Deep Inelastic Charged Current Neutrino Nucleus Reaction

Mohammad Sajjad Athar

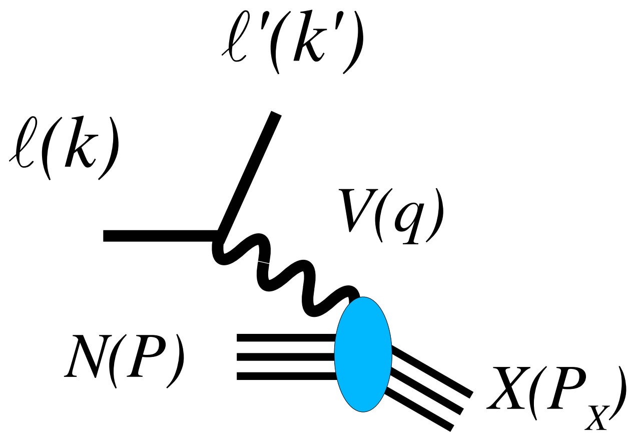
Department of Physics

Aligarh Muslim University, Aligarh

In Collaboration with

I. Ruiz Simo

M. J. Vicente Vacas



For the basic lepton–nucleon inelastic scattering process,

$$\ell(k) + N(P) \rightarrow \ell'(k') + X(P_X)$$

the matrix element is given by

$$-i\mathcal{M} = \bar{u}(k')\left(-\frac{ie}{2\sqrt{2}sin\theta_W}\right)\gamma^\mu(1-\gamma_5)u(k) \\ -i\frac{\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2}\right)}{q^2 - M_W^2} < X|J^\nu|N > \left(-\frac{ie}{2\sqrt{2}sin\theta_W}\right)$$

The general expression of the cross section for the reaction $\nu_l + N \rightarrow l^- + X$

$$\sigma = \frac{1}{v_{rel}} \frac{2m_\nu}{2E_\nu} \frac{2M}{2E(\mathbf{p})} \int \frac{d\mathbf{k}'}{(2\pi)^3} \times$$
$$\frac{2m_l}{2E_l} \prod_{i=1}^{N} \int \frac{d\mathbf{p}'_\mathbf{i}}{(2\pi)^3} \prod_{l\epsilon f} \frac{2M_l}{2E'_l} \times$$
$$\prod_{j\epsilon b} \frac{1}{2E_j} \bar{\Sigma}\Sigma |\mathcal{M}|^2 (2\pi)^4 \delta^4(p + k - k' - \sum_{i=1}^{N} p'_i)$$

$$|\mathcal{M}|^2 = \frac{G_F^2}{2} \left(\frac{M_W^2}{q^2 - M_W^2} \right)^2 |\bar{u}(k')\gamma^\mu (1-\gamma_5) u(k)|^2$$
$$< X|J_\mu|N >< X|J_\nu|N >^*$$

$$|\bar{u}(k')\gamma^\mu(1-\gamma_5)u(k)|^2 = 8\left(L_S^{\mu\nu} + L_A^{\mu\nu}\right) \frac{1}{2m_l} \frac{1}{2m_\nu}$$

We may take one of the 2π from the expression (csection) (denominator) of σ and define the hadronic tensor as

$$W_{\mu\nu} = \frac{1}{2\pi}\sum_{s_p}\sum_{X}\sum_{s_i}\prod_{i=1}^{N}\int\frac{d\mathbf{p_i'}}{(2\pi)^3} \prod_{l\epsilon f}\frac{2m_l}{2E_l'}\prod_{j\epsilon b}\frac{1}{2E_j} < X|J_\mu|N >< X|J_\nu|N >^* (2\pi)^4\delta^4(p+q-p')$$

The differential cross section in the rest frame of the nucleon is expressed as

$$\frac{d^2\sigma_{\nu\bar{\nu}}^N}{d\Omega' dE'} = \frac{{G_F}^2}{(2\pi)^2}\frac{|\mathbf{k}'|}{|\mathbf{k}|}\left(\frac{m_W^2}{q^2-m_W^2}\right)^2 L_{\nu\bar{\nu}}^{\alpha\beta}W_{\alpha\beta}^N$$

The most general form of the hadronic tensor $W^N_{\alpha\beta}$ is expressed as:

$$W^N_{\alpha\beta} = \left(\frac{q_\alpha q_\beta}{q^2} - g_{\alpha\beta}\right) W_1^{\nu(\bar{\nu})} + \frac{1}{M^2}\left(p_\alpha - \frac{p.q}{q^2} q_\alpha\right) \left(p_\beta - \frac{p.q}{q^2} q_\beta\right) W_2^{\nu(\bar{\nu})} - \frac{i}{2M^2}\epsilon_{\alpha\beta\rho\sigma} p^\rho q^\sigma W_3^{\nu(\bar{\nu})} + \frac{1}{M^2} q_\alpha q_\beta W_4^{\nu(\bar{\nu})} + \frac{1}{M^2}(p_\alpha q_\beta + q_\alpha p_\beta) W_5^{\nu(\bar{\nu})} + \frac{i}{M^2}(p_\alpha q_\beta - q_\alpha p_\beta) W_6^{\nu(\bar{\nu})}$$

W^N_i: structure functions which depend on the scalars q^2 and $p.q$

Bjorken variables x and y are defined as

$$x = \frac{Q^2}{2M\nu}, \quad y = \frac{\nu}{E_\nu}, \quad Q^2 = -q^2, \quad \nu = \frac{p.q}{M}$$

The differential scattering cross section (in the limit of lepton mass $m_l \to 0$) as

$$\frac{d^2\sigma^{\nu(\bar{\nu})}}{dxdy} = \frac{{G_F}^2 M E_\nu}{\pi}\Big[xy^2 F_1^{\nu(\bar{\nu})}(x,Q^2) + (1 - y - \frac{xyM}{2E_\nu})F_2^{\nu(\bar{\nu})}(x,Q^2) \pm xy(1-y/2)F_3^{\nu(\bar{\nu})}(x,Q^2)\Big]$$

+ (-) sign stands for the neutrino (antineutrino)

$F_i^{\nu,\bar{\nu}}(x,Q^2)$ are dimensionless structure functions defined as

$$F_1^{\nu(\bar{\nu})}(x,Q^2) = MW_1^{\nu(\bar{\nu})}(\nu,Q^2)$$
$$F_2^{\nu(\bar{\nu})}(x,Q^2) = \nu W_2^{\nu(\bar{\nu})}(\nu,Q^2)$$
$$F_3^{\nu(\bar{\nu})}(x,Q^2) = \nu W_3^{\nu(\bar{\nu})}(\nu,Q^2)$$

• In the Bjorken limit ($Q^2 \to \infty$, $\nu \to \infty$, x finite),

$F_i^{\nu(\bar{\nu})}(x, Q^2)$ are independent of Q^2 and depend only on the single dimensionless variable x

• They satisfy Callan-Gross relation $2xF_1(x) = F_2(x)$.

The cross section is described in terms of two independent structure functions $F_2(x)$ and $F_3(x)$.

• Then in the quark parton model of deep inelastic scattering, these structure functions are determined in terms of parton distribution functions for quarks and antiquarks.

The structure functions $F_2(x)$ and $F_3(x)$ are given as:

$$F_2^{\nu p} = 2x[d(x)+s(x)+\bar{u}(x)+\bar{c}(x)+b(x)+\bar{t}(x)]$$

$$F_2^{\nu n} = 2x[u(x)+s(x)+\bar{d}(x)+\bar{c}(x)+b(x)+\bar{t}(x)]$$

$$F_2^{\bar{\nu} p} = 2x[u(x)+c(x)+\bar{d}(x)+\bar{s}(x)+t(x)+\bar{b}(x)]$$

$$F_2^{\bar{\nu} n} = 2x[d(x)+c(x)+\bar{u}(x)+\bar{s}(x)+t(x)+\bar{b}(x)]$$

$$F_3^{\nu p} = 2[d(x)+s(x)-\bar{u}(x)-\bar{c}(x)+b(x)-\bar{t}(x)]$$

$$F_3^{\nu n} = 2[u(x)+s(x)-\bar{d}(x)-\bar{c}(x)+b(x)-\bar{t}(x)]$$

$$F_3^{\bar{\nu} p} = 2[u(x)+c(x)-\bar{d}(x)-\bar{s}(x)+t(x)-\bar{b}(x)]$$

$$F_3^{\bar{\nu} n} = 2[d(x)+c(x)-\bar{u}(x)-\bar{s}(x)+t(x)-\bar{b}(x)]$$

Parton Distribution Functions

i. MSTW distributions

ii. CTEQ distributions

iii. GRV/GJR distributions

iv. ALEKHIN distributions

These structure functions satisfy some Sum Rules:

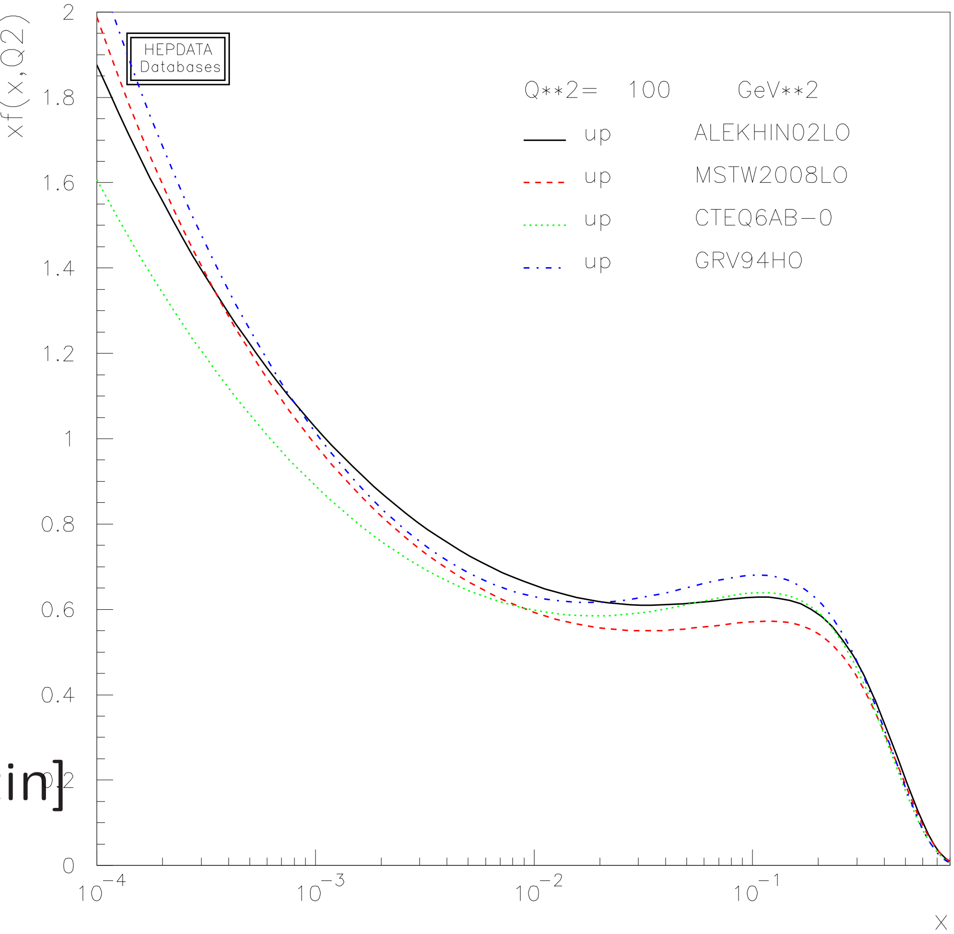
Adler Sum Rule:

$$\int_0^1 \left(F_2^{\nu n}(x) - F_2^{\nu p}(x)\right) \frac{dx}{x} = 2$$

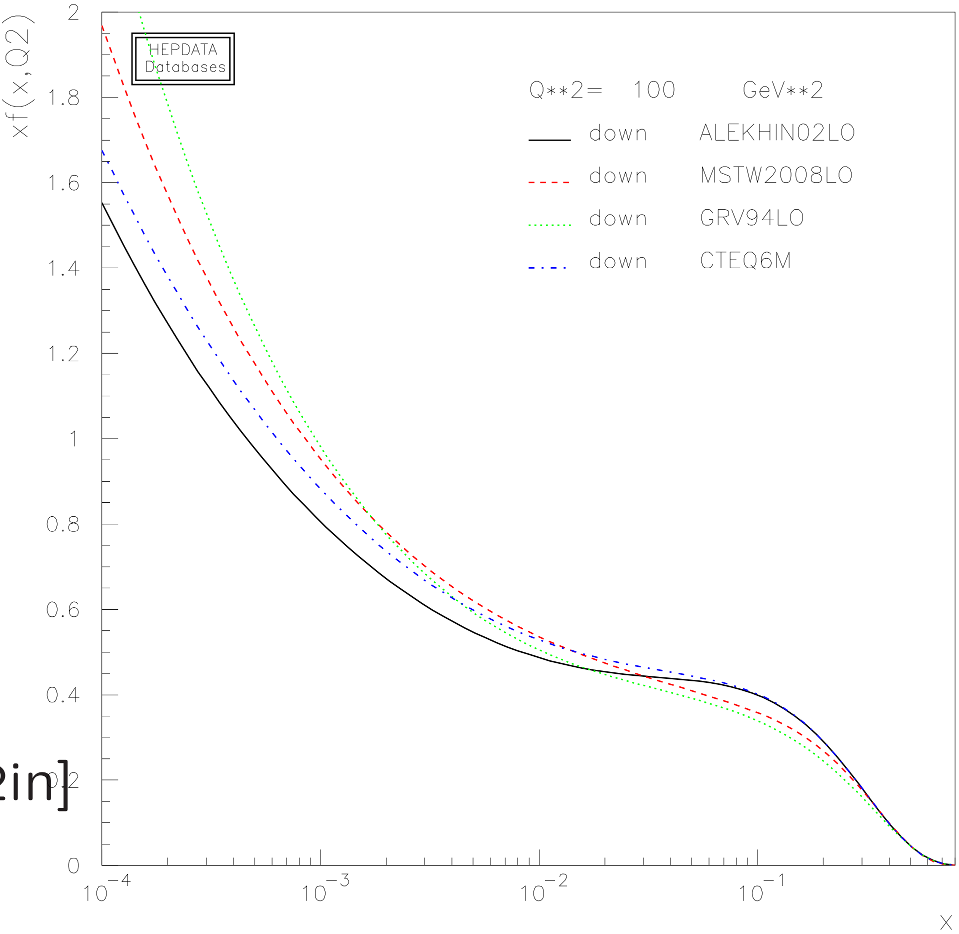
Gross-Llewellyn Smith Sum Rule:

$$\int_0^1 F_3^{\nu N}(x) dx = 3$$

[2in]

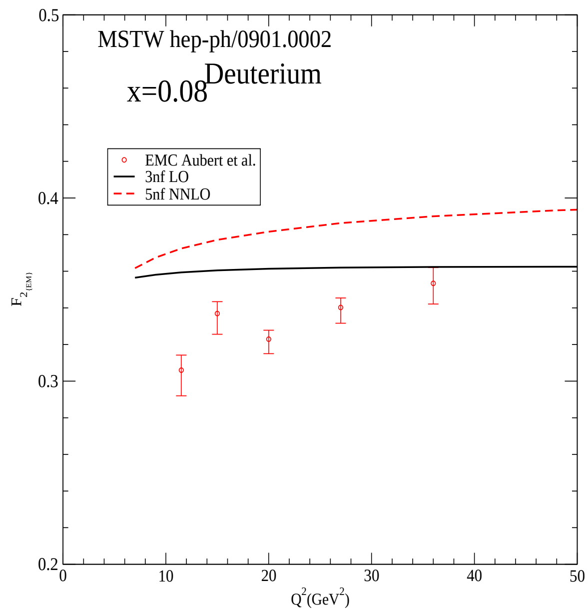


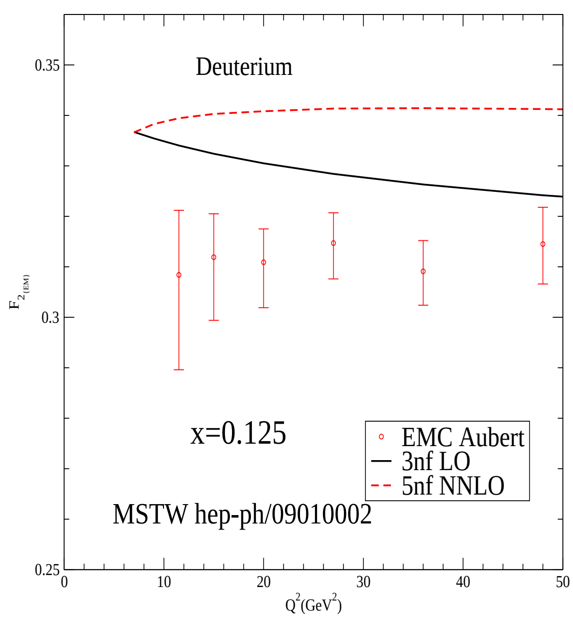
xf(x,Q2)
HEPDATA
Databases
Q**2= 100 GeV**2
up ALEKHIN02LO
up MSTW2008LO
up CTEQ6AB−0
up GRV94HO
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0
10^{-4}
10^{-3}
10^{-2}
10^{-1}
X

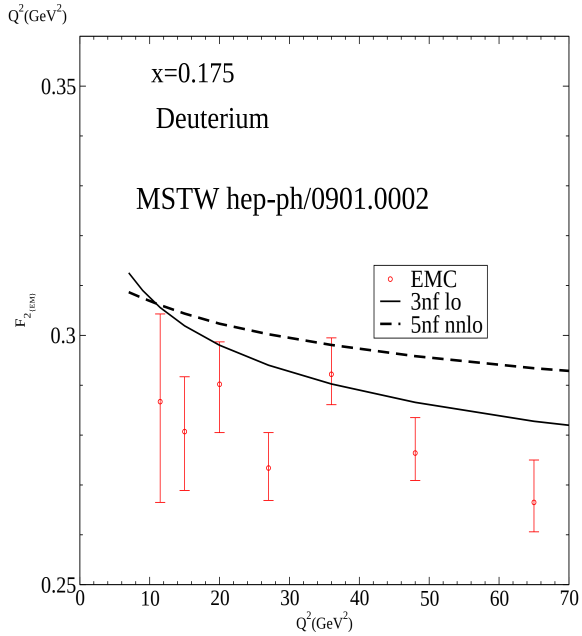

HEPDATA
Databases
Q**2= 100 GeV**2
down ALEKHIN02LO
down MSTW2008LO
down GRV94LO
down CTEQ6M
xf(x,Q2)
X

[2in]

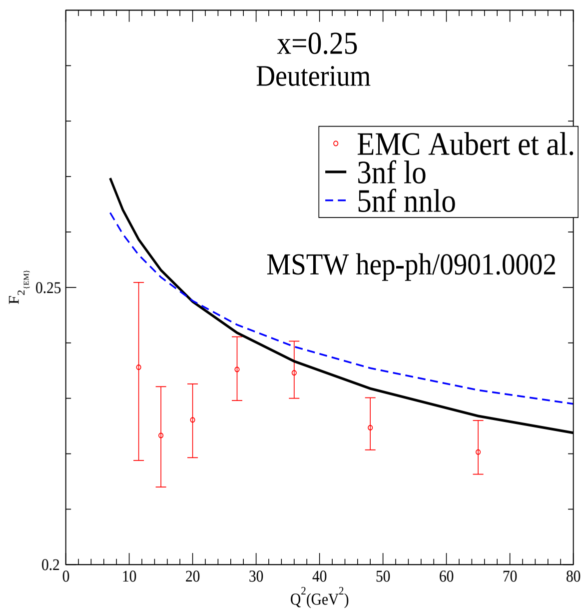
$F_{2_{EM}}$ in Deuterium Target

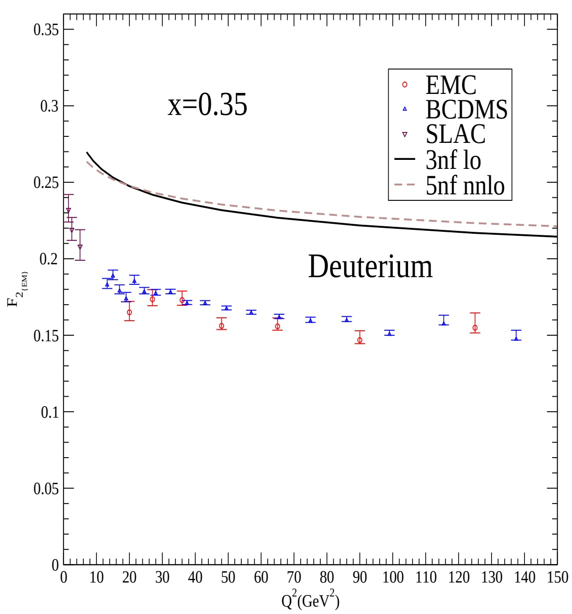


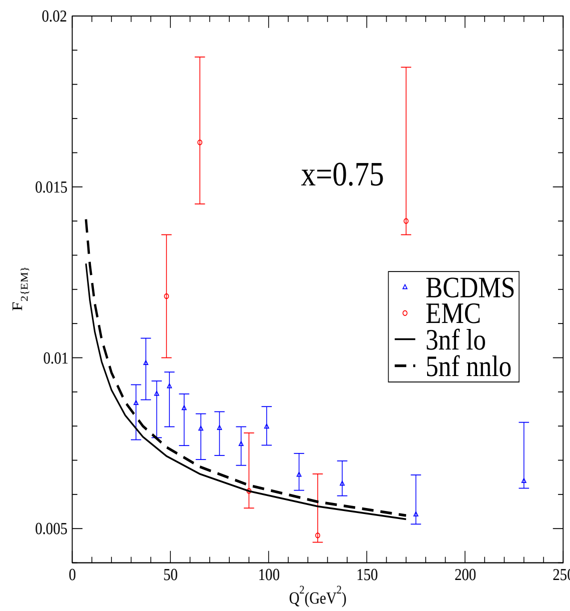


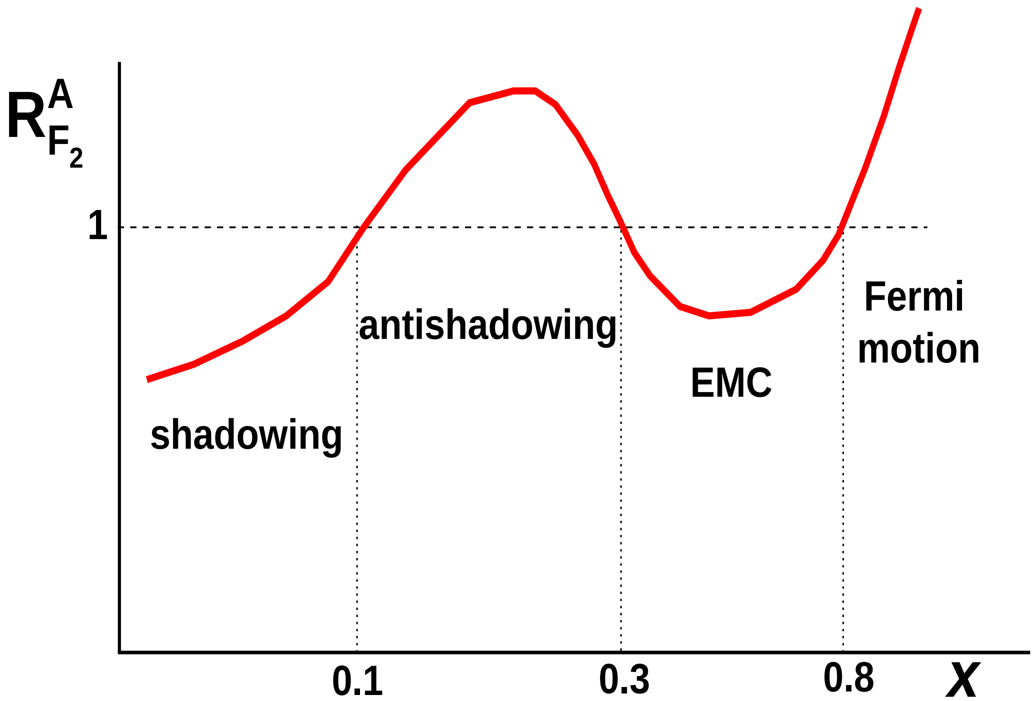


$F_{2_{EM}}$ in Deuterium Target









The fact that the nuclear structure functions in nuclei are different from the superposition of those of their constituent nucleons is a well known phenomenon.

$$R_{F_2} = \frac{F_2^A(x, Q^2)}{AF_2^{Nucleon}(x, Q^2)}$$

• I. Schienbein et al., Phys.Rev.D77:054013 (2008).

• M. Tzanov et al., Phys.Rev.D74:012008 (2006).

• M. Hirai et al., Phys.Rev.C76:065207 (2007).

• Kulagin and Petti PRD76 (2007) 094023

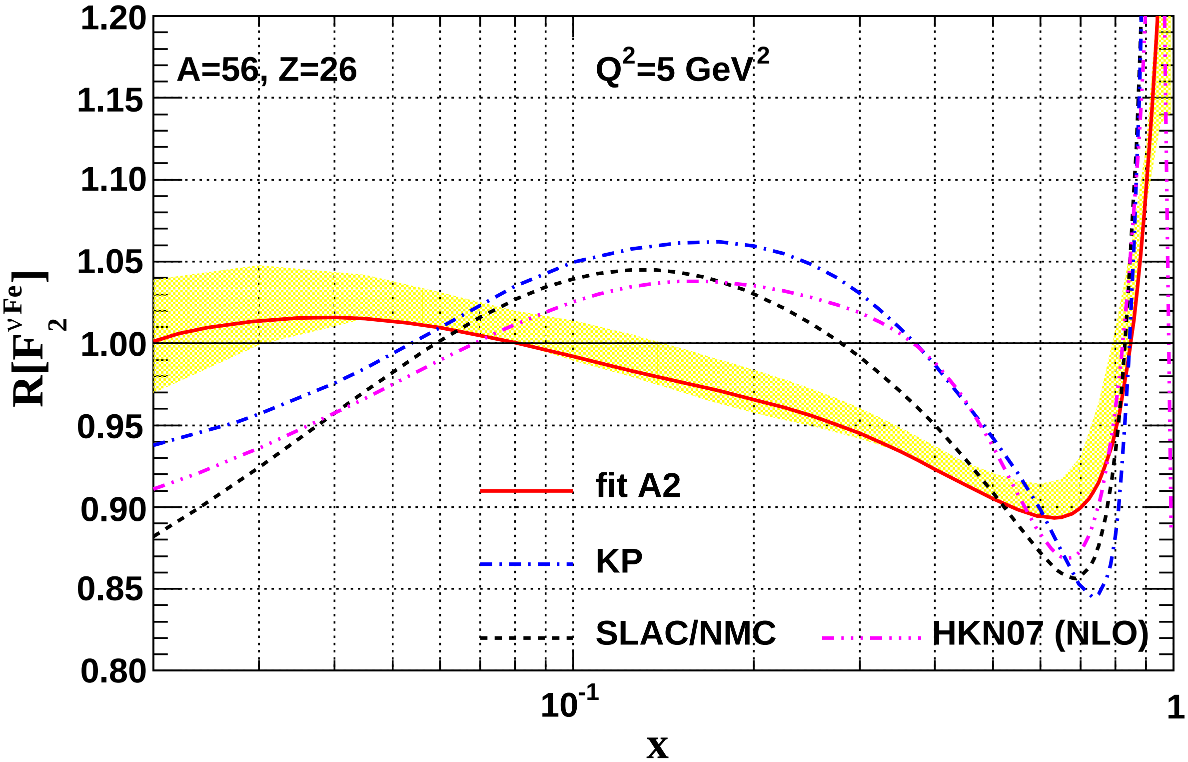
• Ankowski and Sobczyk PRC77 (2008) 044311

• Athar, Singh and Vicente-Vacas PLB 668(2008) 133

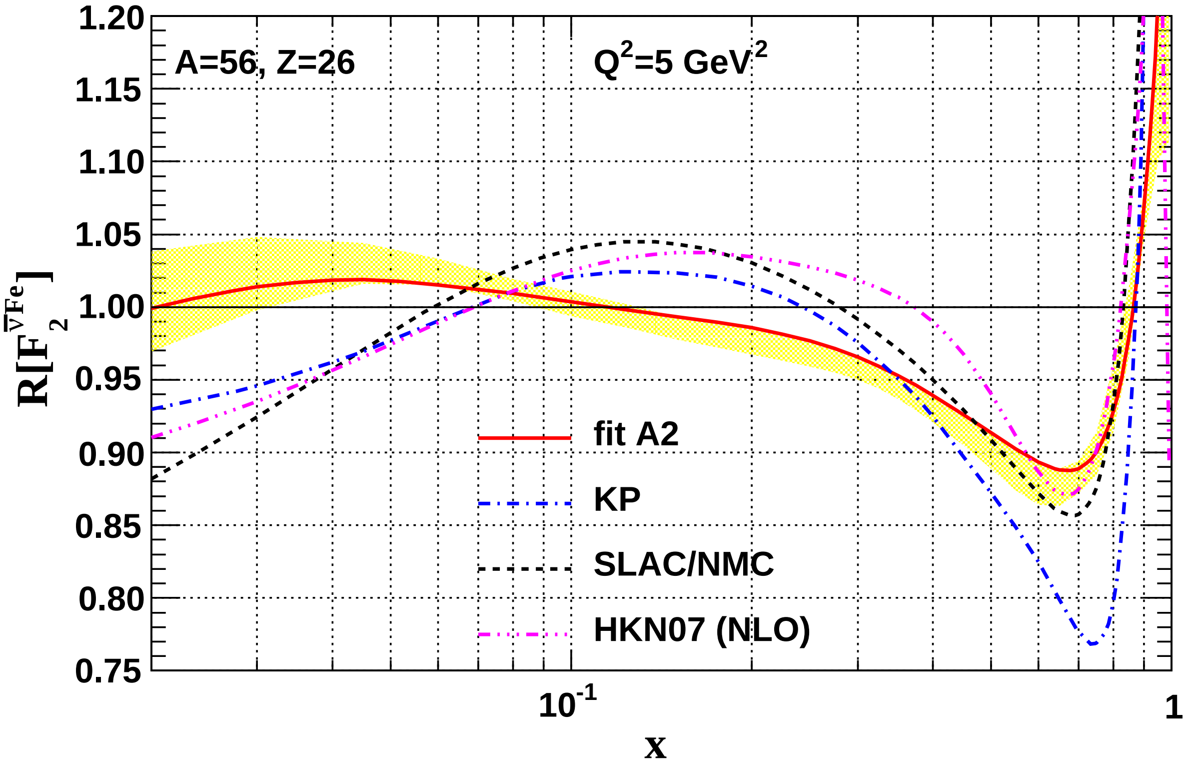
• M. Sajjad Athar, I. Ruiz Simo, S. K. Singh and M. J. Vicente Vacas, (in Preparation)

This work is based on the model developed by

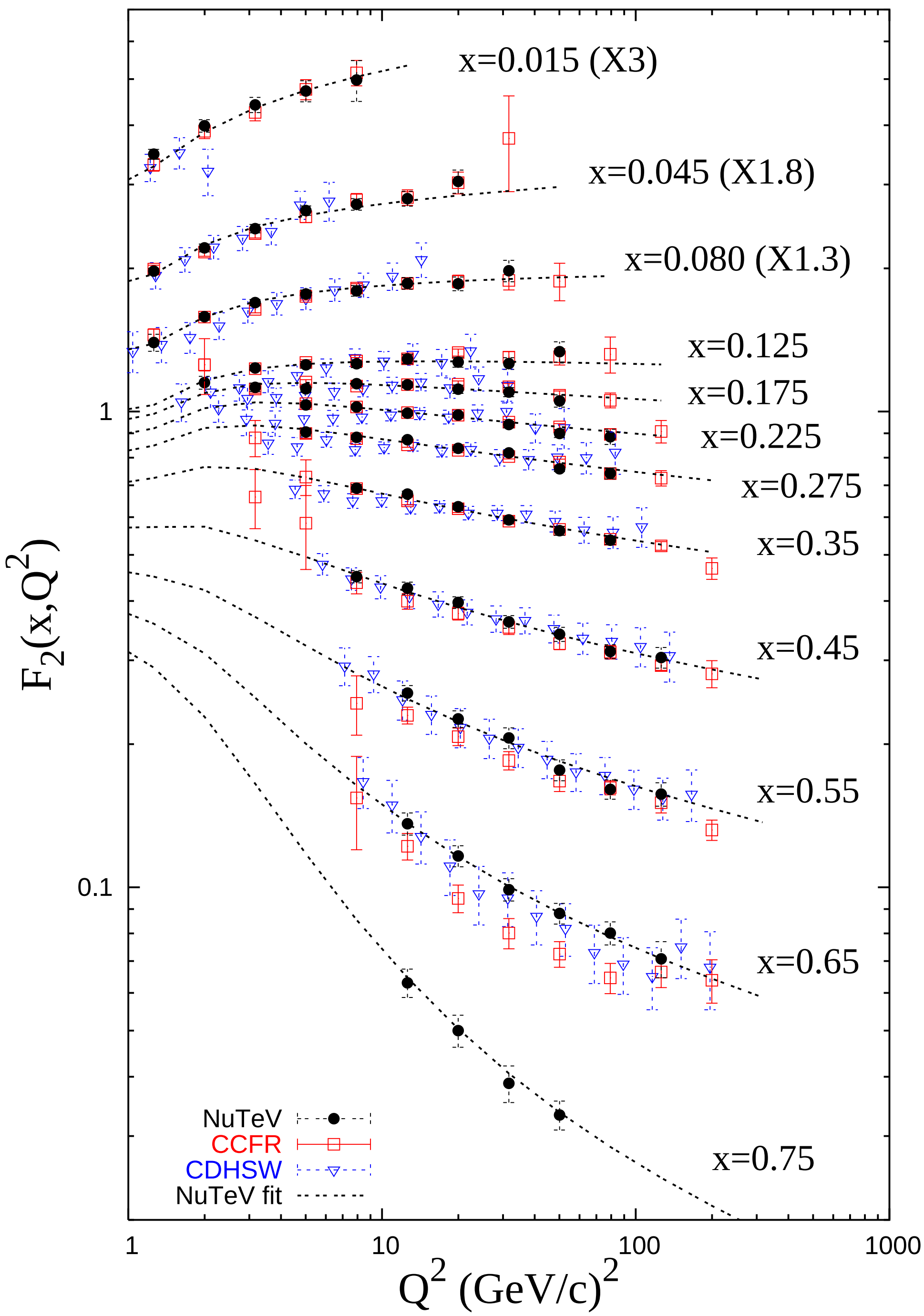
E. Marco, E. Oset and Fernandez de Cordoba, NPA 611 (1996) 484



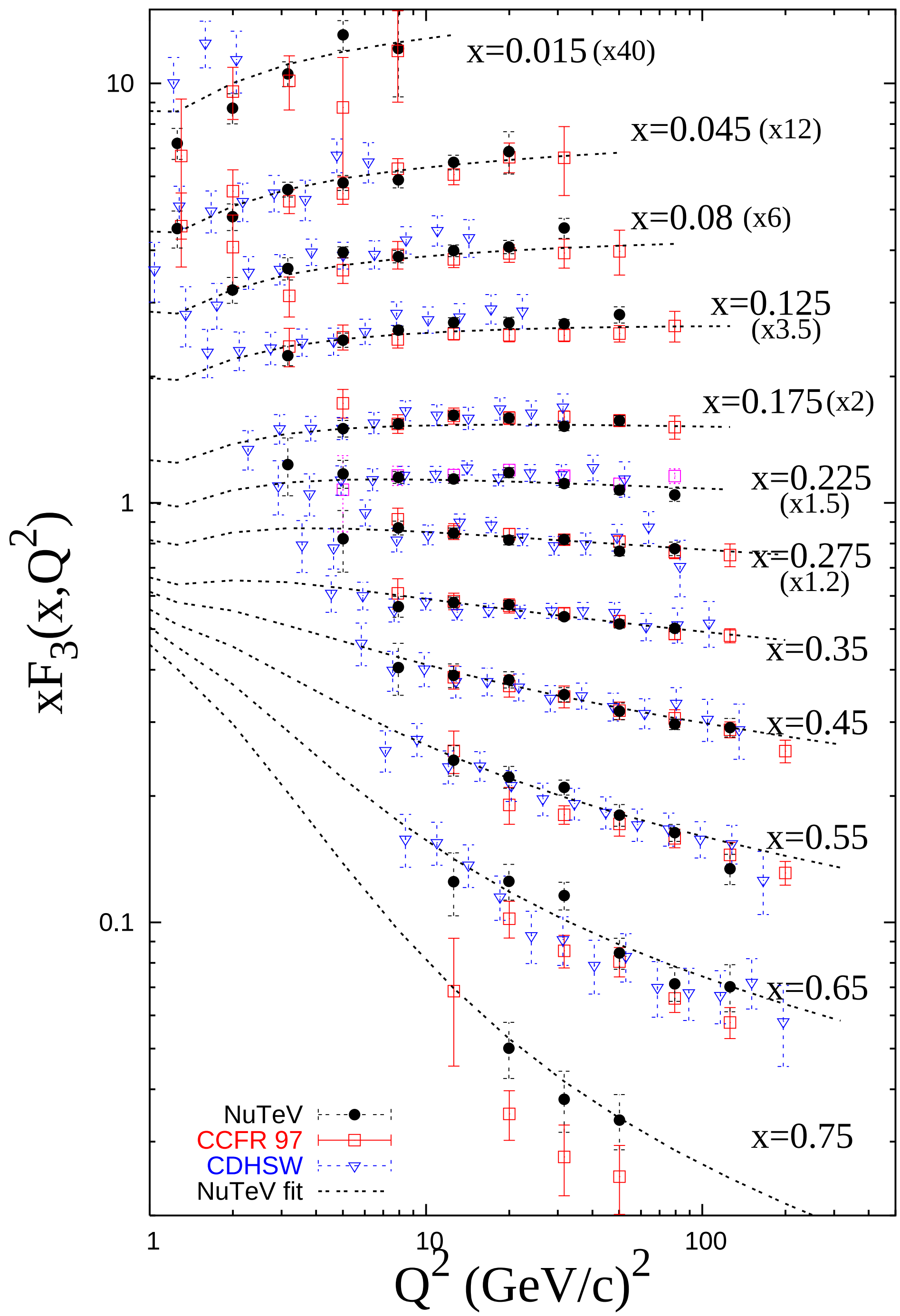
A=56, Z=26
Q^2=5 GeV^2
fit A2
KP
SLAC/NMC
HKN07 (NLO)
R[F_2^νFe]
x
1.20
1.15
1.10
1.05
1.00
0.95
0.90
0.85
0.80
10^-1
1



A=56, Z=26
Q^2=5 GeV^2
fit A2
KP
SLAC/NMC
HKN07 (NLO)
1.20
1.15
1.10
1.05
1.00
0.95
0.90
0.85
0.80
0.75
10^-1
1
x
R[F_2^ν̄Fe]



x=0.015 (X3)
x=0.045 (X1.8)
x=0.080 (X1.3)
x=0.125
x=0.175
x=0.225
x=0.275
x=0.35
x=0.45
x=0.55
x=0.65
x=0.75
NuTeV
CCFR
CDHSW
NuTeV fit
$F_2(x,Q^2)$
Q^2 (GeV/c)^2
1
0.1
1
10
100
1000



$xF_3(x,Q^2)$
Q^2 (GeV/c)^2
x=0.015 (x40)
x=0.045 (x12)
x=0.08 (x6)
x=0.125 (x3.5)
x=0.175 (x2)
x=0.225 (x1.5)
x=0.275 (x1.2)
x=0.35
x=0.45
x=0.55
x=0.65
x=0.75
NuTeV
CCFR 97
CDHSW
NuTeV fit
10
1
0.1
1
10
100

Nuclear effects in neutrino scattering

There are two main nuclear effects:

I. A kinematic effect which arises as the struck nucleon is not at rest but is moving with a Fermi momentum in the rest frame of the nucleus, leading to a Lorentz contraction of the incident flux.

II. The dynamic effects which arise due to Fermi motion, Pauli blocking and strong interaction of the initial nucleon in the nuclear medium.

The expression for the cross section in nuclear medium:

$$\frac{d^2\sigma^A_{\nu,\bar{\nu}}}{d\Omega' dE'} = \frac{G_F^2}{(2\pi)^2} \frac{|\mathbf{k}'|}{|\mathbf{k}|} \left(\frac{m_W^2}{q^2 - m_W^2}\right)^2 L^{\alpha\beta}_{\nu,\bar{\nu}} \, W^A_{\alpha\beta}$$

$W^A_{\alpha\beta}$: nuclear hadronic tensor defined in terms of nuclear hadronic structure functions $W_{iA}(x, Q^2)$

In the rest frame of the nucleus the expression of the differential cross section modifies from

$$\frac{d^2\sigma^N_{\nu,\bar\nu}}{d\Omega' dE'} = \frac{{G_F}^2}{(2\pi)^2}\frac{|\mathbf{k}'|}{|\mathbf{k}|}\left(\frac{m_W^2}{q^2-m_W^2}\right)^2 L^{\alpha\beta}_{\nu,\bar\nu} W^N_{\alpha\beta}$$

to

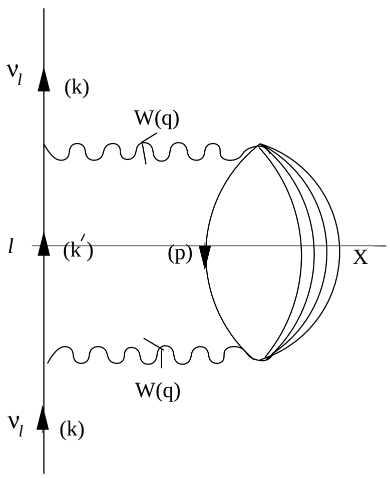
$$\frac{d^2\sigma^A_{\nu,\bar\nu}}{d\Omega' dE'} = \frac{{G_F}^2}{(2\pi)^2}\frac{|\mathbf{k}'|}{|\mathbf{k}|}\left(\frac{m_W^2}{q^2-m_W^2}\right)^2 L^{\alpha\beta}_{\nu,\bar\nu} W^A_{\alpha\beta}$$

$W^A_{\alpha\beta}$ is the nuclear hadronic tensor expressed as:

$$\begin{aligned} W^A_{\alpha\beta} &= \left(\frac{q_\alpha q_\beta}{q^2} - g_{\alpha\beta}\right) W^A_1 + \frac{1}{M_A^2}\left(p_\alpha - \frac{p.q}{q^2}\, q_\alpha\right) \\ &\quad \left(p_\beta - \frac{p.q}{q^2}\, q_\beta\right) W^A_2 - \frac{i}{2M_A^2}\epsilon_{\alpha\beta\rho\sigma} p^\rho q^\sigma W^A_3 \end{aligned}$$

$$\begin{aligned} F_1^A(x_A, Q^2) &= M_A W_1^A(\nu, Q^2) \\ F_2^A(x_A, Q^2) &= \nu W_2^A(\nu, Q^2) \\ F_3^A(x_A, Q^2) &= \nu W_3^{\nu(\bar\nu)}(\nu, Q^2) \end{aligned}$$

In the present formalism the neutrino nuclear cross sections are obtained in terms of neutrino self energy $\Sigma(k)$ in the nuclear medium which also defines the dimensionless nuclear structure functions $F_i^A(x, Q^2)$.

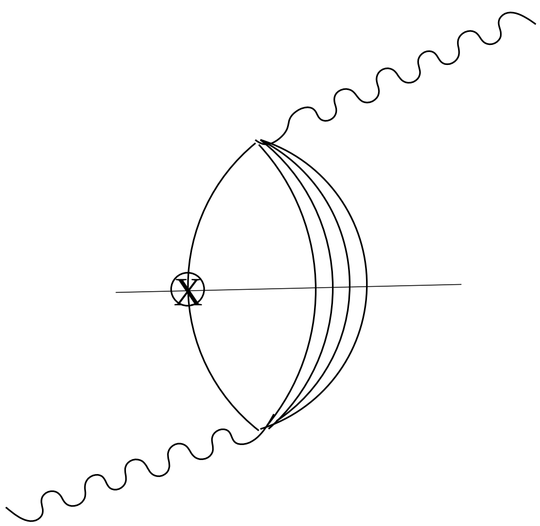


The neutrino self-energy in nuclear matter is given by

$$-i\sum(k) = \int \frac{d^4q}{(2\pi)^4}\bar{u}_\nu(k)\left(-\frac{ie}{2\sqrt{2}sin\theta_W}\gamma^\mu(1-\gamma_5)\right)$$
$$\left(\frac{i(\not{k}'+m_l)}{k'^{\,2}-m_l^2+i\epsilon}\right)\left(-\frac{ie}{2\sqrt{2}sin\theta_W}\gamma^\nu(1-\gamma_5)\right)$$
$$u_\nu(k)\left(-i\frac{\left(g_{\mu\rho}-\frac{q_\mu q_\rho}{M_W^2}\right)}{q^2-M_W^2}\right)(-i)\Pi^{\rho\sigma}(q)i\frac{\left(g_{\sigma\nu}-\frac{q_\sigma q_\nu}{M_W^2}\right)}{q^2-M_W^2}$$

$$\sum(k) = (-i)\frac{G_F}{\sqrt{2}}\frac{4}{m_\nu}\int\frac{d^4k'}{(2\pi)^4}\frac{1}{k^2-m_\nu^2+i\epsilon}$$
$$\left(\frac{m_W}{q^2-m_W^2}\right)^2 L_{\alpha\beta} \quad \Pi^{\alpha\beta}(q)$$

$\Pi^{\alpha\beta}(q)$ is the W self-energy in the nuclear medium and is written as:



$$\Pi^{\alpha\beta}(q) = (-i)\int\frac{d^4p}{(2\pi)^4}\frac{1}{k'^2-m_\mu^2+i\epsilon}\sum_X$$

$$\sum_{s_p,s_i}\prod_{i=1}^{n}\int\frac{d^4p'_i}{(2\pi)^4}\prod_l iG_l(p'_l)\prod_j iD_j(p'_j)$$

$$\left(\frac{-G_F m_W^2}{\sqrt{2}}\right)\times\langle X|J^\alpha|N\rangle\langle X|J^\beta|N\rangle^*$$

$$(2\pi)^4\delta^4(q+p-\Sigma_{i=1}^n p'_i)$$

The nonrelativistic nucleon propagator for a non-interacting Fermi sea in momentum space is given by

$$\begin{aligned} G(p^0, p) &= \frac{1 - n(\vec{p})}{p^0 - \varepsilon(\vec{p}) + i\epsilon} + \frac{n(\vec{p})}{p^0 - \varepsilon(\vec{p}) - i\epsilon} \\ &= \frac{p^0 - \varepsilon(\vec{p}) - i\epsilon + 2i\epsilon n(\vec{p})}{(p^0 - \varepsilon(\vec{p}) + i\epsilon)(p^0 - \varepsilon(\vec{p}) - i\epsilon)} \\ &= \frac{1}{p^0 - \varepsilon(\vec{p}) + i\epsilon} + \\ &\quad \frac{2i\epsilon n(\vec{p})}{(p^0 - \varepsilon(\vec{p}) + i\epsilon)(p^0 - \varepsilon(\vec{p}) - i\epsilon)} \end{aligned}$$

$\frac{1}{x \pm i\epsilon} = \mathcal{P}(\frac{1}{x}) \mp i\pi\delta(x)$

$$G(p^0, p) = \frac{1}{p^0 - \varepsilon(\vec{p}) + i\epsilon} + 2\pi i n(\vec{p})\ \ \delta(p^0 - \varepsilon(\vec{p})) \quad (1)$$

The relativistic Dirac propagator $G^0(p_0, \mathbf{p})$ for a free nucleon is written in terms of the contribution from the positive and negative energy components of the nucleon described by the Dirac spinors $u(\vec{p})$ and $v(\vec{p})$ using their appropriate normalizations as

$$G^0(p_0, \mathbf{p}) = \frac{1}{\not{p} - M + i\epsilon} = \frac{\not{p} + M}{p^2 - M^2 + i\epsilon}$$

$$G^0(p_0, \mathbf{p}) = \frac{M}{E(p)} \left\{ \frac{\sum_r u_r(p)\bar{u}_r(p)}{p^0 - E(\mathbf{p}) + i\epsilon} + \frac{\sum_r v_r(-p)\bar{v}_r(-p)}{p^0 + E(\mathbf{p}) - i\epsilon} \right\}$$

This relativistic nucleon propagator for a nucleon corresponding to noninteracting Fermi sea is then written in analogy with the nonrelativistic case as:

$$G^0(p_0, p) = \frac{\not p + M}{p^2 - M^2 + i\epsilon} + 2\pi i n(\mathbf{p})(\not p + M)\theta(p^0)\delta(p^2 - m^2)$$

$$\begin{aligned} G^0(p_0, \mathbf{p}) &= \frac{M}{E(\mathbf{p})}\left\{\sum_r u_r(p)\bar{u}_r(p)\left[\frac{1 - n(p)}{p^0 - E(\mathbf{p}) + i\epsilon}\right.\right. \\ &+ \left.\left.\frac{n(p)}{p^0 - E(\mathbf{p}) - i\epsilon}\right] + \frac{\sum_r v_r(-p)\bar{v}_r(-p)}{p^0 + E(\mathbf{p}) - i\epsilon}\right\} \end{aligned}$$

The nucleon propagator $G(p_0, \mathbf{p})$ is then calculated by making a perturbative expansion of $G(p_0, \mathbf{p})$ in terms of $G^0(p_0, \mathbf{p})$ by retaining the positive energy contributions only (the negative energy components are suppressed).

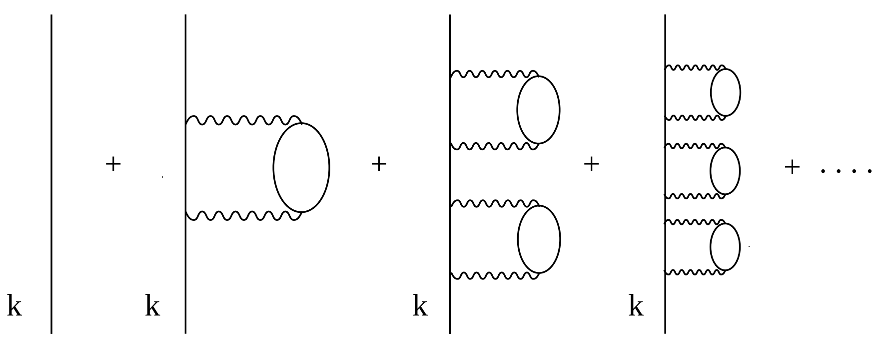


fig. 1

This perturbative expansion is then summed in ladder approximation to give

$$G(p_0,\mathbf{p}) = \frac{M}{E(\mathbf{p})}\sum_r u_r(p)\bar{u}_r(p)\frac{1}{p^0-E(\mathbf{p})}$$
$$+\frac{M}{E(\mathbf{p})}\sum_r \frac{u_r(p)\bar{u}_r(p)}{p^0-E(\mathbf{p})}\sum(p^0,\mathbf{p})$$
$$\frac{M}{E(\mathbf{p})}\sum_s \frac{u_s(p)\bar{u}_s(p)}{p^0-E(p)}+\ldots..$$
$$=\frac{M}{E(\mathbf{p})}\sum_r \frac{u_r(p)\bar{u}_r(p)}{p^0-E(\mathbf{p})-\bar{u}_r(p)\sum(p^0,\mathbf{p})u_r(p)\frac{M}{\mathbf{E(p)}}}$$

This allows us to write the relativistic nucleon propagator in a nuclear medium in terms of the Spectral functions of holes and particles as

$$G(p^0,\mathbf{p}) = \frac{M}{E(\mathbf{p})}\sum_r u_r(\mathbf{p})\bar{u}_r(\mathbf{p})\left[\int_{-\infty}^{\mu} d\,\omega\frac{S_h(\omega,p)}{p^0-\omega-i\eta}\right.$$
$$\left.+\int_{\mu}^{\infty} d\,\omega\frac{S_p(\omega,p)}{p^0-\omega+i\eta}\right]$$

$S_h(\omega,p)$ and $S_p(\omega,p)$ being the hole and particle spectral functions respectively

We use

$$S_h(\omega,\mathbf{p})=\frac{1}{\pi}\frac{\frac{M}{E(\mathbf{p})}Im\Sigma^N(p^0,p)}{(p^0-E(\mathbf{p})-\frac{M}{E(\mathbf{p})}Re\Sigma^N(p^0,p))^2+(\frac{M}{E(\mathbf{p})}Im\Sigma^N(p^0,p))^2}$$

for $p^0\le\mu$

$$S_p(\omega,\mathbf{p})=-\frac{1}{\pi}\frac{\frac{M}{E(\mathbf{p})}Im\Sigma^N(p^0,p)}{(p^0-E(\mathbf{p})-\frac{M}{E(\mathbf{p})}Re\Sigma^N(p^0,p))^2+(\frac{M}{E(\mathbf{p})}Im\Sigma^N(p^0,p))^2}$$

for $p^0>\mu$.

we use local density approximation (LDA) where we do not have a box of constant density, but the reaction takes place at a point $\mathbf{r}$, lying inside a volume element d^3r with local density $\rho_p(\mathbf{r})$ and $\rho_n(\mathbf{r})$ corresponding to the proton and neutron densities at the point $\mathbf{r}$. This leads to the spectral functions for the protons and neutrons to be the function of local Fermi momentum given by

$$k_{F_p}(\mathbf{r}) = \left[3\pi^2\rho_p(\mathbf{r})\right]^{1/3}, k_{F_n}(\mathbf{r}) = \left[3\pi^2\rho_n(\mathbf{r})\right]^{1/3}$$

and therefore the equivalent normalization is

$$2\int \frac{d^3p}{(2\pi)^3}\int_{-\infty}^{\mu} S_h(\omega, p, k_{F_{p,n}}(\mathbf{r}))d\omega = \rho_{p,n}(\mathbf{r})$$

For a symmetric nuclear matter of density $\rho(\mathbf{r})$, there is a unique Fermi momentum given by $k_F(\mathbf{r}) = \left[3\pi^2\rho(\mathbf{r})/2\right]^{1/3}$ for which we obtain

$$4\int \frac{d^3p}{(2\pi)^3}\int_{-\infty}^{\mu} S_h(\omega, p, k_F(\mathbf{r}))d\omega = \rho(\mathbf{r})$$

leading to the normalization condition given by

$$4\int d^3r \int \frac{d^3p}{(2\pi)^3}\int_{-\infty}^{\mu} S_h(\omega, p, \rho(r))\, d\omega = A$$

where $\rho(r)$ is the baryon density for the nucleus which is normalized to A and is taken from the electron nucleus scattering experiments.

For an isospin symmetric nucleus

$$W_A^{\alpha\beta} = 4\int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf{p})}\int_{-\infty}^{\mu} dp^0 S_h(p^0,\mathbf{p},\rho(r)) W_N^{\alpha\beta}(p,q)$$

where $W_N^{\alpha\beta}(p,q)$

$$W_{\alpha\beta}^N = \left(\frac{q_\alpha q_\beta}{q^2} - g_{\alpha\beta}\right) W_1^{\nu(\bar\nu)} + \frac{1}{M^2}\left(p_\alpha - \frac{p.q}{q^2}\, q_\alpha\right) \left(p_\beta - \frac{p.q}{q^2}\, q_\beta\right) W_2^{\nu(\bar\nu)} - \frac{i}{2M^2}\epsilon_{\alpha\beta\rho\sigma}p^\rho q^\sigma W_3^{\nu(\bar\nu)}$$

Taking the xx component, we have

$$W_{xx}^N = \left(\frac{q_x q_x}{q^2} - g_{xx}\right) W_{1N}^{\nu(\bar\nu)} + \frac{1}{M^2}\left(p_x - \frac{p.q}{q^2}\, q_x\right) \left(p_x - \frac{p.q}{q^2}\, q_x\right) W_{2N}^{\nu(\bar\nu)} - \frac{i}{2M^2}\epsilon_{xx\rho\sigma}p^\rho q^\sigma W_{3N}^{\nu(\bar\nu)}$$

Chosing $\mathbf{q}$ along the z-axis

$$W_{xx}^N(x_N,Q^2) = W_{1N}^{\nu(\bar\nu)}(x_N,Q^2) + \frac{1}{M^2} p_x p_x W_{2N}^{\nu(\bar\nu)}(x_N,Q^2)$$

As the nucleus is at rest therefore its three momentum component would be zero and we have

$$W^A_{xx}(x_A, Q^2) = W^{\nu(\bar{\nu})}_{1A}(x_A, Q^2)$$

Using the following relation valid for the nucleon and the nucleus

$$\begin{aligned} F_1^{\nu(\bar{\nu})}(x, Q^2) &= MW_1^{\nu(\bar{\nu})}(\nu, Q^2) \\ F_2^{\nu(\bar{\nu})}(x, Q^2) &= \nu W_2^{\nu(\bar{\nu})}(\nu, Q^2) \end{aligned}$$

We obtain the following expression

$$\mathrm{W}^N_{xx}(x_N, Q^2) = \frac{F_{1N}(x_N,Q^2)}{M} + \frac{1}{M^2} {p_x}^2 \frac{F_{2N}(x_N,Q^2)}{\nu}$$

Similarly for the nucleus

$$W^A_{xx}(x_A, Q^2) = \frac{F_{1A}(x_A, Q^2)}{M_A} = \frac{F_{1A}(x_A, Q^2)}{AM}$$

$$\frac{F_{1A}(x_A,Q^2)}{AM} = 4\int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf{p})}\int_{-\infty}^{\mu} dp^0 S_h(p^0,\mathbf{p},\rho(r)) \times \left[\frac{F_{1N}(x_N,Q^2)}{M} + \frac{1}{M^2}{p_x}^2\frac{F_{2N}(x_N,Q^2)}{\nu}\right]$$

Using the following relationships

$$x_N = \frac{Q^2}{2pq} = \frac{Q^2}{2(p_0q_0 - p_zq_z)},$$

$$x_A = \frac{x}{A} = \frac{1}{A}\frac{Q^2}{2Mq_0},$$

Using Callan Gross relation $F_2(x) = 2xF_1(x)$ one may write this as

$$F_{2A}(x_A,Q^2) = 4\int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf{p})}\int_{-\infty}^{\mu} dp^0 S_h(p^0,\mathbf{p},\rho(r)) \times \left[\frac{x}{x_N}[1 + \frac{2x_N{p_x}^2}{\nu}]F_{2N}(x_N,Q^2)\right]$$

Defining γ as $\quad \gamma = \frac{q_z}{q^0} = \left(1 + \frac{4M^2x^2}{Q^2}\right)^{1/2}$

we get

$$F_3^A(x, Q^2) = 4 \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(p)}$$

$$\int_{-\infty}^{\mu} dp^0 S_h(p^0, p, \rho(r)) \left(\frac{p_0\gamma - p_z}{(p_0 - p_z\gamma)\gamma} \right) F_3^N(x_N, Q^2)$$

This is our main equation which describes the modification on $F_3^A(x, Q^2)$ due to nuclear medium effects

Target Mass Corrections

It has been pointed out that when extracting parton distribution functions in the large-x region, it is crucial to correct the data for effects associated with the nonzero mass of the target.

It is important at high x(x>0.45) and at very low Q^2. It dies out rapidly with increase in Q^2.

Ingo Schienbein et al.

J. of Phys. G 35:053101,2008

$$F_2^{\mathrm{TMC}}(x,Q^2) \simeq \frac{x^2}{\xi^2 r^3} F_2^{(0)}(\xi)\left[1 + \frac{6\mu x\xi}{r}(1-\xi)^2\right]$$

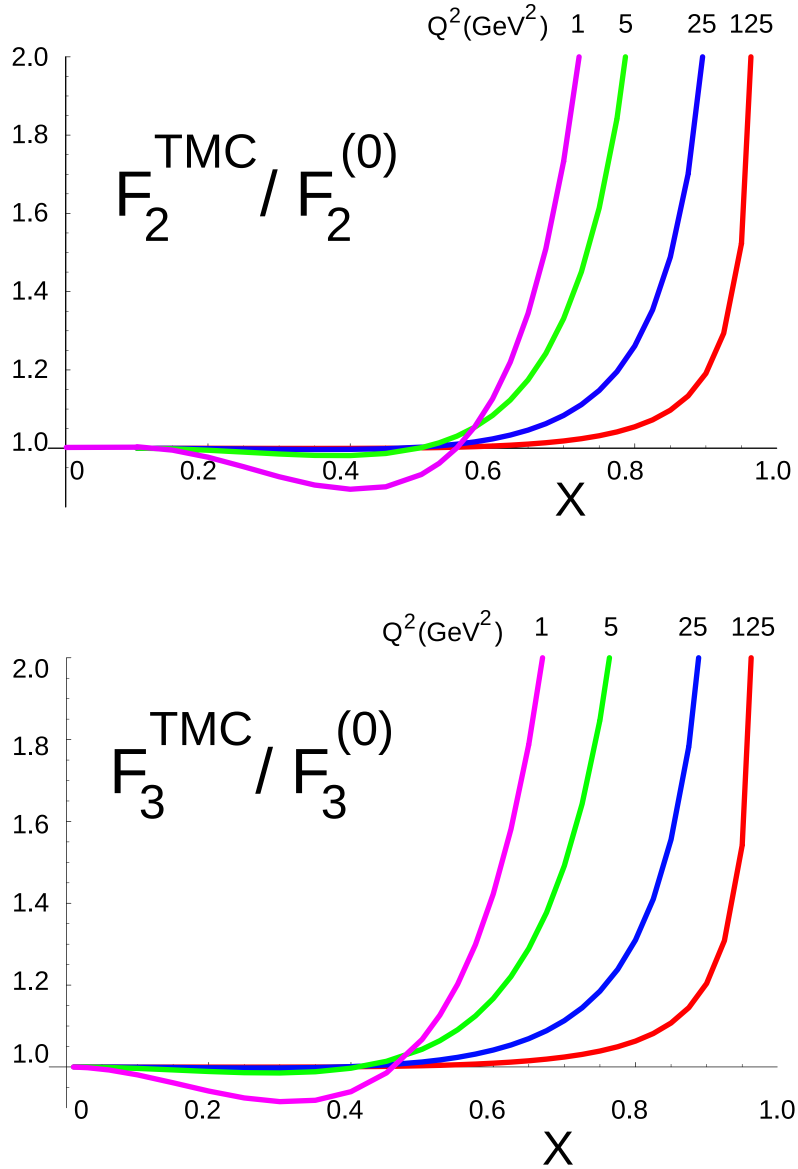
Similarly, $F_3^{\mathrm{TMC}}(x,Q^2)$ is approximated by

$$F_3^{\mathrm{TMC}}(x,Q^2) \simeq \frac{x}{\xi r^2} F_3^{(0)}(\xi)\left[1 - \frac{\mu x\xi}{r}(1-\xi)\ln\xi\right].$$

$$r = \sqrt{1 + \frac{4x^2M^2}{Q^2}}$$

$$\xi = \frac{2x}{1+r}$$

$$\mu = \frac{M^2}{Q^2}$$



Q^2(GeV^2) 1 5 25 125
2.0
1.8
1.6
1.4
1.2
1.0
F_2^TMC / F_2^(0)
0 0.2 0.4 0.6 0.8 1.0
X
Q^2(GeV^2) 1 5 25 125
2.0
1.8
1.6
1.4
1.2
1.0
F_3^TMC / F_3^(0)
0 0.2 0.4 0.6 0.8 1.0
X

Target Mass Correction

It seems that effects in nuclei are minimal even when it is important for nucleons. The reason is the following: For a given x-nucleus we have a wide interval of x-nucleon that contributes and TMC produces an average effect

Effect of Target Mass Correction

Q2	WITH	Without	Difference in %
x= 0.45			
1.2590	0.594435543	0.77166	30
5.0120	0.463959802	0.50072	7
19.950	0.368034699	0.37541	
x= 0.65			
1.259	0.241981271	0.41732	72
5.0120	0.134176562	0.16212	21
19.950	0.0853798181	0.089995	5
80.0	0.061895142	0.062731	
x=0.75			
1.259	0.148675201	0.29576	100
5.0120	0.0608069109	0.78358E-01	28
19.950	0.0312409955	0.33593E-01	8
80.00	0.0204015257	0.020786	

Off shell correction

S. A. Kulagin and R. Petti PHYSICAL REVIEW D 76, 094023 (2007) & Nucl. Phys. A765 (2006) 126.

The structure functions of the bound proton and neutron can differ from those of the free proton and neutron. This effect is related to the analytical continuation of the structure functions to the off-shell region and the dependence on the nucleon invariant mass p^2 .

Kulagin and Petti have studied it phenomenologically by analyzing the data on the ratios of structure functions of different nuclei (EMC effect). It was found that this is independent of Q2 and can be parametrized as

$$\delta f_2 = C_N(x - x_1)(x - x_0)(1 + x0 - x)$$

with the parameters extracted from the fit $C_N = 8.10 \pm 0 : 30(stat) \pm 0 : 53(syst)$, $x0 = 0.448 \pm 0.005(stat) \pm 0 : 007(syst)$ and x_1=0.05.

The off shell effects are important at high x and independent of Q^2

No off shell effects at low x(<0.55)

1. **x=0.55**

I. Around 4-5% reduction for all Q^2

2. **x=0.65**

I. Around 8-9% reduction for all Q^2

3. **x=0.75**

I. Around 10-12% reduction for all Q^2

This is independent of the Structure Function

Shadowing and Anti-shadowing effects

S. A. Kulagin and R. Petti PHYSICAL REVIEW D 76, 094023 (2007) & Nucl. Phys. A765 (2006) 126.

Effect of Shadowing in F_2

1. **x=0.0001 to x=0.015**

I. Q^2=1.259GeV^2 Around 25% reduction

II. Q^2=3.162eV^2 Around 15% reduction

III. Q^2=20GeV^2 Around 4% reduction

2. **x=0.045**

I. Q^2=1.259GeV^2 Around 20% reduction

II. Q^2=3.162eV^2 Around 14% reduction

III. Q^2=20GeV^2 Around 3.6% reduction

3. **x=0.125**

I. Q^2=1.259GeV^2 Around 5% reduction

II. Q^2=3.162eV^2 Around 4% reduction

III. Q^2=20GeV^2 Around 1% reduction

Comments about Shadowing from the different papers

The Shadowing effect which we are getting are consistent with the results of:

(I) Shadowing effects in deep-inelastic lepton-nucleus scattering

G. Piller, Wolfram Weise

Phys.Rev.C42:R1834-R1837,1990.

(II) Mass dependence of nuclear shadowing at small Bjorken-x from diffractive scattering

A. Adeluyi and G. Fai

Phys. Rev. C74:054904,2006.

(III) Resummed QCD power corrections to nuclear shadowing

Jian-wei Qiu, Ivan Vitev

Phys.Rev.Lett.93:262301,2004.

(IV) SHADOWING AND ENHANCEMENT OF QUARK DISTRIBUTIONS IN NUCLEI AT SMALL x.

L.L. Frankfurt, M.I. Strikman

Nucl.Phys.B316:340,1989.

Effects in F_3 at low Q^2 and low x

1. **x=0.0001**

I. Q^2=1.259GeV^2 Around 50% reduction

II. Q^2=3.162eV^2 Around 35% reduction

III. Q^2=20GeV^2 Around 10% reduction

2. **x=0.015**

I. Q^2=1.259GeV^2 Around 40% reduction

II. Q^2=3.162eV^2 Around 30% reduction

III. Q^2=20GeV^2 Around 7-8% reduction

3. **x=0.045**

I. Q^2=1.259GeV^2 Around 12% reduction

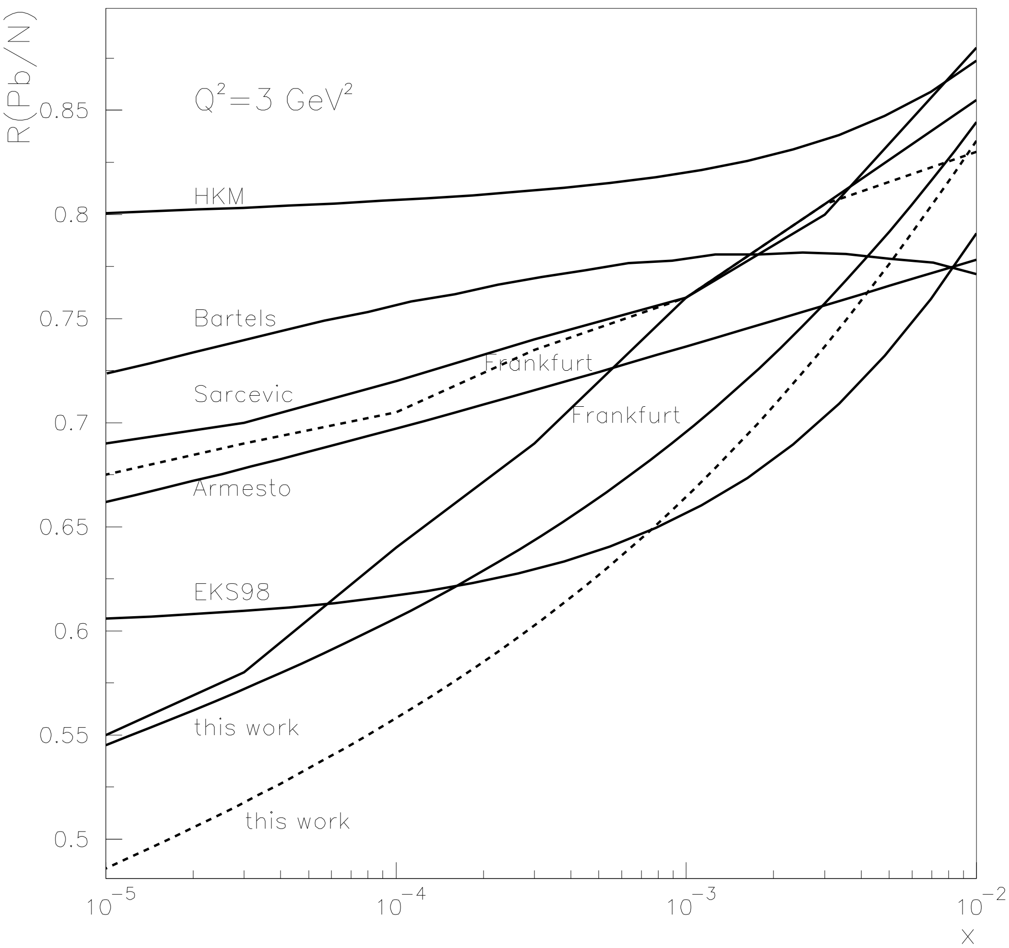
II. Q^2=3.162eV^2 Around 10% reduction

III. Q^2=20GeV^2 Around 4% reduction

4. **x=0.08**

I. Q^2=1.259GeV^2 Around 7-8% enhancement

II. Q^2=3.162eV^2 Around 2-3% enhancement


R(Pb/N)
Q^2=3 GeV^2
HKM
Bartels
Sarcevic
Armesto
EKS98
Frankfurt
Frankfurt
this work
this work
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
10^{-5}
10^{-4}
10^{-3}
10^{-2}
X

QCD corrections to the charged-current structure function F_2 and F_3.

S. Moch, J.A.M. Vermaseren, A. Vogt

e-Print: arXiv:0812.4168 [hep-ph]

The loop corrections for inclusive DIS is required to go from LO framework to NLO, NNLO frameworks.

QCD corrections to the Electromagnetic structure function F(2)

The coefficient functions $C_{n,i}$ for the structure functions F_2 in electromagnetic DIS,

$$x^{-1}F_a = C_{n,\mathrm{ns}} \otimes q_{\mathrm{ns}} + \langle e^2 \rangle \left(C_{n,\mathrm{q}} \otimes q_{\mathrm{s}} + C_{n,\mathrm{g}} \otimes g \right)$$

Mellin Convolution and Coefficient Functions

Mellin Convolution is defined as

$$a \ \otimes \ b = \int_x^1 \frac{dy}{y} a(y) b\left(\frac{x}{y}\right)$$

+-distributions are defined as

$$\int_0^1 dx\, a(x)_+ f(x) = \int_0^1 dx\, a(x) \left\{ f(x) - f(1) \right\}$$

for regular functions $f(x)$.

Convolutions with the distributions D_k are written as

$$x[D_k \otimes f](x) = \int_x^1 dy \, \frac{\ln^k(1-x)}{1-x} \left\{ \frac{x}{y} f\left(\frac{x}{y}\right) - x f(x) \right\} + x f(x) \frac{1}{k+1} \ln^{k+1}(1-x)$$

We start by outlining the general formalism for the NLO and NNLO evolutions of flavour-singlet parton structure functions.

The singlet quark density of a hadron is given by

$$\Sigma(x, \mu_f^2, \mu_r^2) = \sum_{i=1}^{N_f} \left[q_i(x, \mu_f^2, \mu_r^2) + \bar{q}_i(x, \mu_f^2, \mu_r^2) \right]$$

$q_i(x, \mu_f^2, \mu_r^2)$ and $\bar{q}_i(x, \mu_f^2, \mu_r^2)$ represent the number distributions of quarks and anti-quarks, respectively, in the fractional hadron momentum x. The corresponding gluon distribution is denoted by $g(x, \mu_f^2, \mu_r^2)$.

i indicates the flavour of the (anti-) quarks

N_f stands for the number of effectively massless flavours

μ_r and μ_f represent the renormalization and mass-factorization scales.

The singlet quark density and the gluon density are constrained by the energy-momentum sum rule

$$\int_0^1 dx \, x \left[\Sigma(x, \mu_f^2, \mu_r^2) + g(x, \mu_f^2, \mu_r^2)) \right] = 1$$

The singlet components are written in terms of the non-singlet(NS) component $c^{(n)}_{n,\mathsf{NS}}(x)$ and the pure singlet (PS) component $c^{(n)}_{n,\mathsf{NS}}(x)$ and **n** here stands for the Leading order(n=0), first order(n=1), second order(n=2), QCD corrections.

$$c^{(2)}_{n,q}(x) = c^{(2)}_{n,\mathsf{NS}}(x) + c^{(2)}_{n,\mathsf{PS}}(x)$$

The coefficient functions

The coefficient functions at zeroth and first order are given by

Reference: The Third-order QCD corrections to deep-inelastic scattering by photon exchange. J.A.M. Vermaseren, A. Vogt, S. Moch Nucl.Phys.B724:3-182,2005.

$$c^{(0)}_{2,\mathrm{ns}}(x) = \delta(x_1)\ , c^{(0)}_{2,\mathrm{ps}}(x) = c^{(0)}_{2,\mathrm{g}}(x) = c^{(1)}_{2,\mathrm{ps}}(x) = 0$$

and

$$\begin{aligned} c^{(1)}_{2,ns}(x) &= c_f\{4\,D_1 - 3\,D_0 - (9 + 4\,\zeta_2)\,\delta(x_1) \\ &\quad -2\,(1+x)(L_1 - L_0)4\,x_1^{-1}L_0 + 6 + 4\,x\}\ , \end{aligned}$$

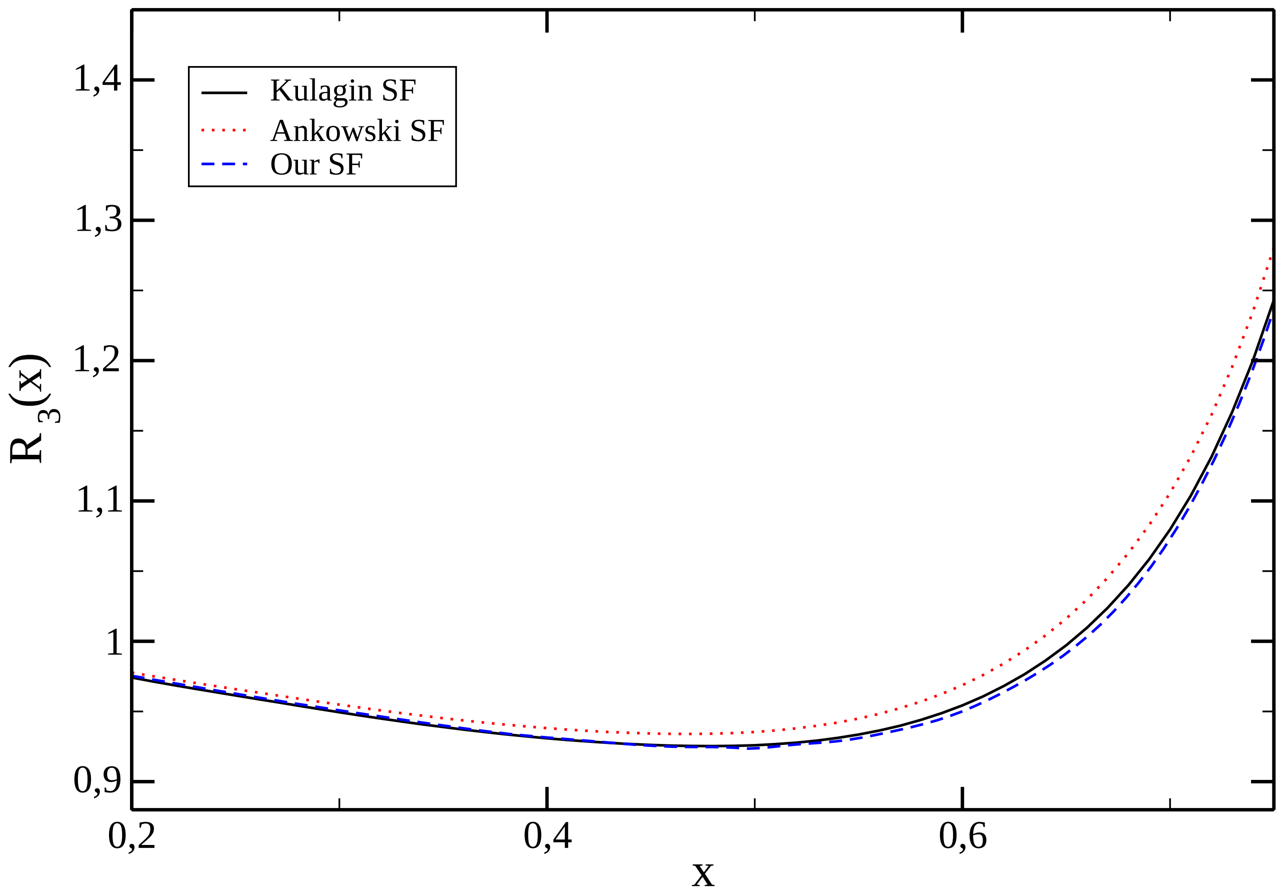
$$\mathrm{c}^{(1)}_{2,\mathrm{g}}(x) = n_f\,\{(2 - 4\,xx_1)(L_1 - L_0) - 2 + 16\,xx_1\}$$

with $c_f = (N_c^2 - 1)/(2N_c) = 4/3$ in QCD. Here and below we use the abbreviations

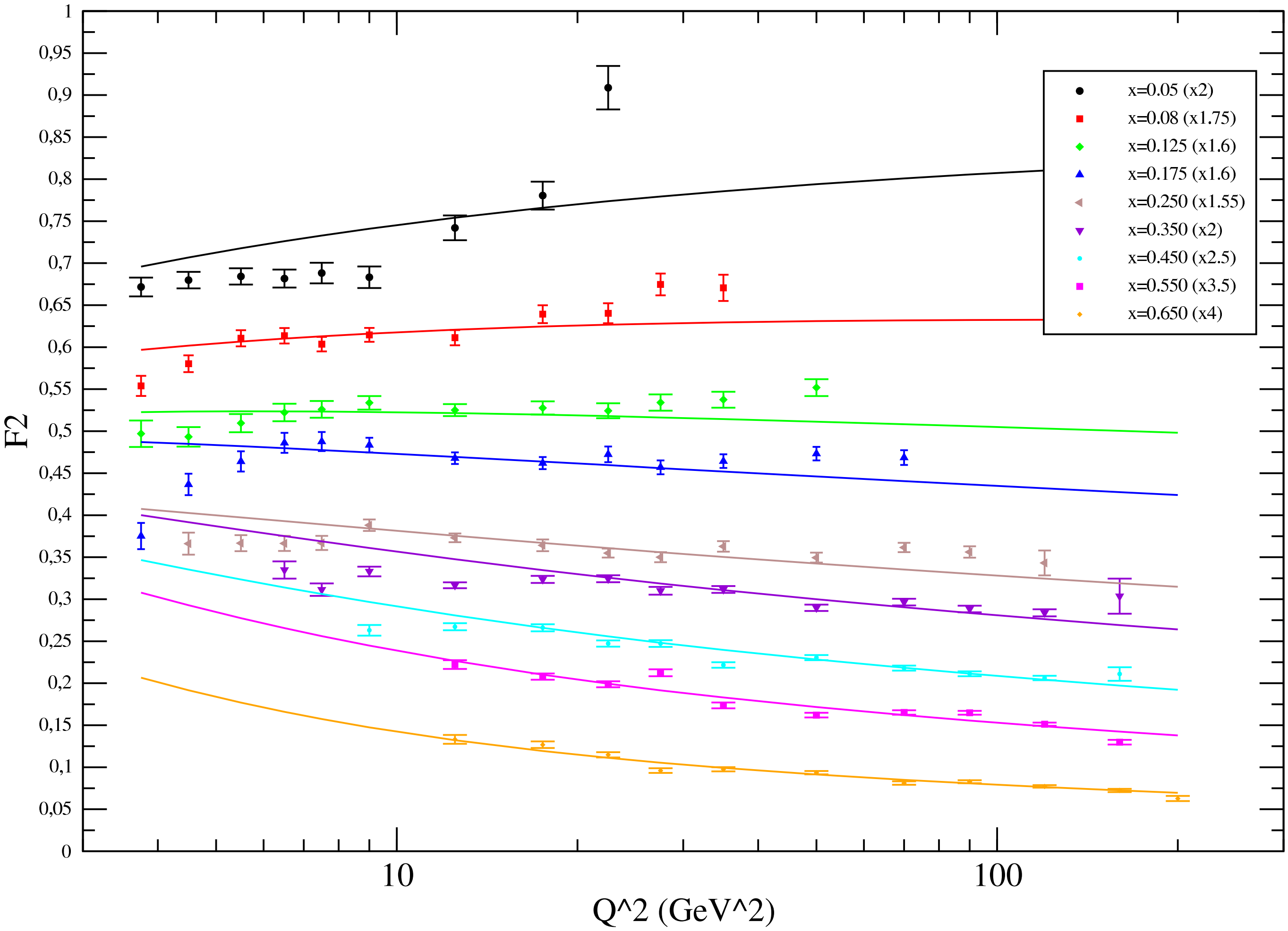
$$x_1 = 1 - x \ , L_0 = \ln x \ , L_1 = \ln x_1 \ , D_k = [x_1^{-1} L_1^k]_+$$

The non-singlet coefficient function entering the electromagnetic F_2 is written as *Reference:* NNLO evolution of deep inelastic structure functions: The Nonsinglet case
W.L. van Neerven, A. Vogt Nucl.Phys.B568:263-286,2000.

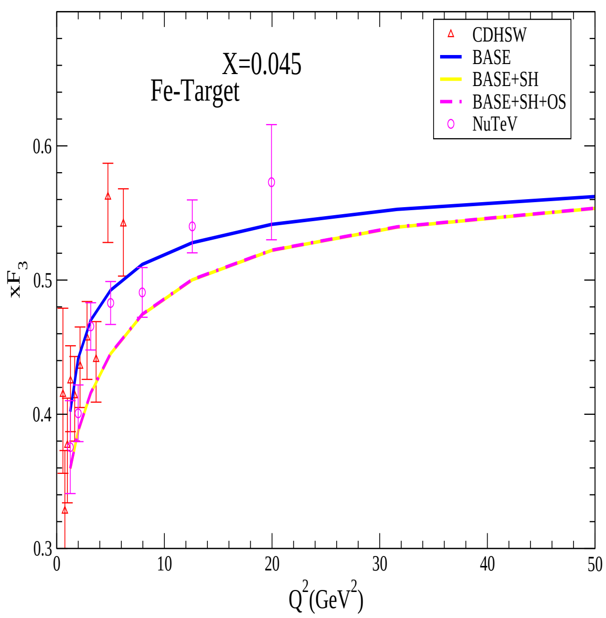
The pure singlet piece has been approximated by using the *Reference:* NNLO evolution of deep inelastic structure functions: The Singlet case.
W.L. van Neerven, A. Vogt Nucl.Phys.B588:345-373,2000.

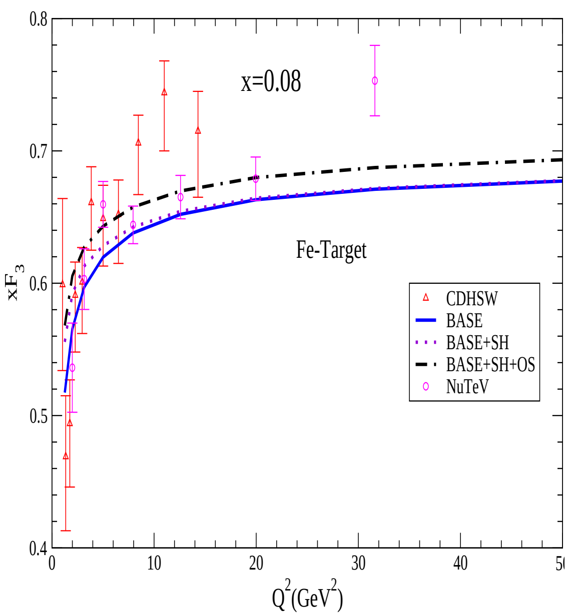


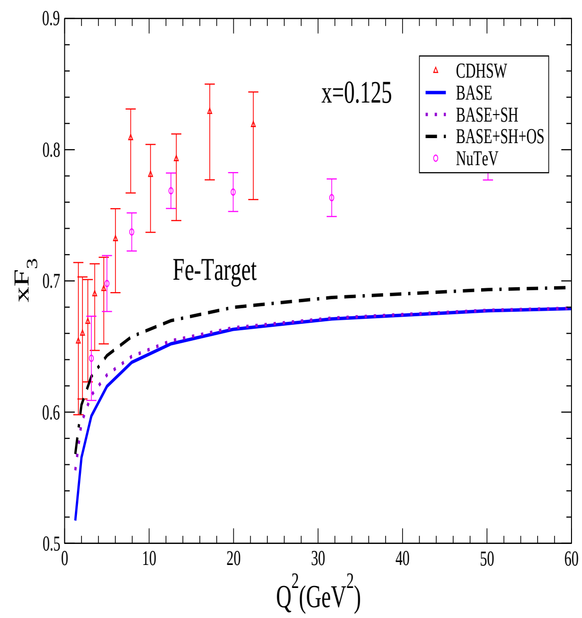
Kulagin SF
Ankowski SF
Our SF
1,4
1,3
1,2
1,1
1
0,9
R_3(x)
0,2
0,4
0,6
x

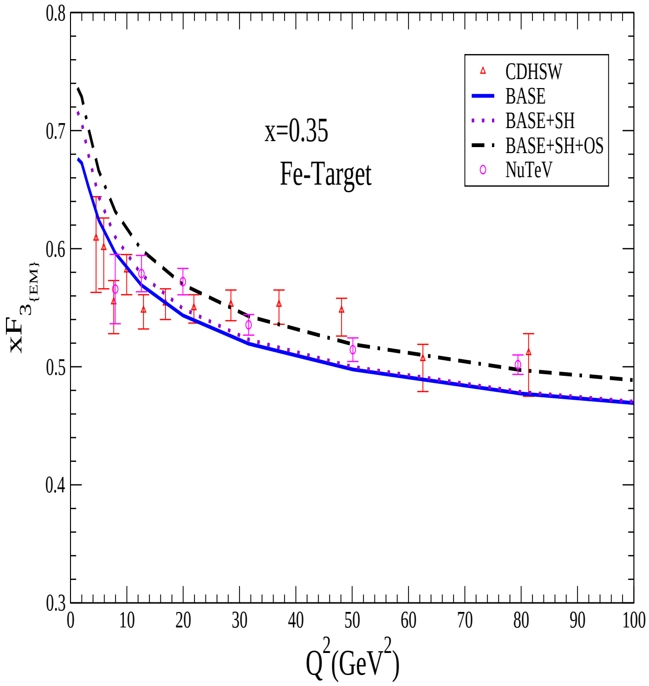


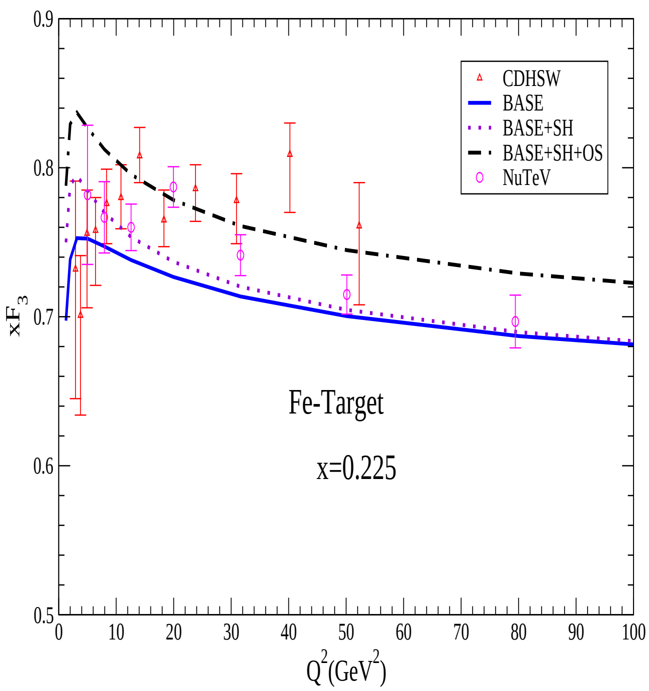
F_{2EM} in Iron: :Experimental Results from Aubert et al.NPB 272 (1986) 15

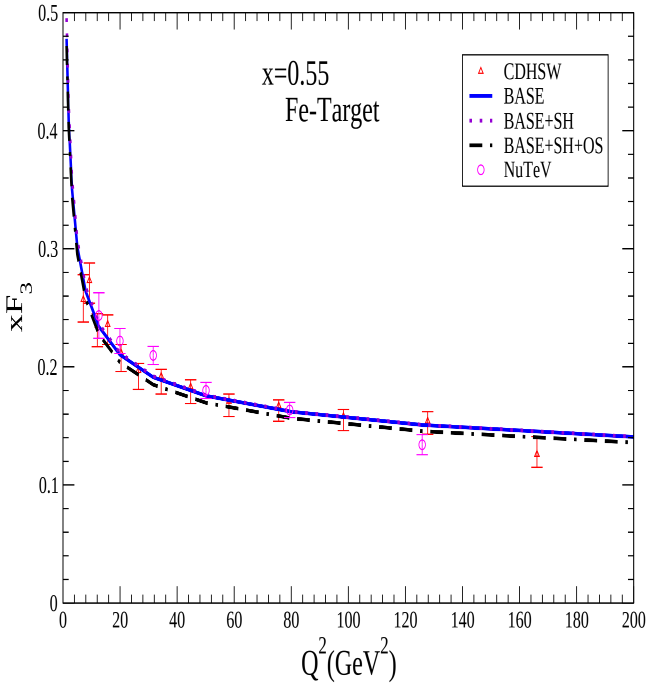

X=0.045
Fe-Target
CDHSW
BASE
BASE+SH
BASE+SH+OS
NuTeV
xF3
Q2(GeV2)

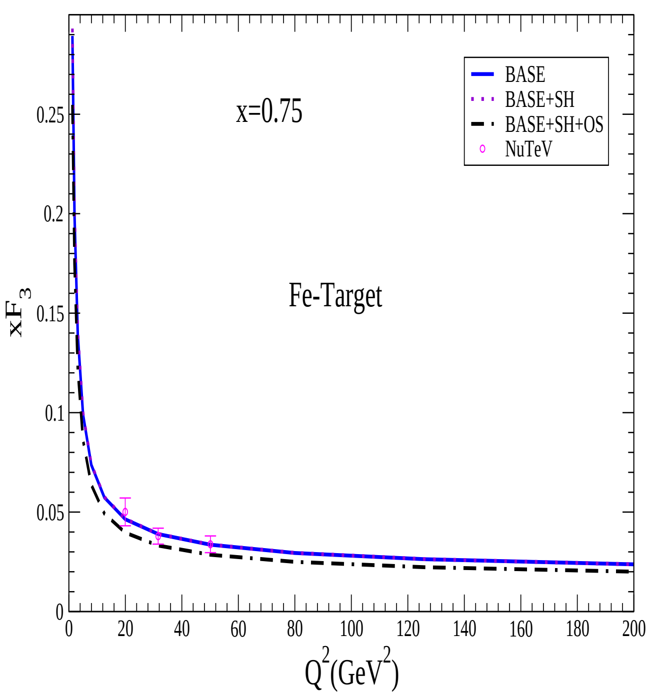

x=0.08
Fe-Target
CDHSW
BASE
BASE+SH
BASE+SH+OS
NuTeV
xF3
Q2(GeV2)

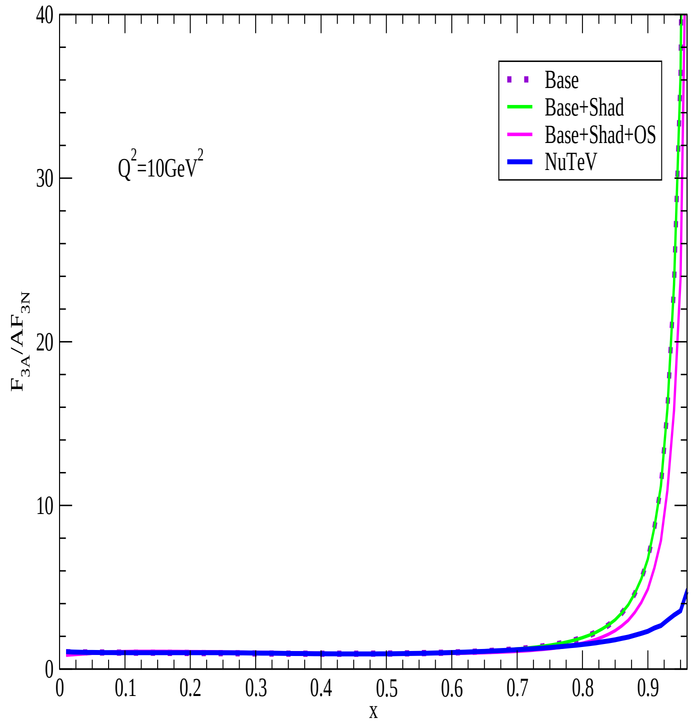

x=0.125
Fe-Target
CDHSW
BASE
BASE+SH
BASE+SH+OS
NuTeV
xF3
Q2(GeV2)

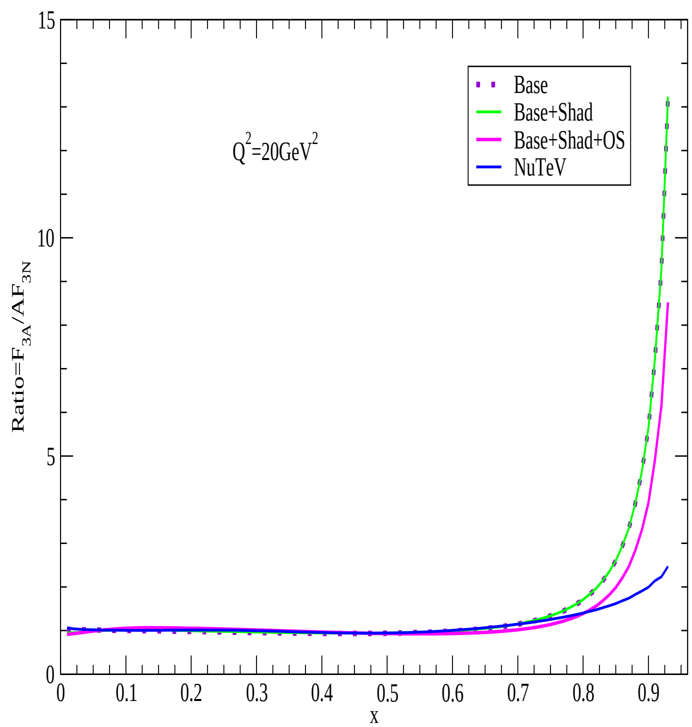

x=0.35
Fe-Target
CDHSW
BASE
BASE+SH
BASE+SH+OS
NuTeV
$xF_{3(EM)}$
$Q^2(GeV^2)$
0.3
0.4
0.5
0.6
0.7
0.8
0
10
20
30
40
50
60
70
80
90
100

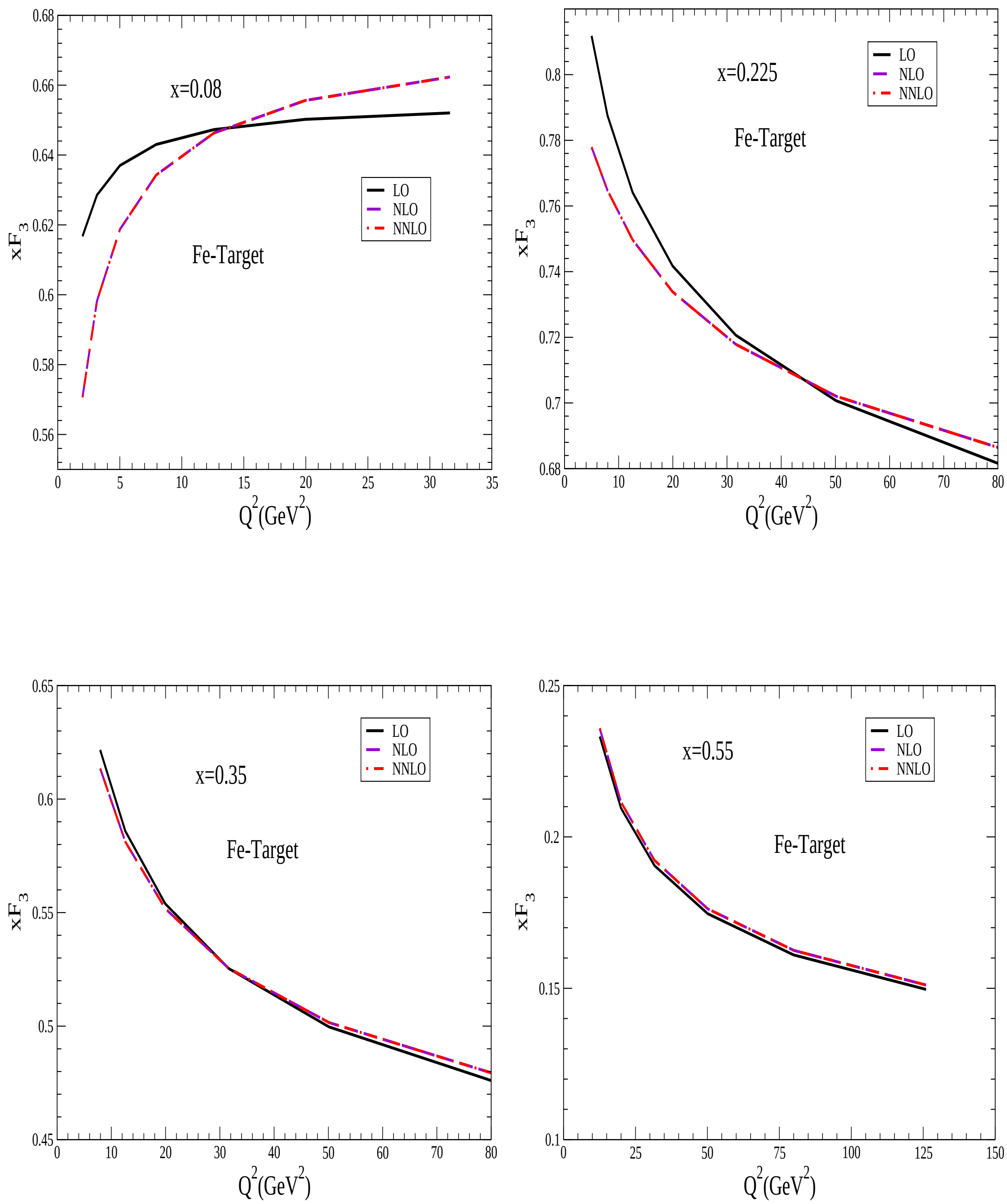

Fe-Target
x=0.225
CDHSW
BASE
BASE+SH
BASE+SH+OS
NuTeV
xF_3
$Q^2(GeV^2)$
0.5
0.6
0.7
0.8
0.9
0
10
20
30
40
50
60
70
80
90
100


x=0.55
Fe-Target
CDHSW
BASE
BASE+SH
BASE+SH+OS
NuTeV
xF3
Q2(GeV2)


x=0.75
Fe-Target
BASE
BASE+SH
BASE+SH+OS
NuTeV
xF3
Q2(GeV2)


Base
Base+Shad
Base+Shad+OS
NuTeV
Q^2=10GeV^2
F_3A/AF_3N
x
0
10
20
30
40
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9


Base
Base+Shad
Base+Shad+OS
NuTeV
Q^2=20GeV^2
Ratio=F_3A/AF_3N
x
0
5
10
15
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9


x=0.08
Fe-Target
LO
NLO
NNLO
xF_3
$Q^2(GeV^2)$
x=0.225
Fe-Target
x=0.35
Fe-Target
x=0.55
Fe-Target

Conclusions

Summary of NME

A. Quasielastic Scattering

1. NME reduces the total cross section due to RPA (20-30%) for ν and $\bar{\nu}$.

2. NME increases σ due to quasielastic-like effects through ΔN $\rightarrow$ NN (5-10%).

3. NME improves the agreement with MiniBooNE results of $< \frac{d\sigma}{dQ^2} >$ for $\bar{\nu}$.

3. NME worsens the agreement with MiniBooNE results of $< \frac{d\sigma}{dQ^2} >$ for ν.

B. One Pion Production

1. NME reduces the total cross section for ν and $\bar{\nu}$ (10-15%).

2. NME does not affect Q^2 dependence so Q^2 disagreement with MiniBooNE results remains.

C. Deep Inelastic Scattering

1. NME on $F_2(x, Q^2)$ and $F_3(x, Q^2)$ lead to reduction at large Q^2.

2. NME results on $F_2(x, Q^2)$ and $F_3(x, Q^2)$ are in qualitative agreement with phenomenological analysis of NuTeV collaboration but not with Hirai et al.

3. NME lead to better agreement with NuTeV and CCFR results.