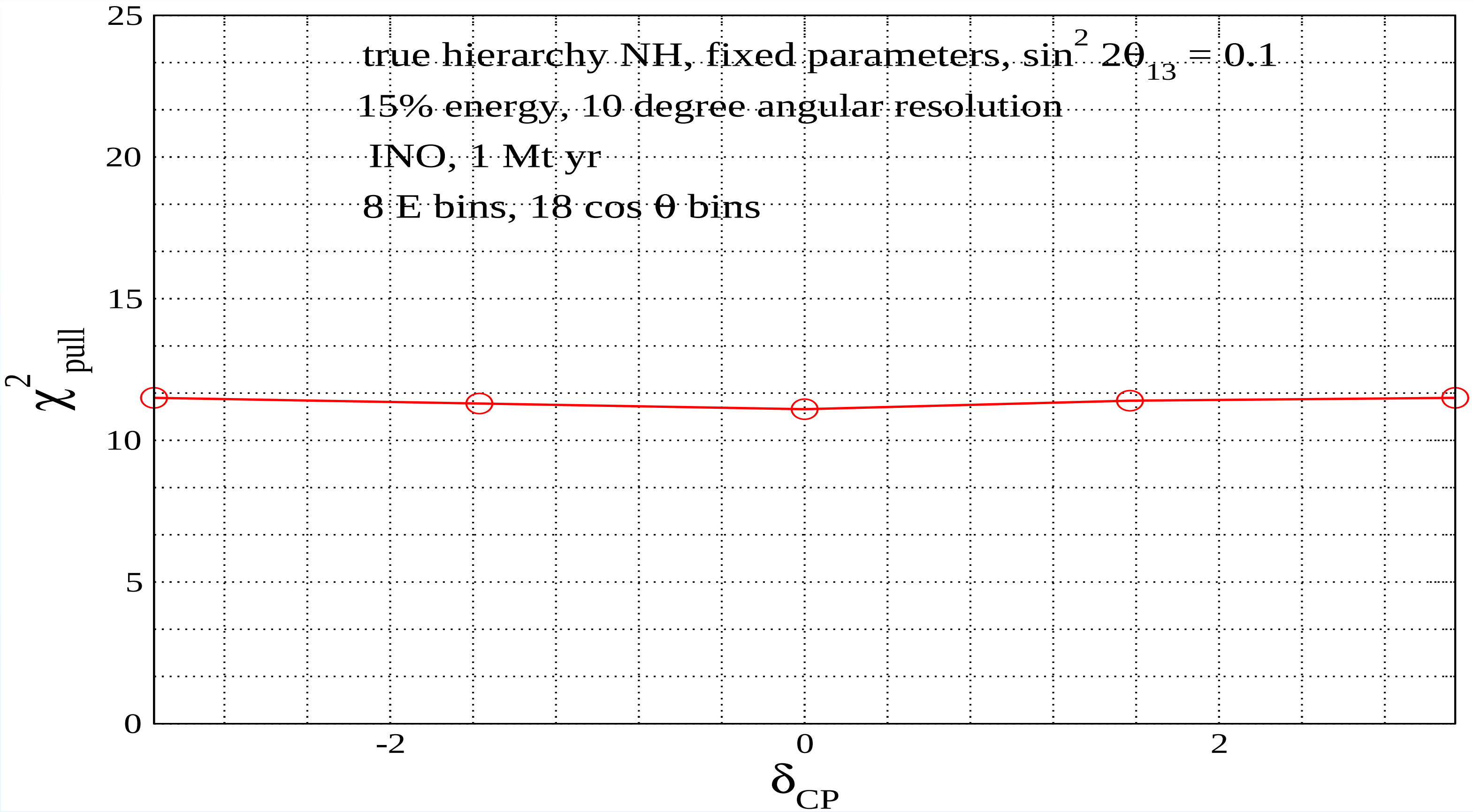
$$\chi^2 = \chi^2_\mu + \chi^2_{\bar{\mu}}$$

- Pull method is used
- Values of theoretical and systematic uncertainties:
  - ➢ Flux normalization error 20 $\%$
  - ➢ Energy dependent tilt factor 5 $\%$
  - ➢ Zenith angle dependence uncertainty 5 $\%$
  - ➢ Overall cross section uncertainty 10 $\%$
  - ➢ Overall systematic uncertainty 5 $\%$
- $\mathrm{N_{expt} = N_{NH}, N_{theory} = N_{IH}}$

Statistical analysis

- Parameters uncertainties are taken care of by Marginalization
- Marginalization in $\mathrm{N}_{\mathrm{theory}}$, Δ_{21}, θ_{12} fixed, other parameters varied in the range:
- $\Delta m^2_{31} = 2.35 \times 10^{-3} - 2.6 \times 10^{-3}$ eV^2
- $\sin^2\theta_{23} = 0.4 - 0.6$
- $\sin^2\theta_{13} = 0.0 - 0.05$ (3σ bound from CHOOZ is $<$ 0.044)
- "True" values of parameters fixed in $\mathrm{N}_{\mathrm{expt}}$

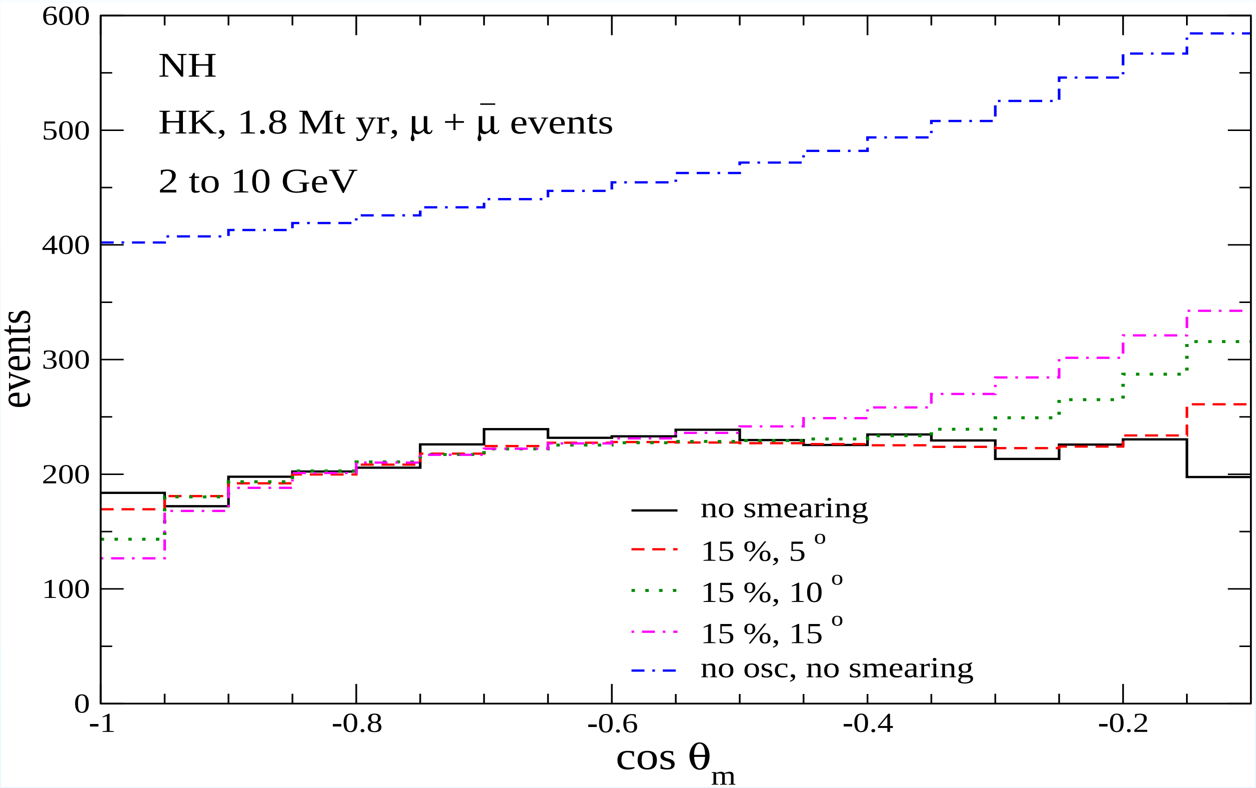
Effect of δ_{CP} on χ^2



- Effect of δ_{CP} on Muon χ^2 insignificant

  R. Gandhi, P. Ghoshal, S.G., P. Mehta, S. Shalgar, S. Umashanakar, PRD, 2007

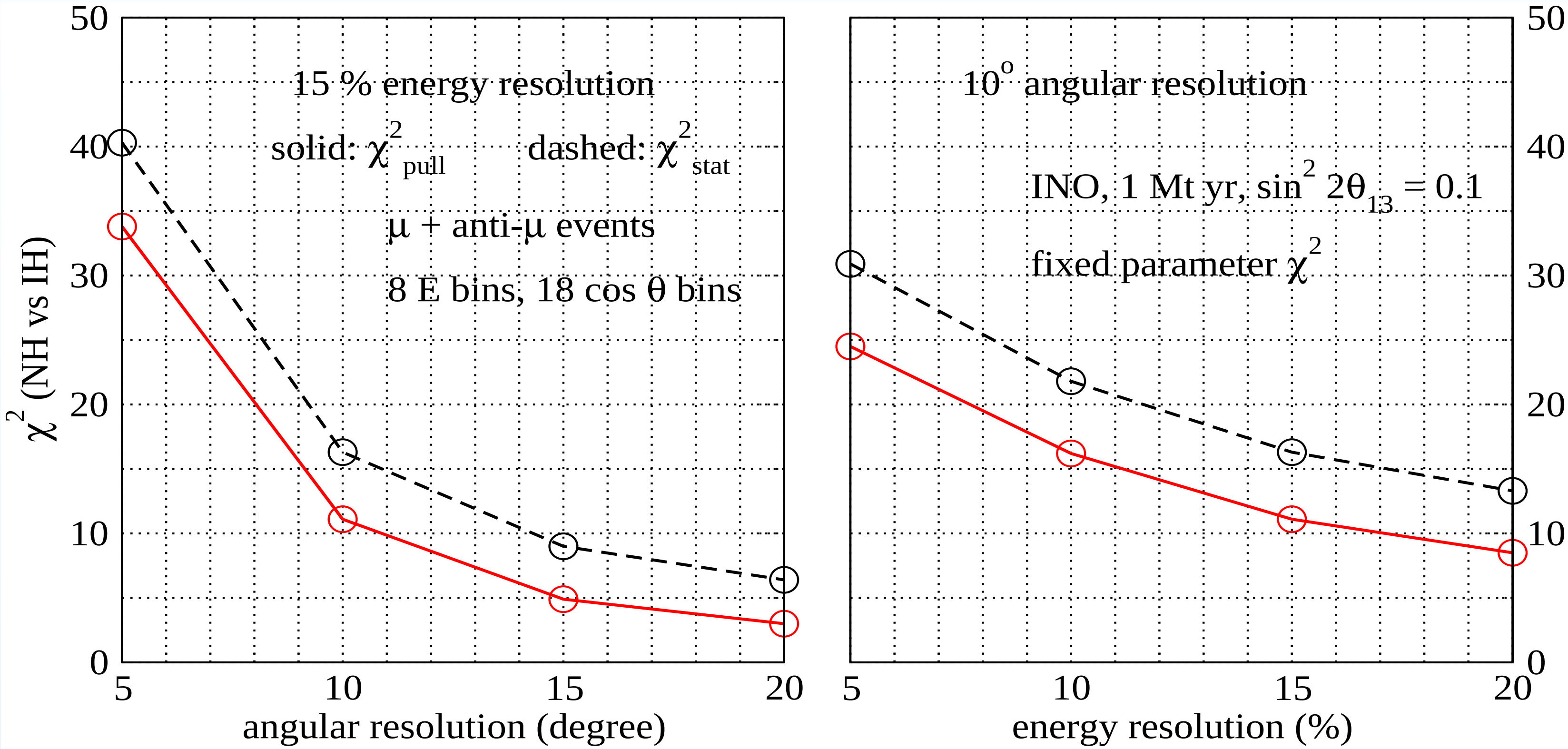
Effect of Smearing on Atmospheric ν Events



- With increased width of smearing the event distribution tends to no oscillation distribution

Effect of Smearing on χ^2

- Effect of smearing on muon-χ^2 in INO



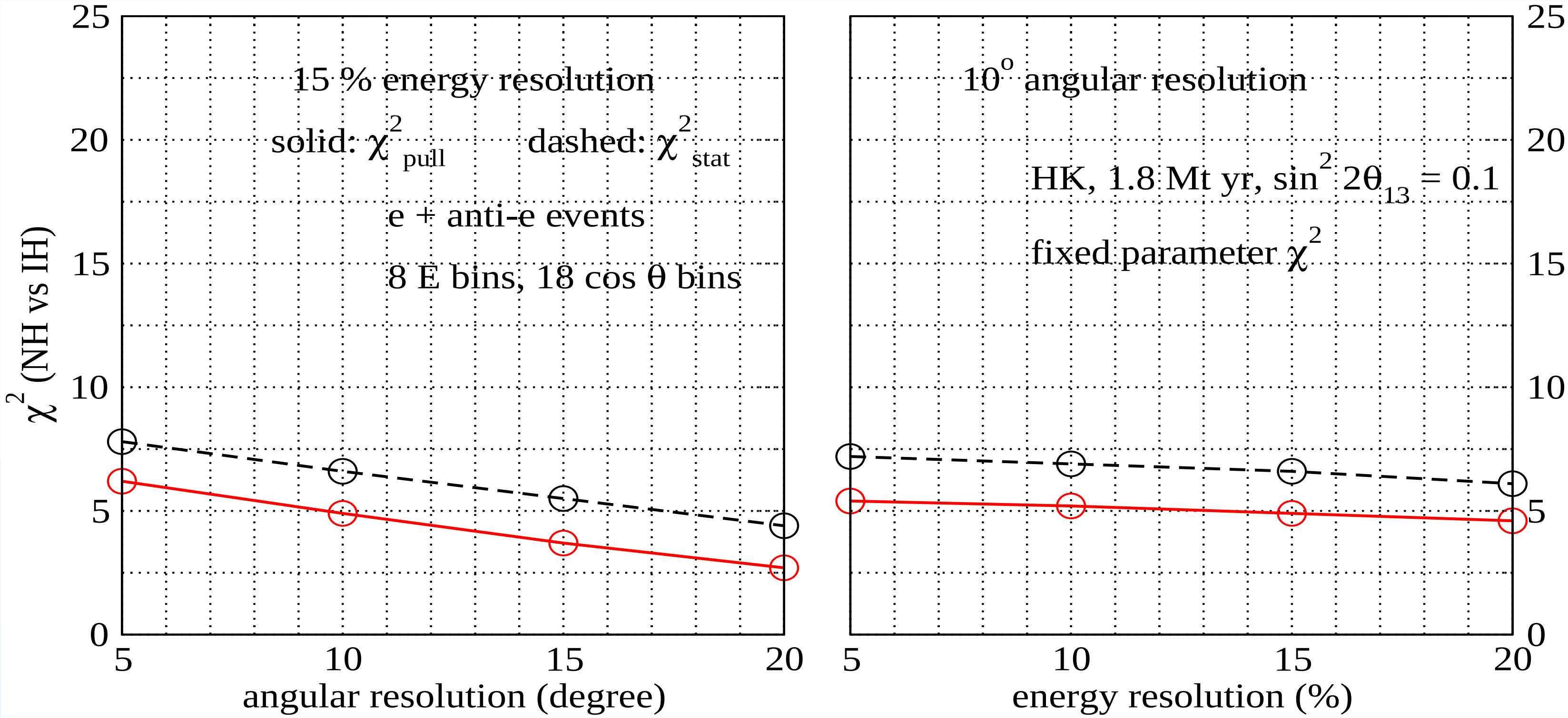
- With increased energy or angular smearing the χ^2 for muon like events decrease.

R. Gandhi, P. Ghoshal, S.G., P. Mehta, S. Shalgar, S. Umashanakar, PRD, 2007

Also, T. Schwetz and S.T. Petcov, Nucl. Phys. B, 2006

Effect of Smearing on χ^2

- Effect of smearing on electron-χ^2

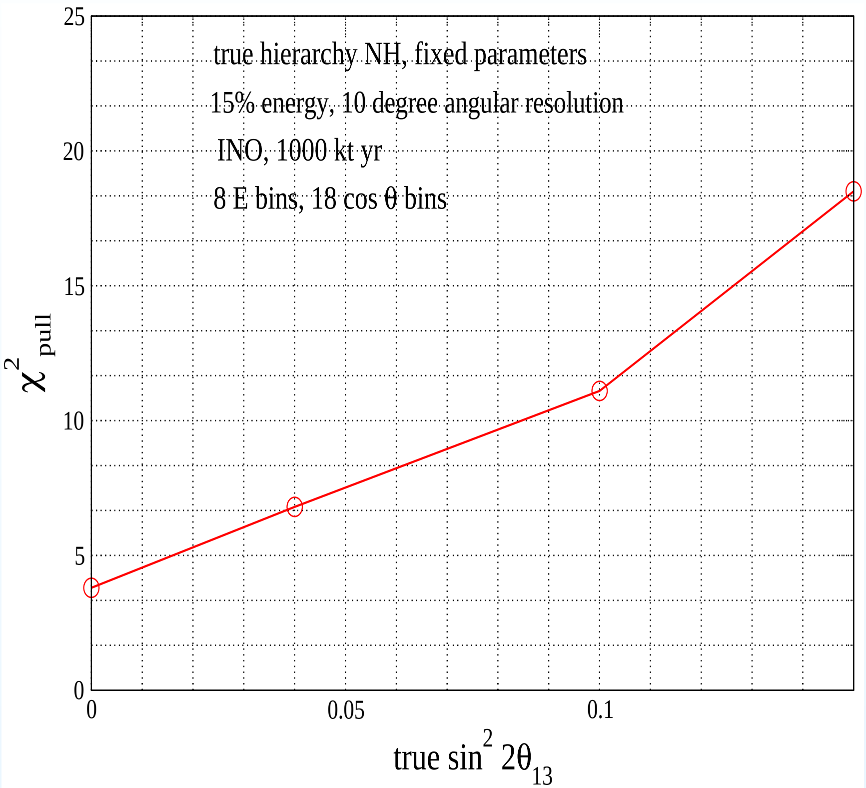


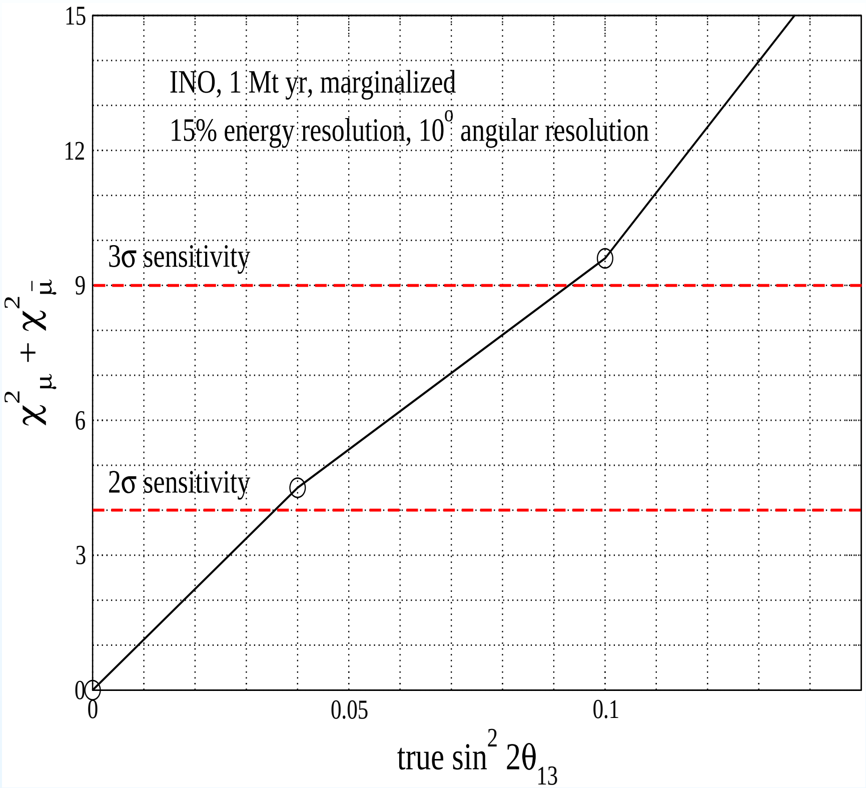
- The effect of smearing is less than that for muon events because the electron survival probability varies less rapidly with energy and zenith angle.

R. Gandhi, P. Ghoshal, S.G., P. Mehta, S. Shalgar, S. Umashanakar, PRD, 2007

Also, T. Schwetz and S.T. Petcov, Nucl. Phys. B, 2006

Results





- Hierarchy Sensitvity reduces with marginalization

R. Gandhi, P. Ghoshal, S.G., P. Mehta, S. Shalgar, S. Umashanakar, PRD, 2007

T. Schwetz and S.T. Petcov, Nucl. Phys. B, 2006

A. Samanta, 2006

D. Indumathi and M.V.N. Murhty, PRD, 2005

Hierarch Sensitivity: comparative study

- INO: 1 Mtyear (100 kT $\times$ 10 years)
  $\chi^2 = \chi^2_\mu + \chi^2_{\bar{\mu}}$
- HyperKamiokande : 1.8 Mtyear (544 kT $\times$ 3.3 years)
  $\chi^2 = \chi^2_{\mu+\bar{\mu}} + \chi^2_{e+\bar{e}}$
- LiqAr : 1 Mtyear (100 kT $\times$ 10 years)
  $\chi^2 = \chi^2_\mu + \chi^2_{\bar{\mu}} + (\chi^2_e + \chi^2_{\bar{e}})_{1-5GeV} + (\chi^2_{e+\bar{e}})_{5-10GeV}$

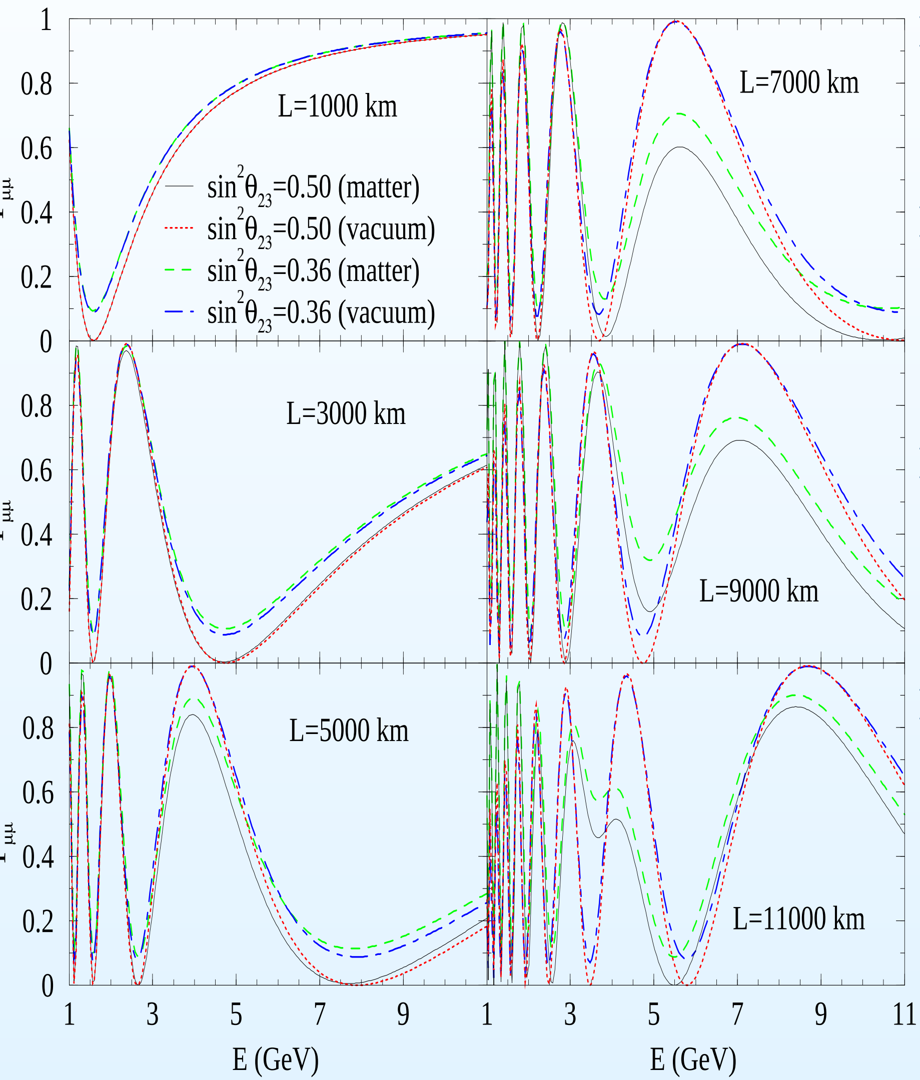
$\sin^2 2\theta_{13}$	$HK\chi^2$	$INO\chi^2$	LiqAr χ^2
0.0	0.0	0.0	0.0
0.04	3.6	4.5	13.8
0.1	5.9	9.6	27.5
0.15	7.1	16.9	

- LiqAr type detector has better energy smearing and partial charge identification of electrons (R. Gandhi et al. 2007,2008)

Deviation of $\sin^2\theta_{23}$ from maximal value

- $D \equiv 1/2 - \sin^2\theta_{23}$
- |D| gives the deviation of $\sin^2\theta_{23}$
- sgn(D) gives the octant of $\sin^2\theta_{23}$
- Current 3σ limits:
  - $|D| < 0.16$ at 3σ from the SK data
  - No robust information on sgn(D)

Can Earth matter effects determine $|D|$?



$$P_{\mu\mu}^{m} = 1 - P_{\mu\mu}^{m\ 1} - P_{\mu\mu}^{m\ 2} - P_{\mu\mu}^{m\ 3}$$

$$P_{\mu\mu}^{m\ 1} = c_{13}^{2\ m}\sin^2 2\theta_{23}\sin^2\left[1.27(\Delta m_{31}^2 + A + \Delta_{31}^m)L/2\right.$$

$$P_{\mu\mu}^{m\ 2} = s_{13}^{2\ m}\sin^2 2\theta_{23}\sin^2\left[1.27(\Delta m_{31}^2 + A - \Delta_{31}^m)L/2\right.$$

$$P_{\mu\mu}^{m\ 3} = \sin^4\theta_{23}\sin^2 2\theta_{13}^m\sin^2(1.27\Delta_{31}^m L/E)$$

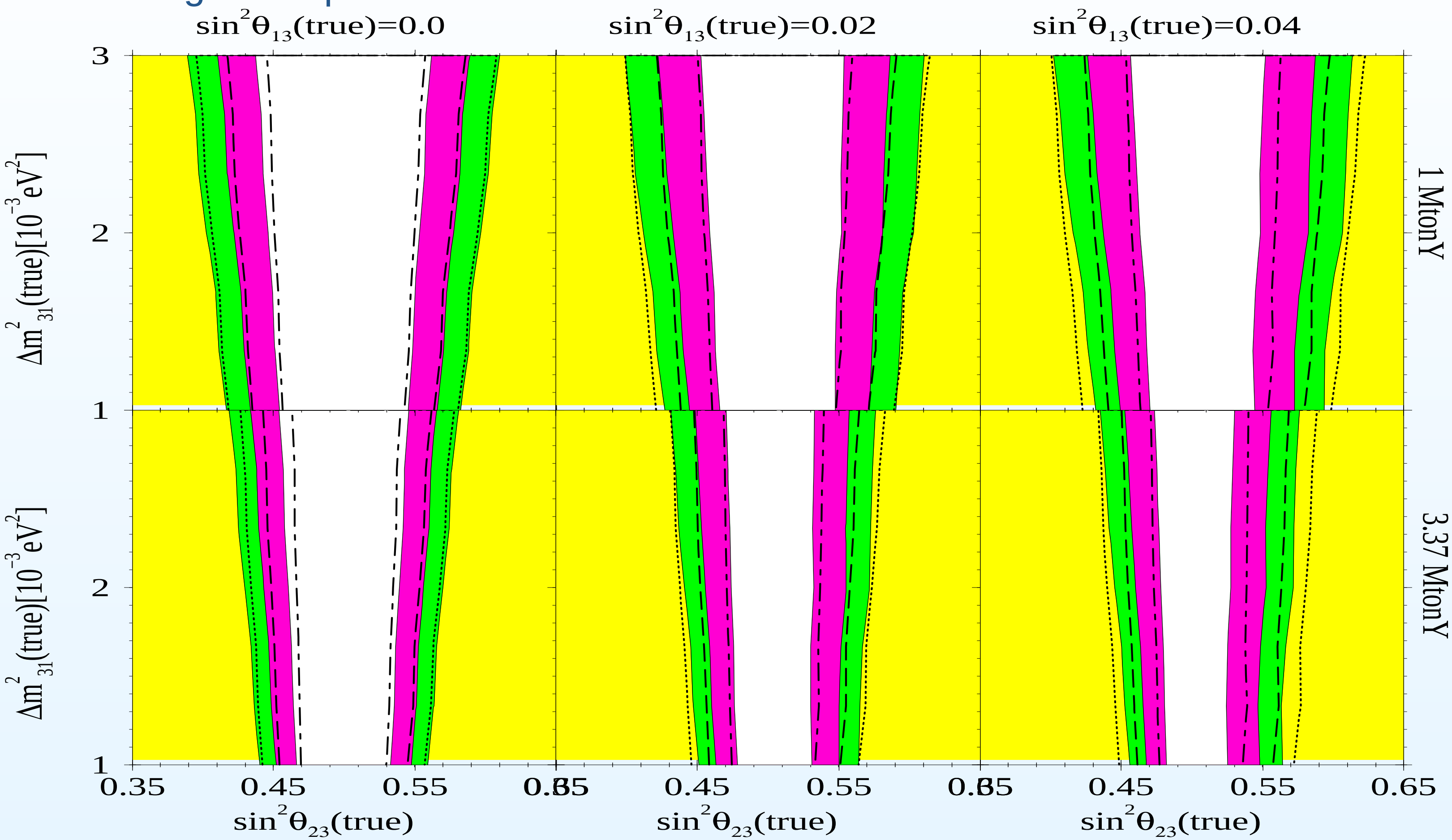
- Dependence on θ_{23} in the form $\sin^4\theta_{23}$
- Octant sensitivity ?

S.Choubey. and P. Roy hep-ph/0509197

Also Indumathi et al. hep-ph/0603264

Can Earth matter effects determine $|D|$?

- Using atmospheric neutrinos in INO



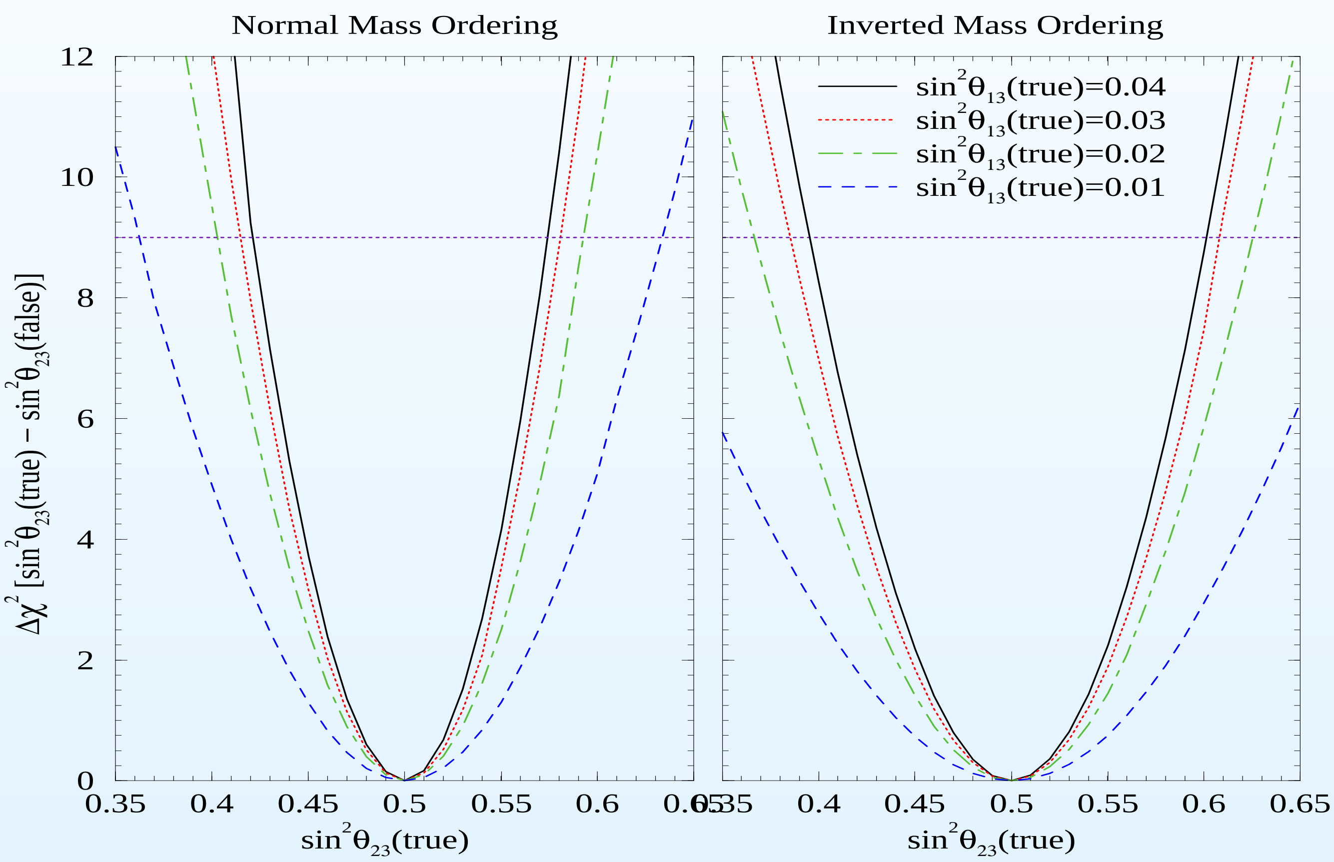
- $|D|$ can be measured to $\sim 17\%(20\%)$ at 3σ for $s^2_{13} = 0.04(0.00)$ with 1 MtonY exposure and 50% detector efficiency

S.Choubey. and P. Roy hep-ph/0509197

Resolving the octant ambiguity in INO

- Using atmospheric neutrinos in INO
- For every non-maximal $\sin^2\theta_{23}$(true) there exists a $\sin^2\theta_{23}$(false)

$$\sin^2\theta_{23}\text{(false)} = 1 - \sin^2\theta_{23}\text{(true)}$$



S.Choubey. and P. Roy hep-ph/0509197

Comparing the Octant Sensitivity of Experiments

- Long baseline experiments
  No octant sensitivity
  - LBL+atmospheric — Huber et al hep-ph/0501037
  - LBL accelerator + reactor — Minakata et al hep-ph/0601258
- Atmospheric neutrinos in water Cerenkov detectors
  $\sin^2\theta_{23}$(false) can be excluded at 3σ if:

$$\sin^2\theta_{23}(\text{true}) < 0.36 \text{ or } > 0.62$$

Gonzalez-Garcia et al, hep-ph/0408170

- Atmospheric neutrinos in large magnetized iron detectors
  $\sin^2\theta_{23}$(false) can be excluded at 3σ if:

$$\sin^2\theta_{23}(\text{true}) < 0.36 \text{ or } > 0.63 \text{ for } \sin^2\theta_{13}(\text{true}) = 0.01,$$

$$\sin^2\theta_{23}(\text{true}) < 0.40 \text{ or } > 0.59 \text{ for } \sin^2\theta_{13}(\text{true}) = 0.02,$$

$$\sin^2\theta_{23}(\text{true}) < 0.41 \text{ or } > 0.58 \text{ for } \sin^2\theta_{13}(\text{true}) = 0.03,$$

$$\sin^2\theta_{23}(\text{true}) < 0.42 \text{ or } > 0.57 \text{ for } \sin^2\theta_{13}(\text{true}) = 0.04.$$

S.Choubey. and P. Roy hep-ph/0509197

CPT and Lorentz Violation

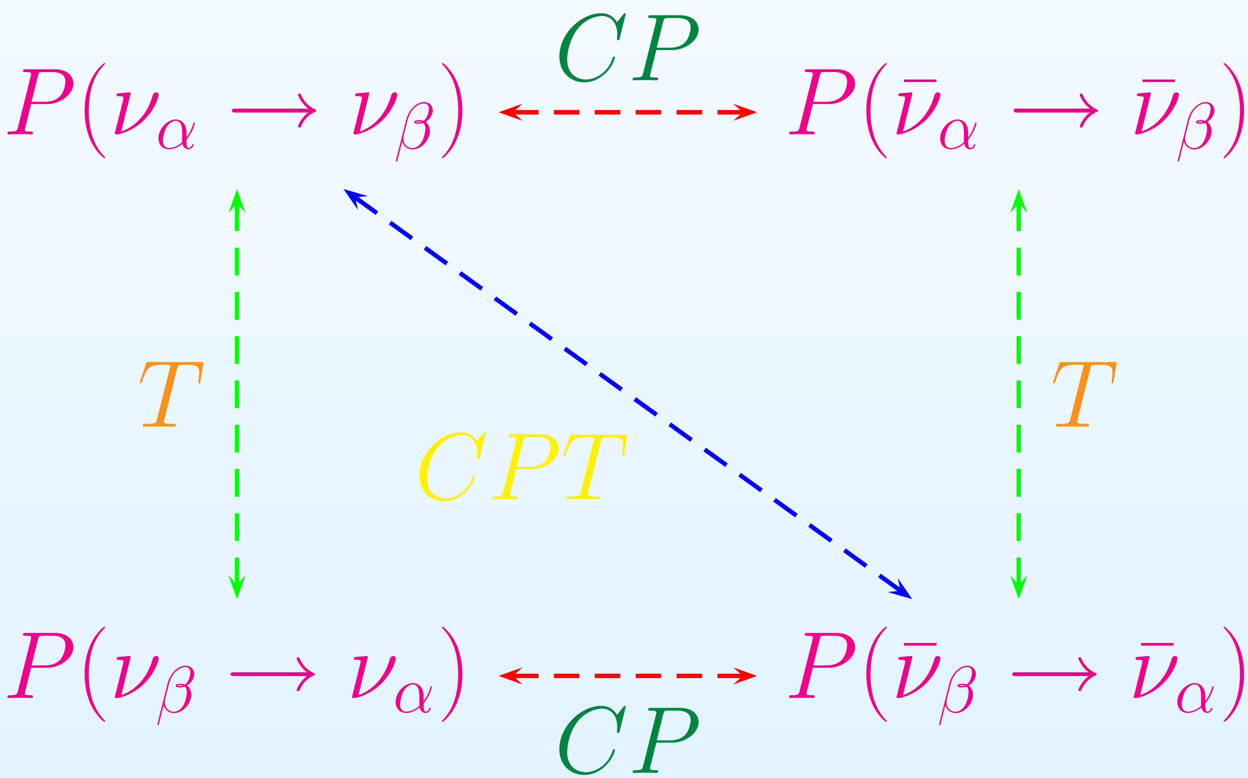
- CPT Invariance and Lorentz symmetry are pillars of modern physics. Tests of fundamental principles of invariance are important due to the far-reaching consequences of their violations.
- String or other unified theories may INDUCE small violations of CPT and Lorentz symmetry into the SM at low energies naturally, which can be tested at levels reachable by high precision experiments.

D. Colladay and V. A. Kostelecky, PRD 55, 6760 (1997); PRD 58, 116002 (1998)

S. R. Coleman and S. L. Glashow, PRD 59, 116008 (1999)

CP, T & CPT in ν oscillations

ν oscillations are sensitive to violation of Discrete symmetries : CP, T and CPT.



Violations of discrete symmetries . . .

- If CP is violated then

$$\mathrm{P}(\nu_\alpha \to \nu_\beta) \neq \mathrm{P}(\bar{\nu}_\alpha \to \bar{\nu}_\beta), \qquad \beta \neq \alpha$$

Violations of discrete symmetries . . .

- If CP is violated then

$$P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta), \qquad \beta \neq \alpha$$

- If T is violated then

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Violations of discrete symmetries . . .

- If CP is violated then

$$P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta), \qquad \beta \neq \alpha$$

- If T is violated then

$$P(\nu_\alpha \to \nu_\beta) \neq P(\nu_\beta \to \nu_\alpha), \qquad \beta \neq \alpha$$

- If CPT is violated then either

$$P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\beta \to \bar{\nu}_\alpha), \qquad \beta \neq \alpha$$

or,

$$P(\nu_\alpha \to \nu_\alpha) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\alpha)$$

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- Also, MATTER EFFECTS ➢ apparent (extrinsic) CP & CPT violation even if mass matrix is CP conserving

Atm ν, INO and CPT violation

- CPT violating term b gives Hamiltonian of the form

$$A = \frac{m^2}{2p} + b$$

which gives 2-flavour vacuum survival probability

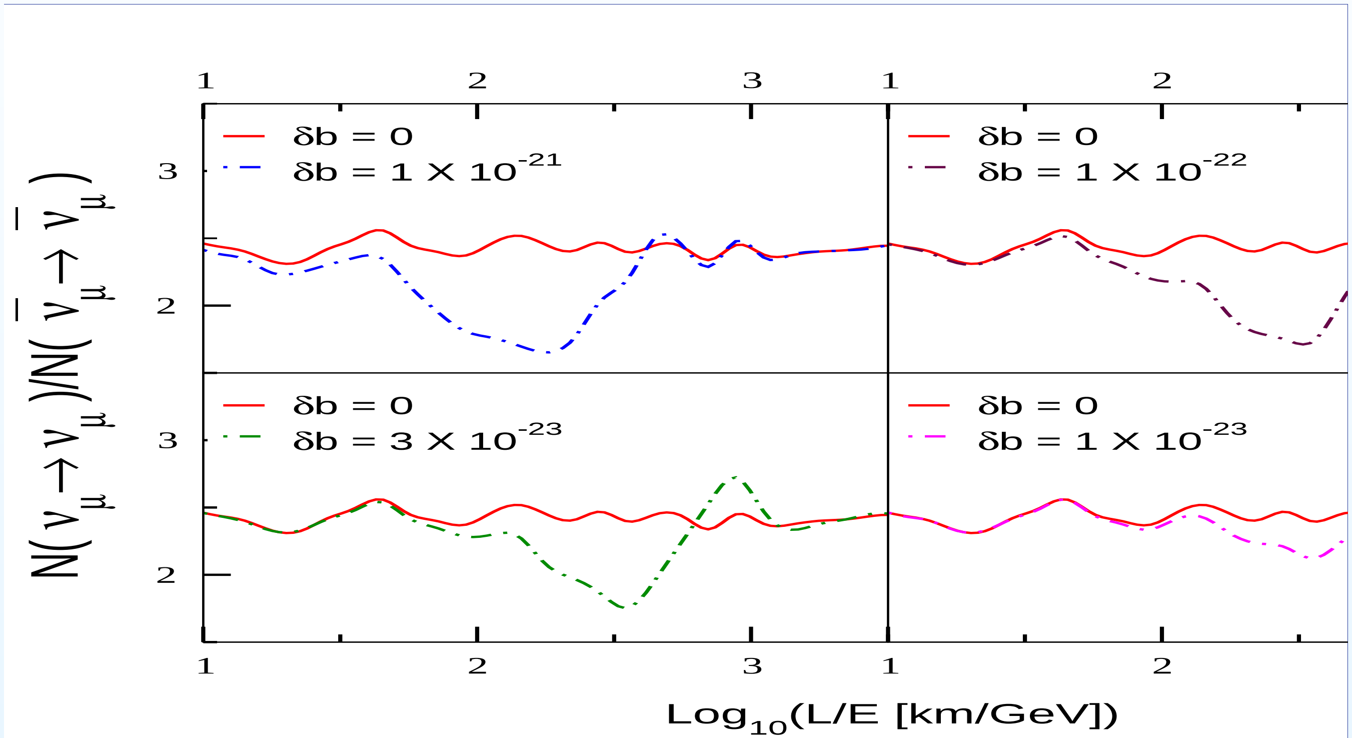
$$P_{\alpha\alpha}(L) = 1 - \sin^2 2\theta \sin^2[(\frac{\delta m^2}{4E} + \frac{\delta b}{2})L]$$

where $\alpha = \mu, \tau$ & $\delta m^2, \delta b$ are eigenvalue differences.

- For anti-neutrinos, $b \longrightarrow -b$. Hence

$$\Delta P^{CPT}_{\alpha\alpha} = -\sin^2 2\theta \sin[\frac{\delta m^2 L}{2E}] \sin(\delta b L)$$

Atm ν, INO and CPT violation



Gandhi et al., PLB, 2004

Detector and Physics Simulation

- Nuance Event Generator
  - Generates of atmospheric neutrino events inside the INO detector
- GEANT Monte Carlo Package
  - Simulates the detector response for the neutrino events
- Event Reconstruction
  - Fits the raw data to extract neurtrino energy and direction
- Physics Performance
  - Analysis of reconstructed events to extract physics.