Some Miniprojects with Megaresults

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Plan of the Talk

I Introductory Remarks

- Inward Bound Journey (SM of HEP)
- 4-way Road Map for HEP
- Two special remarks on INO

II Miniprojects with Megaresults

1. Discovery of New Principles of Acceleration
   (Laser-Plasma Accelerators)
2. Are Neutrinos Majorana Particles?
   (Neutrinoless Double Beta Decay)
3. Mossbauer Effect with Neutrinos
   (A Revolutionary Proposal by R. RAGHAVAN)

The ROAD MAP for HEP in India must be based on a 4-LANE HIGHWAY

1. A vigorous participation of Indian groups in international experiments, accelerator-based as well as non-accelerator based.
2. Creation of an underground laboratory for Neutrino Physics and other non-accel^r particle physics
3. Construction of a high-energy accelerator in India
4. A programme for the search for NEW methods of acceleration that can take HEP beyond the TeV energies.

We must build ALL these 4 lanes. At this stage, no one can predict which lane leads to gold.

① • While the first one (participation in international expts is being pursued vigorously (& this must continue with increased vigour), that is not true of the other three, so far.

LEP, Tevatron
Heavy Ion Expts at CERN, BNL
LHC, ILC .....

② • After vigorous efforts of many of us during the last 5 years, the 2^{nd} one (Underground Lab Project) is taking shape in the form of INO and I hope it will reach fruition soon enough.

INO Includes NP :
- $0\nu\beta\beta$
- β beam
- Monoenergetic ν beam (K capture)
- Recoilless ν emission & absorption (Raghavan)

③ • While our participation in international accelerator expts must continue with full vigour, at the same time, for a balanced growth of experimental HEP, we must have in-house activity. Construction of an accelerator in India, in a suitable energy range (10–20 GeV?) and its utilization for research & student-training will provide this missing link.

Two remarks on INO :

- In contrast to many other Mega projects, INO involves a networking of 20 independent institutions (many Universities & IIT's in addition to pure research institutions such as TIFR, SINP ...)

  We have to learn (from the success story of CERN) the importance of networking, in modern scientific research.

- INO provides an opportunity to India to mount a major HEP expt right here in the country. Our students do not have to go to CERN or FermiLab to see a HEP detector in action or participate in its use. The ICAL of INO is as big as the CMS or ATLAS of LHC (even bigger), altho' not that sophisticated.

  Our college students (you) will be able to contribute to the INO detector through their summer programme.

DISCOVERY OF NEW PRINCIPLES OF ACCELERATION

• Fundamental Units: $\hbar = c = 1$

$$\boxed{E\ (\text{in GeV}) = \frac{0.192}{L\ (\text{in } 10^{-13}\text{ cm})}}$$

• Scales in HEP:

$L \rightarrow$ (cm)	10^{-8}	10^{-13}	10^{-17}	10^{-33}
$E \rightarrow$		200 MeV	2 TeV	10^{19} GeV

Planck Scale
(Scale of Quantum Gravity
& String Theory)

$$\boxed{E_P \sim \sqrt{\frac{1}{G_N}}, \quad L_P \sim \sqrt{G_N}}$$

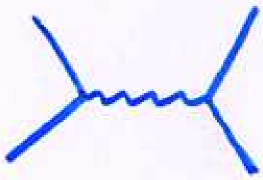
• Accelerator Energies:

LEP : 200 GeV

Tevatron : 2 TeV

LHC : 14 TeV

When does Gravity become as strong as other interactions?

SM :  $\sigma_{SM}(E) \sim \frac{1}{E^2}$

Gravity:  $\sigma_G(E) \sim G_N^2 E^2 \sim \frac{E^2}{M_P^4}$

(where $M_P \sim \frac{1}{G_N^{1/2}}$)

So, $\sigma_G(E) \sim \sigma_{SM}(E)$

for $E \sim M_P \sim 10^{19}$ GeV

$\hookrightarrow G_N^{-1/2}$

Comparison between Gravity & EM

$$\frac{G_N m_p^2 / r^2}{e^2 / r^2} \approx 10^{-36}$$

$$\frac{G_N E^2 / r^2}{e^2 / r^2} \approx 1$$

$$\Rightarrow \quad \frac{E^2}{m_p^2} \approx 10^{36}$$

or, $\boxed{E \approx 10^{18} \text{ GeV}}$

- It is now recognized that known methods of acceleration cannot take us beyond Tens of Tev.
- Hence in order to ensure the continuing vigour of HEP in the 21st Century, it is absolutely essential to discover new principles of acceleration.
- Herein lies an opportunity that our Country should not miss.
- I have been repeatedly emphasizing for the past 20 years that we must initiate a programme whose mission shall be to discover new methods of acceleration.
    - Reference
    - Methods
    - Achievement so far
- I would even say the Future of HEP depends on the discovery of new principles of acceleration.
- Unfortunately, this 4th component in the Road Map seems to have been left out. This must be corrected.

Future of HEP (references)

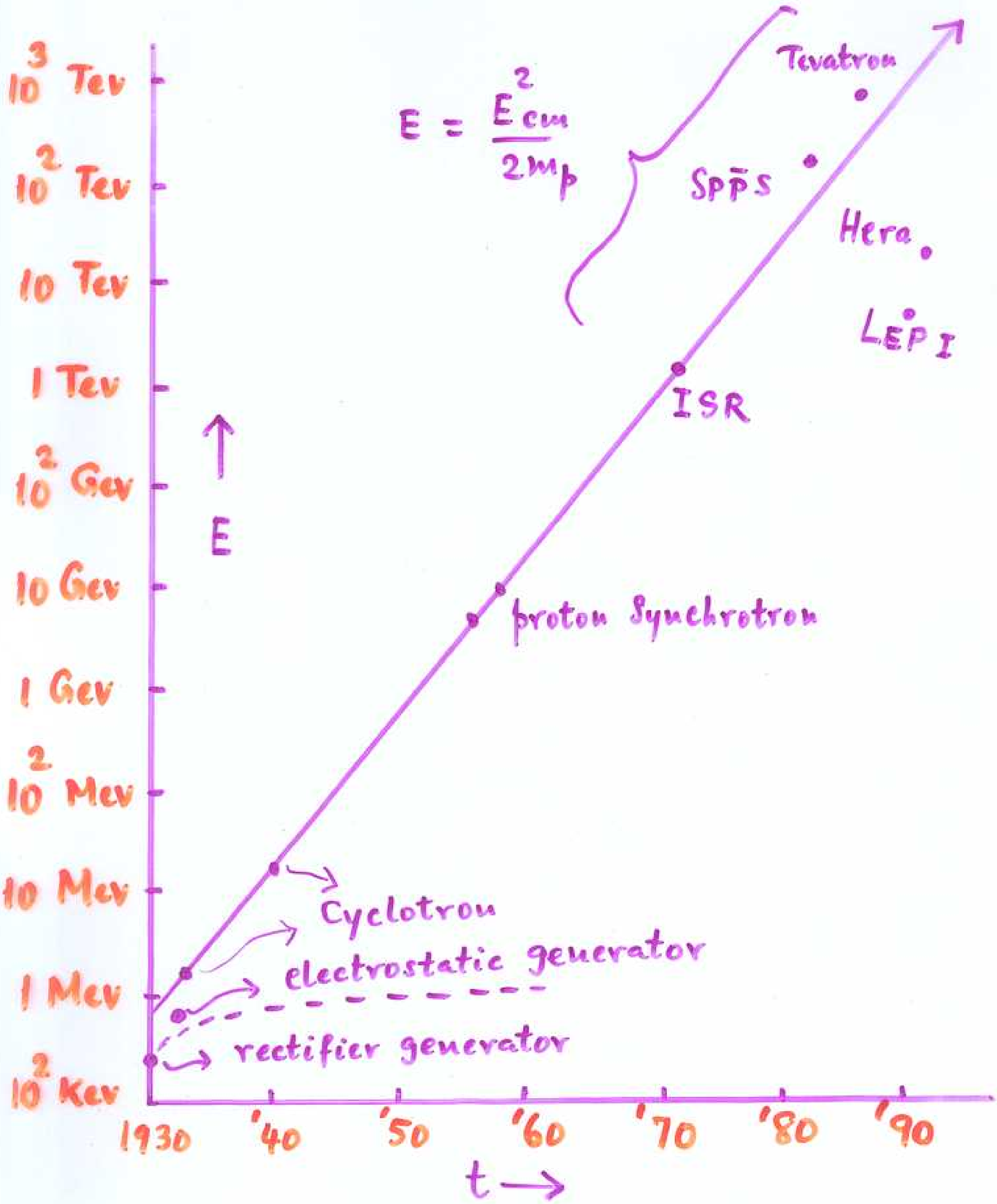
- $\overline{\underline{\text{VIII}}}$ HEP Symposium, SINP, Calcutta, 1986
- WHEPP, SNBC, Calcutta, 1991
- $\overline{\underline{\text{XII}}}$ HEP Symposium, Guwahati, 1996
- Beam Physics Schools, CAT, Indore
- Plasma Physics School, SINP
- Plasma Physics Workshop, IPR, Gandhinagar

HEP for the next 100 years

- Energy of Accelerators increases by a factor of 10 in every 6 years.

- $(19-3) \times 6 = 96$ years required to reach Planck scale.

$\uparrow$ 10^{19} $\uparrow$ 10^{3}



The Livingston Chart
(Energy growth of accelerators)

Laser-Plasma Accelerators

* C. Joshi & his Colleagues at UCLA have succeeded in using this method to produce an accelerating electric field of 2·8 GeV/m. which is the largest coherent man-made accelerating field yet produced, and is 30 times larger than the limit imposed by RF breakdown in conventional accelerators.

(Nature, 1994, 368, 527)

** Nakajima etal have achieved an ultrahigh accelerating field of 30 Gev/m in a laser-produced plasma (Phys. Rev. Lett. 1995, 74, 4428)

*** SLAC

1 GeV in 3.3 cm Table-top Accelerator

- Lawrence Berkeley National Lab, together with Oxford University have accelerated electrons to more than 1 GeV in only 3.3 cm.

  W. P. Leemans et al, Nature Physics 2, 699 (2006)
  (CERN Courier Nov 2006)

- This is the highest energy achieved with laser-wakefield acceleration, which harnesses the electric field of a plasma wave driven by a laser beam.

- A laser pulse travelling through a plasma excites plasma waves in its wake, setting up electric fields that accelerate electrons from the plasma.

- This produces accelerating gradients of 10-100 GV/m compared to 10-50 MV/m for standard RF based systems.

- The main problem has been, to sustain the laser-wakefield acceleration over the distances needed to reach energies greater than 100 MeV and to control the energy spread. This has been solved, to a certain extent.

More details of the Laser-Wakefield Accelerator

- A high voltage discharge forms a plasma channel in a hydrogen-filled capillary in a sapphire block.

  The wakefield of a laser pulse sent through the channel then accelerates the plasma electrons.

- Capillary dimensions : 3·3 cm long
  250 μm diameter
- Density of plasma : 4×10^{18} per c.c

- 12 TW laser $\Longrightarrow$ 0.48 GeV beam ($\Delta E < 5\%$)
  40 TW laser $\Longrightarrow$ 1 GeV beam ($\Delta E < 2.5\%$)

- Energy measured with 1.2 T spectrometer

- Trying for 10 GeV

- There may still be many hurdles to overcome. They will be overcome and table top accelerators will come. The route to particle acceleration beyond tens of TeV by new methods will be opened.
- But where are we in this country? Should we not start small groups of bright young experimenters in various institutions (research institutes & universities), whose mission shall be to discover new methods of particle acceleration?

Standard Model of HEP
≡ Dynamics of 12 Gauge Fields

Electroweak Dynamics
(γ, W^+, W^-, Z)

Chromodynamics
(QCD)
$(G_\alpha \quad \alpha = 1 \ldots 8)$

Electrodynamics

- All the past & present accelerators are based on Electrodynamics.
- Standard model technology in a very primitive stage; (rubbing glass rod on cat's fur!)
- Electrodynamics does not stand alone; it is only part of the unified EW Dynamics.
- Deeper implications (Faraday, Maxwell ...)
- Our understanding of QCD is at an even more primitive stage (colour confinement).
- But chromodynamics will be mastered & chromodynamic technology also will come.
- Electrodynamic technology led to acceleration upto Tev (10^3 Gev). By releasing the forces of the SM & putting them to work, Planckian energies (10^{19} Gev) may be reached.

$$
\begin{tikzcd}
\boxed{\begin{array}{c} \text{EM} \\ \equiv 1 \text{ g.f.} \end{array}} \arrow[r] \arrow[d] & \boxed{\begin{array}{c} \text{SM} \\ \equiv 12 \text{ g.f.} \end{array}} \arrow[d] \\
\boxed{0 \longrightarrow 1 \text{ Tev}} & \boxed{1 \text{ Tev} \longrightarrow ?}
\end{tikzcd}
$$

By releasing the forces of the Standard Model and putting them to work, the goal of acceleration upto Planckian energies may be achieved even earlier than any prediction.

"Prediction is a difficult art,
especially when it concerns the Future".

ARE NEUTRINOS

MAJORANA PARTICLES?

Dirac and Majorana

- Dirac introduced the concept of antiparticles while trying to solve the −ve energy problem in the famous relativistic eq for the electron that he had discovered in 1928.
- Now we know that the existence of antiparticles is one of the most important consequences of combining quantum mechanics with special relativity.
- For every particle there exists an antiparticle.
- However, some particles could be self-conjugate, in the sense that particle and antiparticle could be the same. (Of course, such particles have to be electrically neutral.)
- Among the elementary particles of the Standard Model, γ and the Z boson are self-conjugate. Both these are bosons.

- The possibility of a self-conjugate fermion was first pointed out by Majorana in 1937, and hence they are called Majorana fermions while the other fermions (with distinct particles & antiparticles) are called Dirac fermions.
- Among the fermions of the Standard Model, only neutrinos are electrically neutral and hence qualify to be Majorana particles.
- But it is still an open question whether ν's are Majorana particles or Dirac particles, in other words, whether ν's are self-conjugate particles or not. $\boxed{\nu \overset{?}{=} \bar{\nu}}$

Standard Model

Fermions			$\boxed{Q}$	Bosons	$\boxed{Q}$
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$	0 -1	γ	0
				Z	0
				$W^\pm$	± 1
$\begin{pmatrix} u \\ d \end{pmatrix}$	$\begin{pmatrix} c \\ s \end{pmatrix}$	$\begin{pmatrix} t \\ b \end{pmatrix}$	$2/3$ $-1/3$	G_i $(i = 1 \ldots 8)$	0

Composite particles:

$$\pi^0 \sim \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) = \pi^0$$

$$K^0 \sim d\bar{s}$$

$$\bar{K}^0 \sim \bar{d}s$$

$$n \sim udd$$

$$\bar{n} \sim \bar{u}\bar{d}\bar{d}$$

$$\boxed{\nu \overset{?}{=} \bar{\nu}}$$

- This question can be shown to be a pure semantic one if neutrinos are massless. In that case, one can prove, by renaming the left-handed ν and the right-handed $\bar{\nu}$ suitably, that there is no physical distinction between Dirac and Majornana neutrinos.
- This is sometimes called "the confusion theorem".
- But, after the discovery of neutrino mass in the last decade, we know that such a distinction exists.
- Hence, physicists must determine the category (Dirac vs Majorana) to which neutrino belongs. We have to answer the above question.

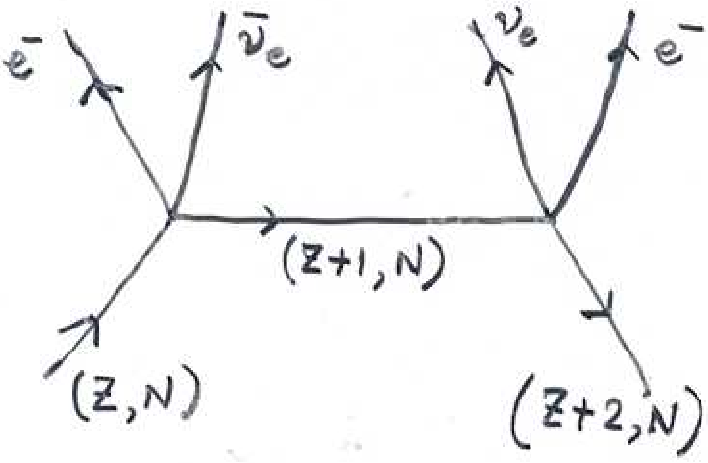
Sea-saw, $\cancel{L}$ and Cosmology

- There are important reasons why theoreticians prefer Majorana neutrinos.
- If ν's are Majorana particles, there exists an elegant mechanism called see-saw to explain why the neutrino masses, although not zero, are so tiny.
- Further, if ν's are Majorana particles, lepton number L is not conserved. This opens the door to generate an excess of leptons over antileptons in the early Universe; this can subsequently generate an excess of baryons over antibaryons, thus explaining how after annihilation of most of the particles with antiparticles, a finite but small residue of particles was left, to make up the present Universe.
- Hence the fundamental importance of the question whether neutrinos are Majorana particles is clear.

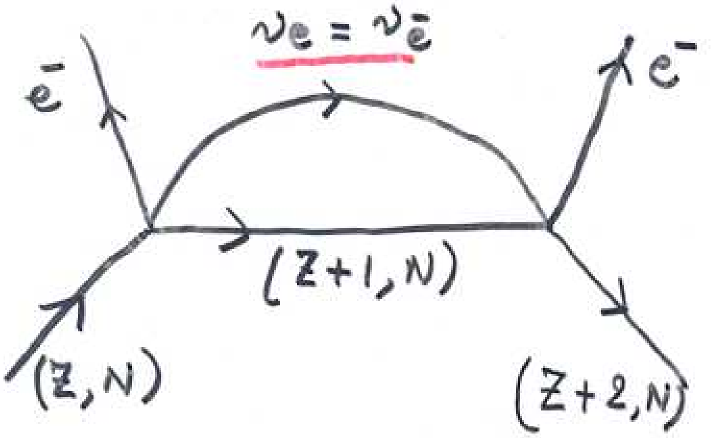
NDBD

- Inspite of the great attractiveness of the idea that ν's could be Majorana particles, it is only a theoretical idea. It has to be either confirmed or refuted by experiment.
- At the present stage, the only experiment that can answer this question is the neutrinoless double beta decay (NDBD).
- In NDBD, the two Majorana neutrinos that are virtually produced can annihilate each other leaving only two electrons in the final state, thus violating lepton number L by 2 units ($\Delta L = 2$).
- So establishment of the existence of NDBD would be proof of the Majorana nature of ν and violation of L.

- Usual double beta decay (DBD), in which two ν's along with two e^- are emitted, itself is a very rare process because it is doubly weak as compared to the standard beta decay and further the phase space is suppressed by the large number of particles in the final state.
- Inspite of its rarity, this decay (DBD) has been experimentally detected and now well studied.
- However, as we already stated, the signature for the Majorana character of the ν is the absence of the two ν's in the final state ($E_1 + E_2 = Q$).
- This process without the ν's in the final state (NDBD) has not yet been detected.
- Although NDBD had reduced number of particles in the final state and has more phase space, it is a much rarer process than the 2ν decay because the decay amplitude is proportional to the tiny neutrino mass. (This is consistent with the confusion theorem.)



Double Beta Decay



Neutrinoless
Double Beta Decay

Amplitude for NDBD

$$\boxed{A = m_{ee}\, M_{nuclear}}$$

- The amplitude for NDBD is proportional to a neutrino mass factor m_{ee} which is a linear combination of the masses of the three types of neutrinos that are known to exist.
- This lin. combination also involves the ν mixing angles and the very important CP violating phases.
- Of course this has to be multiplied by the relevant nuclear matrix element, to get the decay amplitude.
- Thus if NDBD is detected and the rate is measured and if the nucl. m.E is known, one can extract important information on the ν parameters.
- 1) m_{ee} contains the overall scale of the ν masses and this mass-scale is not obtainable from ν oscillation expts.

  2) Some of the ~~CP~~ phases (the Majorana phases) do not even occur in the oscillation formulae.
- However the success of such an extraction of ν parameters from NDBD depends very crucially on our knowledge of the nucl. m.E and hence nuclear theory will play an essential role in NDBD activity.

Can the NDBD neutrino mass factor be zero?

$$m_{ee} \stackrel{?}{=} 0$$

Unfortunately the answer is yes.

- The particular linear combination of neutrino masses that enters here may be zero (or very small) for some reason and so the rate for NDBD may be vanishingly small.
- In this unfortunate case, NDBD expts will not throw any light on whether neutrinos are Majorana particles.
- One may have to go to processes such as

$$\underline{\mu^-} + \text{nucleus} \rightarrow \underline{\mu^+} + \text{another nuclear state}$$

- The amplitude for this reaction is proportional to a different linear combination of ν masses, (call it $m_{\mu\mu}$), which hopefully is not zero.
- But this lepton number violating reaction will be even harder to study.

- Is it possible to enhance the rates for the lepton number violating processes by some mechanism, for instance by shining an intense laser?
- Such ideas are being considered and one of them might work.
- But, for the present, let us return to NDBD.

Search for NDBD

- In spite of the many expts that have been mounted for NDBD search, none has borne fruit so far.
- However, in 2004, Klapdor and his group reported observing the NDBD of ^{76}Ge.
- This created quite an excitement, because of the importance of such a discovery.
- But this result was soon controverted by many critical physicists.

- I was reading Galileo's biography when this news came. So, my mind went back by 400 years to the time when Galileo was facing his sceptical opponents who refused to believe that Galileo really saw the things in the heavens that he claimed to have seen through his spyglass (telescope).
- Of course times have changed and the situation is quite different. In any case, there is no religious dogma concerning the nature of the neutrino!
- It is very important to settle the issue by independant experiments.

Some history

- The National Workshop "Neutrinos in Nuclear, Particle and Astrophysics (NUPA 04)", held at IIT, Kharagpur in Feb 2004 was the first workshop covering the impact of the recent discoveries in Neutrino Physics in all the 3 fields.
- In that workshop, i stressed the importance of the NDBD expt and suggested that the DBD theorists P.K. Raina and P.K. Rath must organize and coordinate the activity that could lead to mounting a NDBD expt in the INO Cavern.
- Their magnificent enthusiasm led to two focussed Workshops DBD 05 at IIT, kgp in March 05 and NDBD 05 at Lucknow in Nov 05.
- More importantly, they have succeeded in bringing two excellent senior experimenters Vivek Datar and R G Pillay and their groups into the NDBD project. This augurs well for the ultimate success of the project.
- Utpal Sarkar and VKB Kota took the next step of bringing the experimenters and theorists together at NDBD 07 at Ahmedabad in Feb 2007.
- We are here for NDBD 07 at Mumbai in Oct 2007, thanks to Vandana Nanal and her Organizational Committee.

Final remarks relevant to the Roadmap ahead

1. The fundamental importance of NDBD must be stressed again and again.

- Repetition of this manthra is not a waste of time, since the most important unknown in all of neutrino physics is the answer to the question: whether ν is Majorana or Dirac.
- The answer will have a bearing on HEP as well as cosmology.
- At present, NDBD expt is the only way to answer it.

2. Hence, at the India-based Neutrino Observatory (INO), NDBD must be pursued with full vigour.

- Although the major focus of the INO as of now is to construct the magnetized iron calorimeter (ICAL) to be used for atmospheric ν's (Phase I) and ν's from muon storage rings (Phase II), parellel processing of NDBD must go on.
- NDBD activity must soon gain sufficient strength so that the NDBD detector can become as important as ICAL to INO, (if not more important).
- But this requires considerable spadework and R and D.

3. It is absolutely essential to scout for good experimenters and augment the NDBD group.

- We must scan the whole spectrum of likely candidates in research institutions and universities inside and outside the country and attract them to NDBD.
- Also, we must not restrict ourselves to HEP and NP experimenters only.
- We must interact with atomic physicists, condensed matter physicists, material scientists, chemists, engineers ....
- All of these can make useful and important contributions to mounting a viable NDBD expt in the country.
- Students must form an important component of the human resource that we seek for NDBD.
- The task is a gigantic one, but it can be done and it must be done.

4. Finally we must mention that expts on the search for dark matter have reached very high sensitivities and are becoming capable of detecting it, if it really exists in the form and abundance generally expected by cosmologists and astrophysicists.

- Hence we must include the possibility of the Indian NDBD project leading to an Indian Dark Matter project in the future.

A Majorana puzzle

- Dirac eq. has negative energy solns
- Dirac solved the negative energy problem by filling the -ve energy sea (Dirac sea) using Pauli exclusion principle.
- But, then he had to explain the possible vacancy or hole in the sea. So, he had to predict the antiparticle.

  Hole = antiparticle (with charge, energy, momentum & spin direction reversed in sign)

  This is the famous Hole Theory of Dirac

- What happens to all this, for Majorana particle?

  Majorana particle also satisfies Dirac eq.

  So there are negative energy solns.

  How is the problem to be solved?

MOSSBAER EFFECT

WITH

NEUTRINOS

Recoilless Resonant Capture of $\bar{\nu}$'s from ^3H Decay

R. S. Raghavan, hep-ph/0601079

Or, Mössbauer Effect in neutrinos

$$^3H \to \bar{\nu}_e + {}^3He + e^- \text{ (bound)} \qquad (1)$$

$\bar{\nu}_e$: (18.6 keV)

$$\bar{\nu}_e + {}^3He + e^- \text{ (orbital)} \to {}^3H \qquad (2)$$

(18.6 keV)

- Embed ^3H and ^3He in fcc metal tritides

$\Rightarrow$ Recoilless resonant capture of $\bar{\nu}_e$

$$\frac{\Delta E}{E} \sim 2 \times 10^{-7}$$

$$\boxed{\sigma_{res} \sim 5 \times 10^{-32}\ cm^2 \gg \sigma(\bar{\nu}_e + p) \sim 10^{-42}\ cm^2}$$

- Ultraprecise, very low E & very high σ (res)
- gravitational red-shift of $\bar{\nu}_e$
- flavour oscillations in table-top expts with (1 gm - 1 kg materials)
- This is the route to Precision Neutrino Phys.
- This will revolutionize Neutrino Physics.

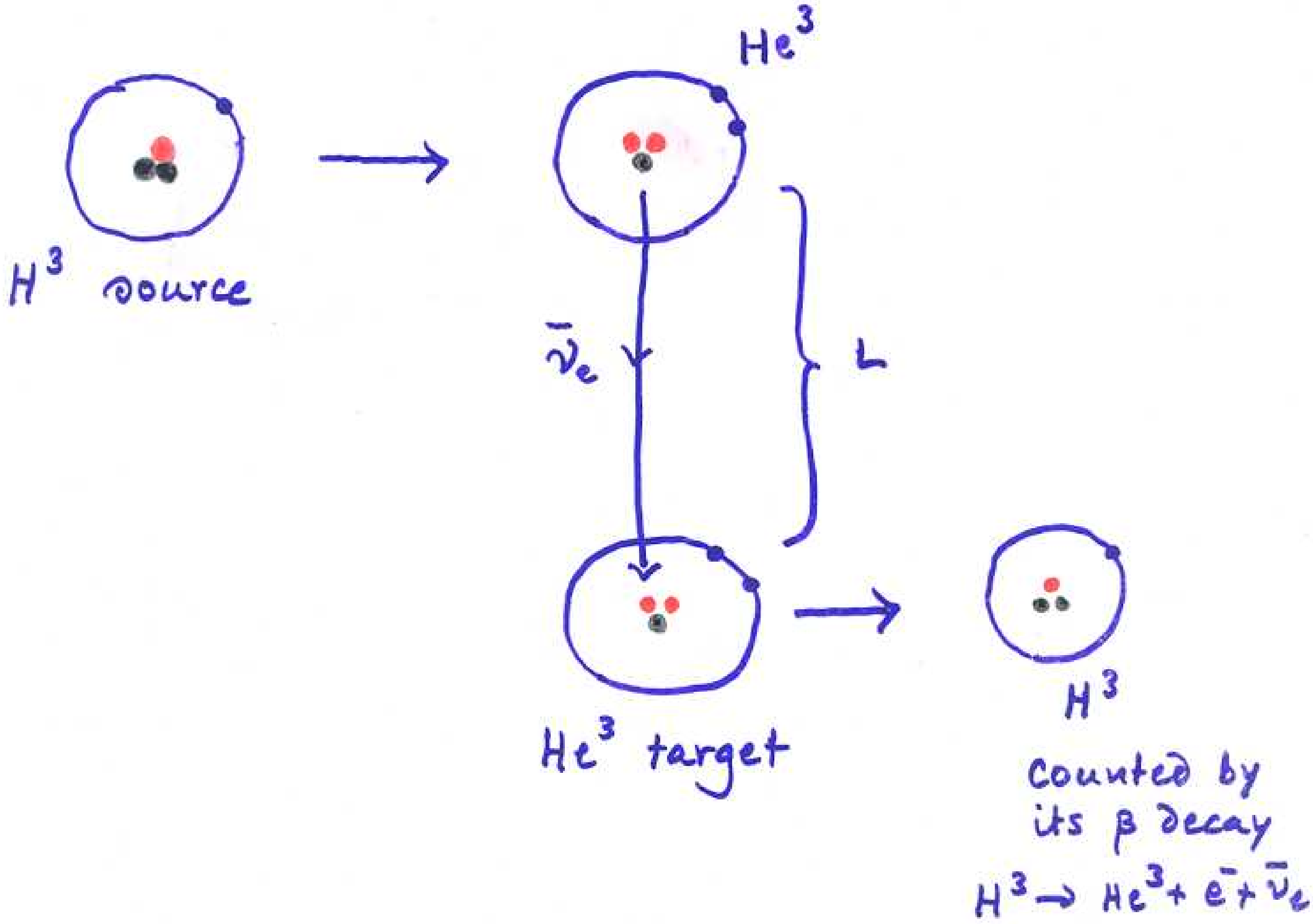
$$\sigma = \frac{4\pi}{k^2} \frac{\Gamma^2}{(E - E_0)^2 + \Gamma^2}$$

$$= \frac{4\pi}{k^2} \quad \text{for } E = E_0$$

$$= 4\pi \lambda^2 \quad (\lambda = \text{de Broglie wavelength})$$

Precision Neutrino Physics with Mossbauer Neutrinos

$$H^3 \rightleftarrows (He^3 + e^-)_B + \bar{\nu}_e$$



Neutrino Oscillations

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - 4 s_{12}^2 c_{13}^2 c_{12}^2 c_{13}^2 \sin^2 \frac{\Delta m_{21}^2 L}{4E}$$

$$- 4 s_{13}^2 c_{12}^2 c_{13}^2 \sin^2 \frac{\Delta m_{31}^2 L}{4E}$$

$$- 4 s_{13}^2 s_{12}^2 c_{13}^2 \sin^2 \frac{\Delta m_{32}^2 L}{4E}$$

s_{12}, s_{13}, Δm_{21}^2, Δm_{31}^2 & Δm_{32}^2 can be determined very precisely.

$$\Delta m_{31}^2 \equiv m_3^2 - m_1^2 = m_3^2 - m_2^2 + m_2^2 - m_1^2$$

$$= \underbrace{\Delta m_{32}^2}_{\pm 2 \times 10^{-3}\ eV^2} + \underbrace{\Delta m_{21}^2}_{+7 \times 10^{-5}\ eV^2}$$

$\Delta m_{32}^2 = +2 \times 10^{-3}\ eV^2$ $\qquad$ $\Delta m_{32}^2 = -2 \times 10^{-3}\ eV^2$

- We must emphasize the importance of ingenious ideas to take neutrino physics further.
- What we need are a hundred crazy ideas. Maybe one of them will work and help us to solve the neutrino mystery.
- In particular, I am almost sure that somebody, somewhere in the world, is bound to succeed in Mossbauer Effect with neutrinos. Why can't that be somebody here in this Country?

Directed Monoenergetic ν Beam?

- Take ν's from polarized nuclei that undergo 2-body β-decay

(a) Bound-state β^- decay

$$^3H \rightarrow \bar{\nu}_e + {}^3He + e^- \text{ (bound)}$$

(b) Electron Capture

$$^3He + e^- \text{(orbital)} \rightarrow {}^3H + \nu_e$$

- Let z be the direction of the polarizing magnetic field.
- Do angular momentum Conservation:

Initial State

	J	J_z
^3H	$\frac{1}{2}$	$+\frac{1}{2}$
$1s\ e^-$	$\frac{1}{2}$	$+\frac{1}{2}$
Total	1	+1

(by hf int)

Final State

	J	J_z
^3He	$\frac{1}{2}$	$+\frac{1}{2}$
Filled 1s shell	0	0
$\bar{\nu}_e$ (S wave)	$\frac{1}{2}$	$+\frac{1}{2}$
Total	1	+1

In allowed β decay, only S wave $\bar{\nu}_e$ participates.

$kR \approx 18.6\text{ keV} \times 1\text{ F} \approx \frac{18.6\text{ keV}}{192\text{ MeV}} \sim 10^{-4}$

Spin of $\bar{\nu}$ is along $+z$ axis.

$$\vec{\sigma}\cdot\hat{p} = +1 \implies \boxed{\hat{p} \text{ is along } +z}$$

$\bar{\nu}$ are monoenergetic & unidirectional.

- Pulsed polarizing fields
- ν communication even thro' the Earth to the antipodes

The Ultimate Neutrino Device

Solution of the puzzle

$$\psi = \begin{pmatrix} u \\ \dfrac{\vec{\sigma}\cdot\vec{p}}{E+m}\, u \end{pmatrix} e^{i\vec{p}\cdot\vec{r}}$$

Dirac wf for a particle with well-defined $\vec{p}$

2 component spinor

$$\psi = \begin{pmatrix} u \\ \dfrac{\vec{\sigma}\cdot\vec{p}}{p}\, u \end{pmatrix} e^{i\vec{p}\cdot\vec{r}}$$

for $m = 0$ so that $E = p$

More generally, if energy is well-defined, but direction of $\vec{p}$ not well-defined,

$$\psi = \begin{pmatrix} F(\vec{r}) \\ -i\dfrac{\vec{\sigma}\cdot\vec{\nabla}}{p}\, F(\vec{r}) \end{pmatrix}$$

where $F(\vec{r})$ is an arbitrary 2-component object.

$\therefore$ Pure S wave not possible for relativistic particles.

$\boxed{\nu\text{'s}}$: $\boxed{i\dfrac{\vec{\sigma}\cdot\vec{\nabla}}{p}\,\psi = \psi}$ This is the correct way of writing the helicity eq. for the ν. (not, $\vec{\sigma}\cdot\hat{p} = 1$)

$$\Longrightarrow \quad \psi = \begin{pmatrix} \left(1 + i\dfrac{\vec{\sigma}\cdot\vec{\nabla}}{p}\right) G(\vec{r}) \\ \left(1 + i\dfrac{\vec{\sigma}\cdot\vec{\nabla}}{p}\right) G(\vec{r}) \end{pmatrix}$$

where $(\nabla^2 + p^2)\, G(\vec{r}) = 0$

$\boxed{\text{not a pure S wave}}$

A correct understanding of ν requires both QM and relativity.

To say that spin of ν is pointing in the direction of its motion is not entirely correct