
Nuclear Effects in Neutrino Nucleus Reaction

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OUTLAY

Nuclear Effects in ν -A Cross Section

I. Quasielastic Reaction

II. Inelastic Reaction

Incoherent Pion Production

Coherent Pion Production

III. Deep-Inelastic Reaction

IV. Conclusion

Open Questions in ν Physics

- what are the masses of neutrinos?
- what is the mass hierarchy?
- more precisely measure remaining oscillation pars:
 - what are the precise values of Δm_{12}^2 , θ_{12} , $|\Delta m_{23}^2|$ and θ_{23}
 - is θ_{13} non-zero?
- is CP violated in the sector?
- if we want to address these questions, first need to understand how ν s interact with matter ... to estimate signal, backgrounds

to see maximum oscillation effects need to have low ν energy

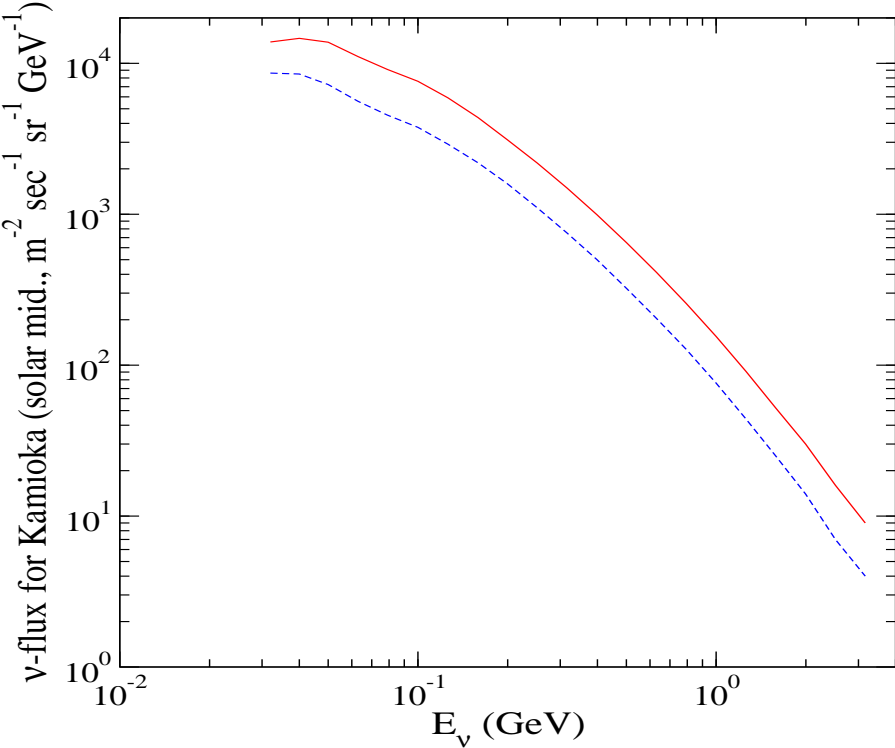
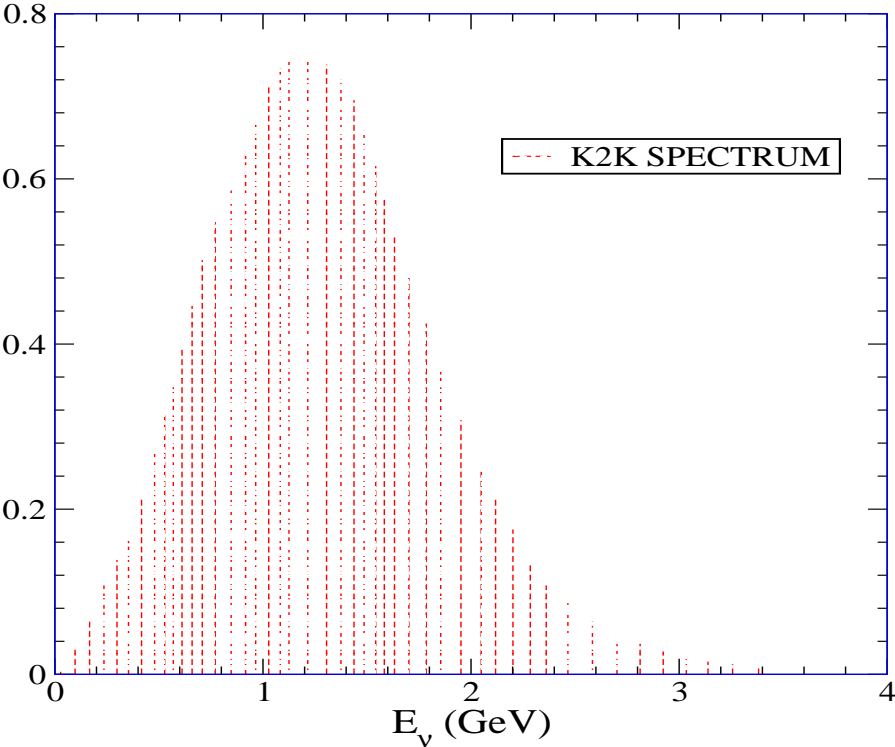
Energy Region of Interest

$$E_\nu < 3 \text{ GeV}$$

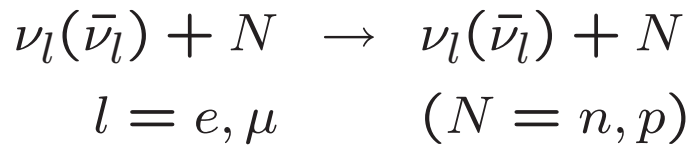
At:

1. K2K
2. MiniBooNE
3. T2K
4. β -Beam
5. Atmospheric

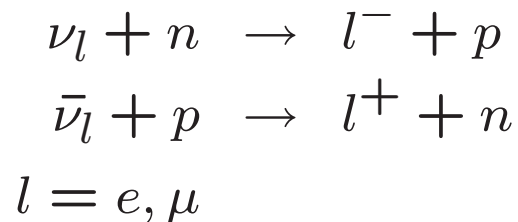
K2K and Atmospheric ν Spectrum



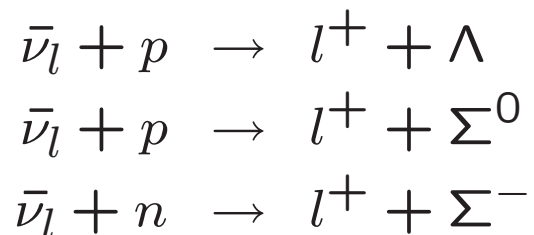
- Elastic Reaction



- Quasielastic Reaction

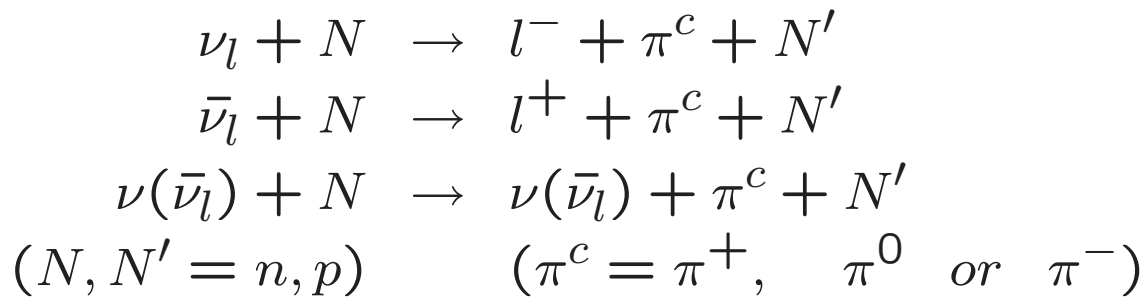


- Quasi-elastic $\Delta S = 1$ hyperon (Υ) production processes allowed in the neutrino(anti-neutrino) induced reactions are

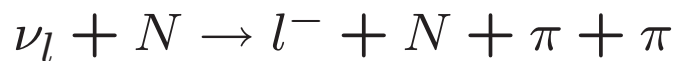


Inelastic Reaction

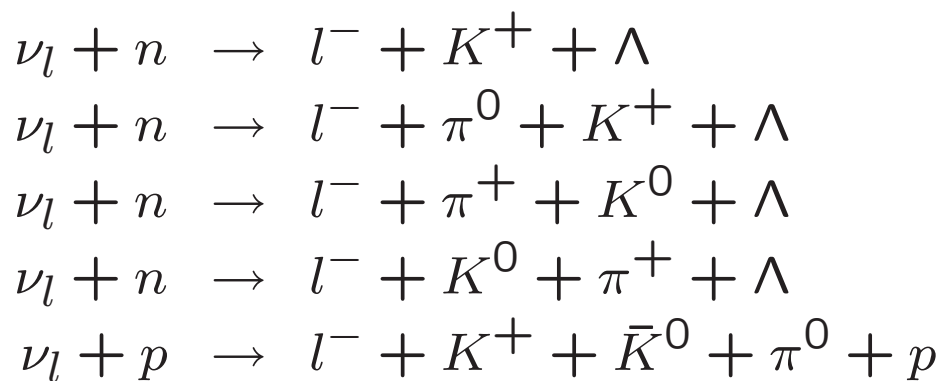
•Incoherent Pion Production



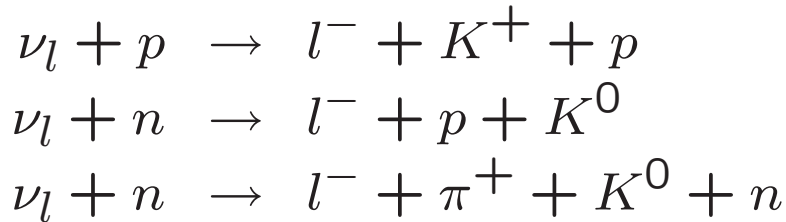
•Two pion production



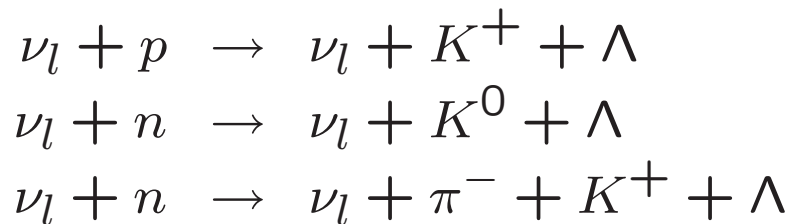
•Charged current $\Delta S = 0$ reactions induced by neutrinos(anti-neutrinos):



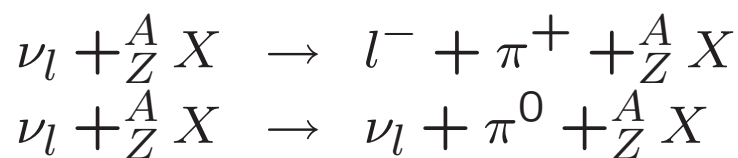
- Charged-current $\Delta S = 1$ reactions induced by neutrinos(anti-neutrinos):



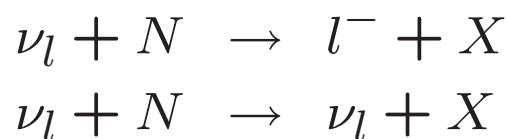
- Neutral-current $\Delta S = 0$ reactions induced by neutrinos(anti-neutrinos):



Coherent Pion Production



Deep-Inelastic Reaction



Energy Region and Targets of Some of the Older Experiments

	Energy	Target	QE	1 π	1 π Coh	2 π
Bonetti	1.5-6.0	$CF_3Br - C_3H_8$	✓			
Lerche	1-10.0	C_3H_8		✓		
Armenise	1-6.0	$CF_3Br - C_3H_8$	✓			
Pohl	1-8.5	$CF_3Br - C_3H_8$	✓			
Bolognese	1-7.5	$CF_3Br - C_3H_8$		✓		
Isiksal	3.5	$CF_3Br - C_3H_8$			✓	
Grabosch	3-30	CF_3Br			✓	
Ammosov	4.0-18.4	CF_3Br		✓		
Grabosch	3.5-12.0	CF_3Br		✓		
Brunner	3.0-11.0	CF_3Br	✓			
Belikov	3.0-30.0	Al	✓			
Belikov	3.0-25.0	Al	✓			
Belikov	3.5-20.0	Al	✓			
Fanourakis	1-4	H	✓			
Baker	0.7-2.2	D_2	✓			
Kitagaki	0.5-14.50	D_2		✓		✓
Day	0.75-5.55	D_2				✓

Various Nuclei being used or planned to be used in
the different detectors

Nuclei	Detector	Experiments
$^{12}_C$	Scintillator, Mineral Oil	NO ν A, MiniBooNE, K2K, MINERVA
$^{16}_O$	Water Cerenkov	T2K, SK-III, UNO, Hyper-K, K2K, MEMPHYS
$^{40}_{Ar}$	Liquid Argon TPC	ICARUS
$^{56}_{Fe}$	Iron Calorimeter	MINOS, INO, MINERVA
$^{208}_{Pb}$	Emulsion	OPERA, MINERVA

Common Inputs In Neutrino Event Generator

All the neutrino event generators use some nuclear model to estimate σ but inclusion of nuclear effects is mainly limited to Quasi Elastic reactions

Common Theoretical Inputs to all ν Event Generators:

- Llewellyn Smith free nucleon QE x-section
- Rein and Sehgal Resonance x-section
- Standard DIS formula for high W , Q^2 .

Inputs which are different for various ν Event Generators:

- ✓ Treatment of Nuclear Effects
- ✓ Joining of Resonance and DIS
- ✓ Treatment of FSI

Neutrino Event Generators

NUANCE

- Q.E. scattering comprises both C.C. and N.C. neutrino interactions with nucleons
- Relativistic Fermi gas model of Smith and Moniz

NUANCE v2

- Dipole form factors, $M_v = 0.84$, $M_A = 1.0\text{GeV}$

NUANCE v3

- Non-dipole form factors, π absorption model tuned on π data and $M_A = 1.03\text{GeV}$

NEUGEN

- Dipole Form Factors, $M_A = 1.032\text{GeV}$
- Relativistic Fermi gas model of Gaisser and O'Connell

NEUT

- Dipole Form Factors, $M_A = 1.1\text{GeV}$
- Fermi gas model of Smith and Moniz

Resonance Processes

- ✓ Rein and Sehgal model is used.
- ✓ Nuclear medium effects have not been taken for Resonance Production

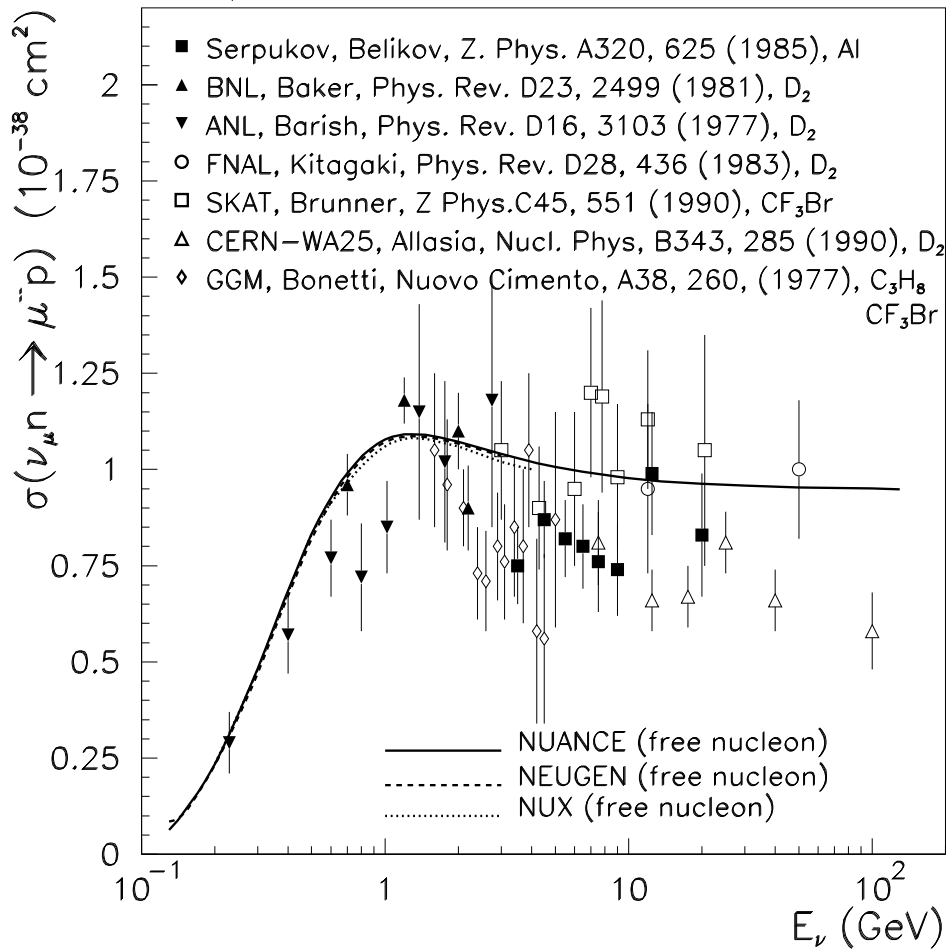
An ad hoc suppression of pion production:

- ✓ 20% for $I_3 = \pm \frac{1}{2}$ Resonance Excitations
- ✓ 10% for $I_3 = \pm \frac{3}{2}$ Resonance Excitations

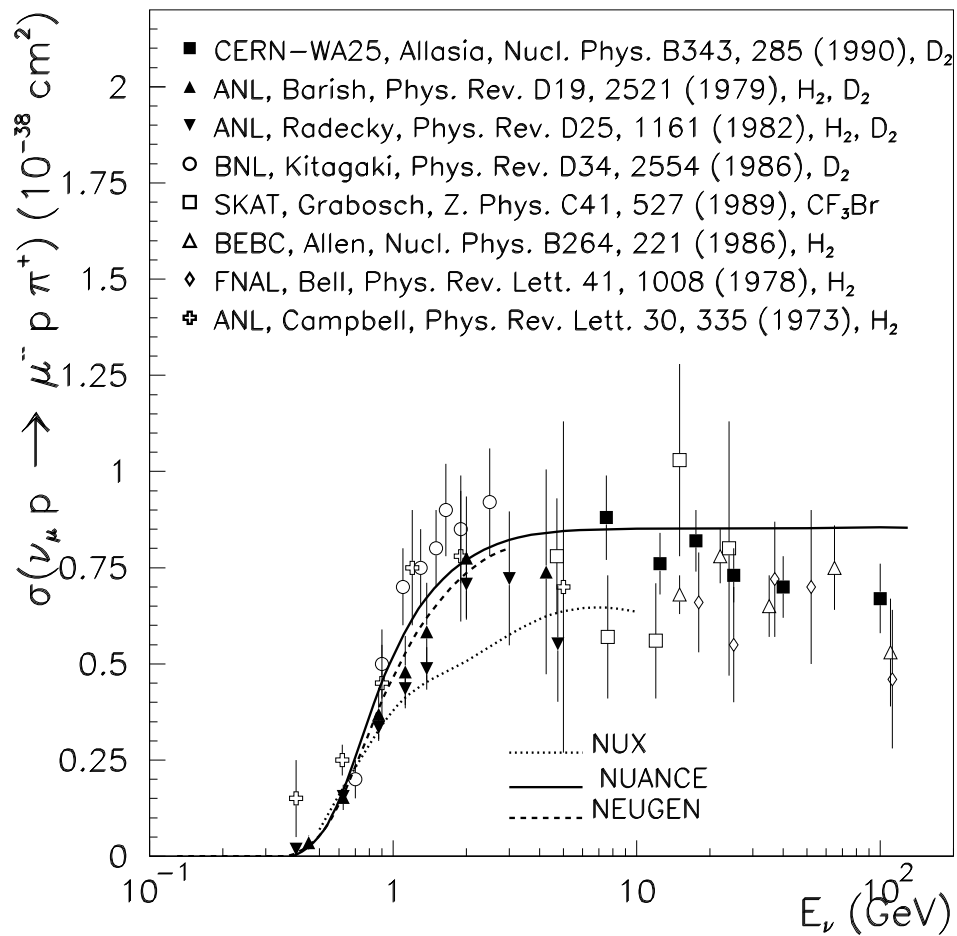
Coherent Processes

- ✓ Rein and Sehgal model is used.
- ✓ Nuclear Medium Effects on π Production is included through effective π -Nucleus Scattering

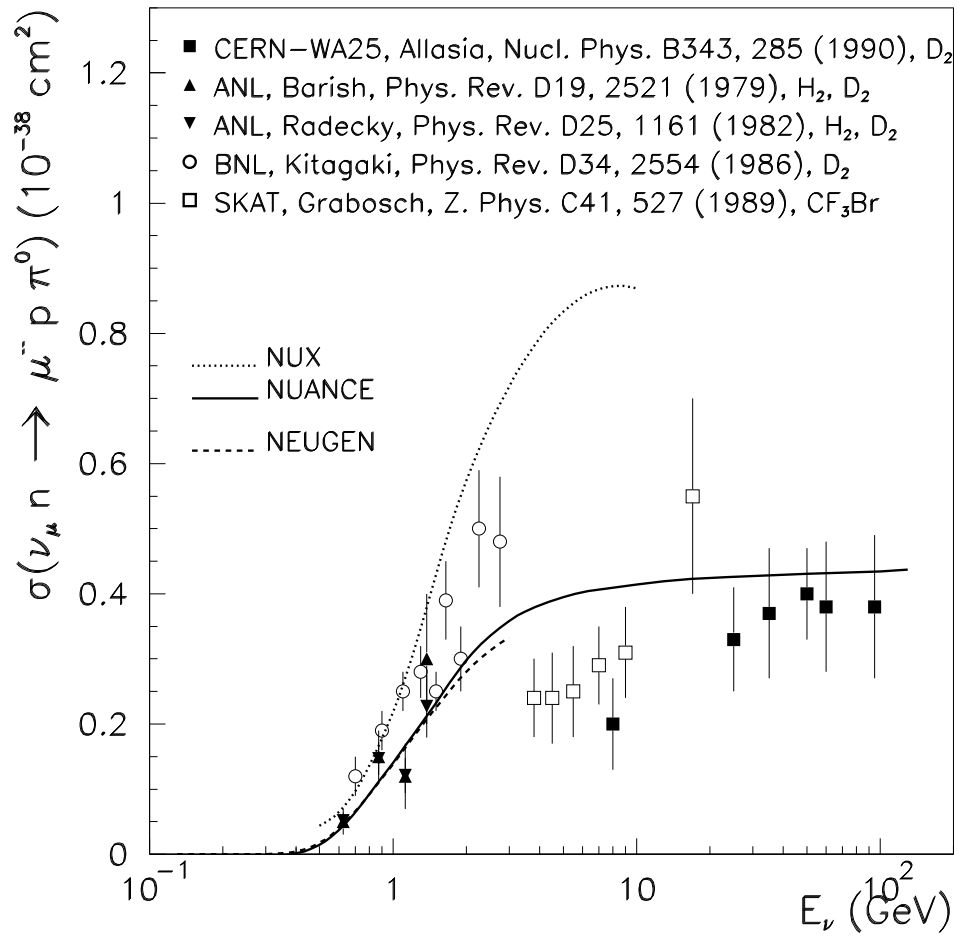
CC ν_μ Quasi-Elastic Cross Section



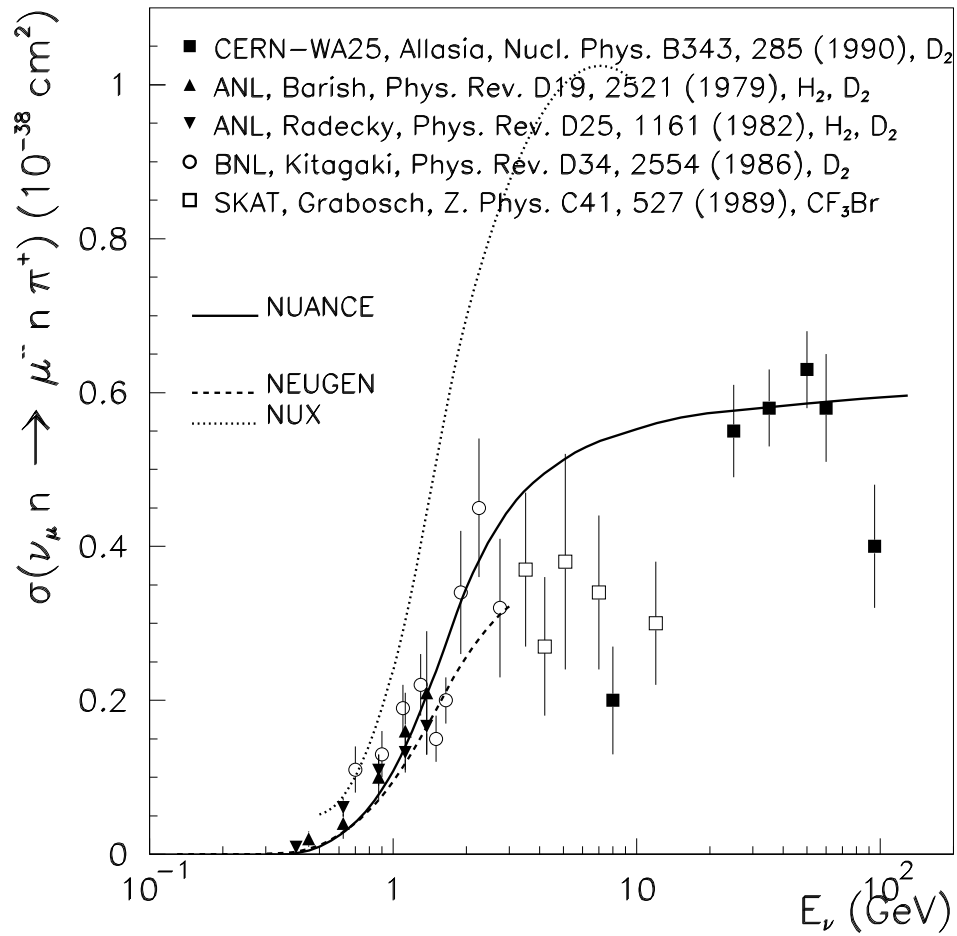
CC Single Pion Production



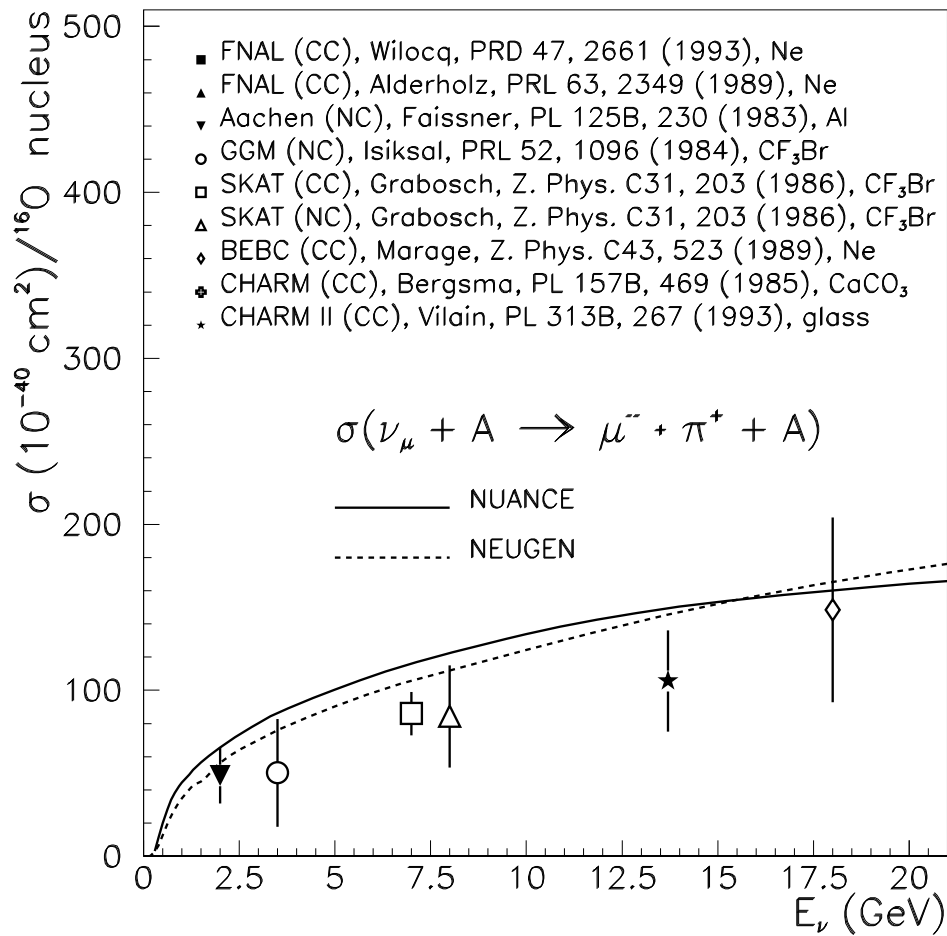
CC Single Pion Production

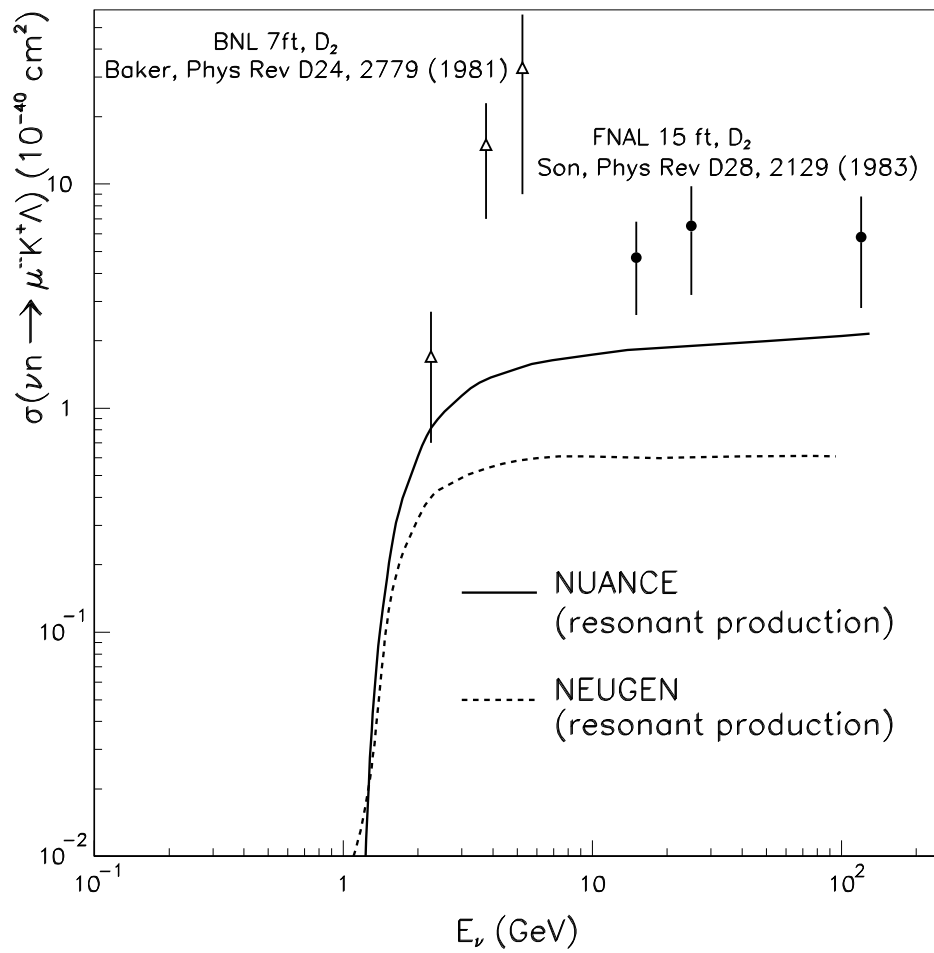


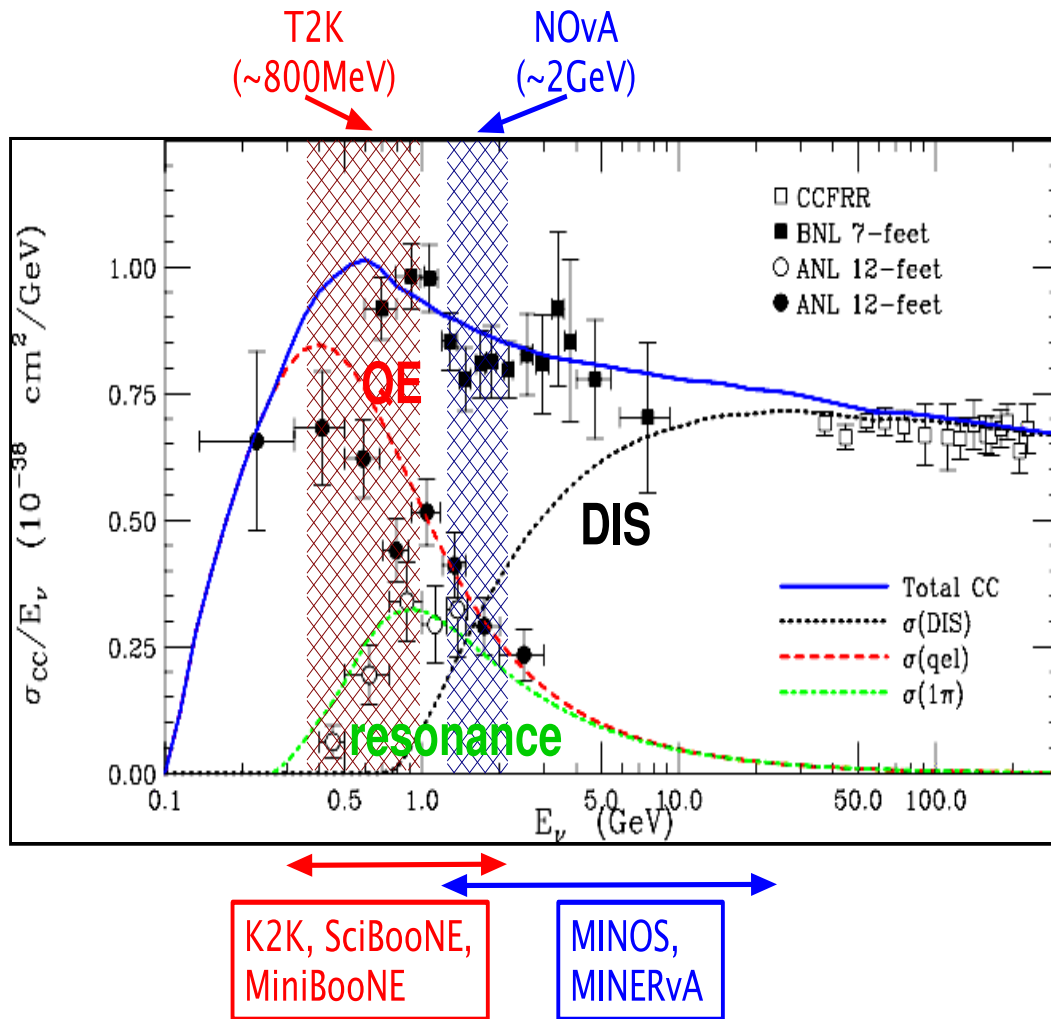
CC Single Pion Production



CC Coherent Pion Production Cross Section







K2K, MiniBooNE, SciBooNE, and MINERA experiments

Experiment	$\langle E_\nu \rangle$	target	run period
K2K	1.3 GeV	C_8H_8 , H_2O	1999-2004
MiniBooNE	0.8 GeV	CH_2	2002-present
SciBooNE	0.8 GeV	C_8H_8	2007-2008
MINERA	4.2, 5.7 GeV	He, CH	2009+
	6.1, 8.5 GeV	C, Fe, Pb	2009+

Axial dipole mass

$M_A = 1.03\text{GeV}$ World Average

$M_A = 1.20 \pm 0.12\text{GeV}$ K2K, SciFi, H_2O

$M_A = 1.14 \pm 0.11\text{GeV}$ K2K, SciBar, ^{12}C

$M_A = 1.23 \pm 0.20\text{GeV}$ MiniBooNE, ^{12}C

$M_A = 1.05 \pm 0.02 \pm 0.06\text{GeV}$ NOMAD, ^{12}C

MiniBooNE collaboration have recently measured NC coherent π^0 fraction, $19.5 \pm 2.7\%$, which is 35% lower than the most commonly used model. While K2K sees no evidence for the CC equivalent of this process in their low energy data.

Quasielastic Lepton Production

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O. Lalakulich, E.A. Paschos et al., Nucl. Proc. Suppl. 159, 133 (2006), PRD 74,014009 (2006)

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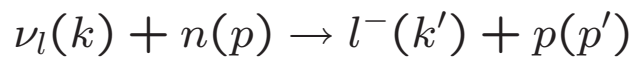
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Quasielastic Charged Current Reaction

The basic ν_l -neutron reaction taking place in ${}^A X$ nucleus is



The double differential cross section $\sigma_0(E_e, |\vec{k}'|)$

$$\begin{aligned} \sigma_0(E_l, |\vec{k}'|) &= G_F^2 \cos^2 \theta_c \frac{|\vec{k}'|^2}{8\pi E_{\nu_e} E_e} \frac{M_n M_p}{E_n E_p} \\ &\quad \bar{\Sigma} \Sigma |T|^2 \delta[q_0 + E_n - E_p] \end{aligned}$$

Matrix Element

$$T = \frac{G_F}{\sqrt{2}} \cos \theta_c l_\mu J^\mu$$

$$l_\mu = \bar{u}(k') \gamma_\mu (1 - \gamma_5) u(k)$$

$$\begin{aligned} J^\mu &= \bar{u}(p') \left[F_1^V(q^2) \gamma^\mu + F_2^V(q^2) i \sigma^{\mu\nu} \frac{q_\nu}{2M} \right. \\ &\quad \left. F_A^V(q^2) \gamma^\mu \gamma_5 + F_P^V(q^2) q^\mu \gamma_5 \right] u(p) \end{aligned}$$

$F_1^V(q^2)$, $F_2^V(q^2)$, $F_A^V(q^2)$ and $F_P^V(q^2)$

are isovector form factors.

$F_1^V(0) = 1.0$, $F_2^V(0) = 3.7059$, Dipole mass $M_v = 0.84\text{GeV}$, $F_A(0) = -1.26$.

$$F_1^V(q^2) = F_1^p(q^2) - F_1^n(q^2)$$

$$F_2^V(q^2) = F_2^p(q^2) - F_2^n(q^2)$$

$$F_A^V(q^2) = F_A(q^2)$$

$$F_1^{p,n}(q^2) = \left(1 - \frac{q^2}{4M^2}\right)^{-1} \left[G_E^{p,n}(q^2) - \frac{q^2}{4M^2} G_M^{p,n}(q^2) \right]$$

$$F_2^{p,n}(q^2) = \left(1 - \frac{q^2}{4M^2}\right)^{-1} \left[G_M^{p,n}(q^2) - G_E^{p,n}(q^2) \right]$$

where

$$G_E^p(q^2) = \left(1 - \frac{q^2}{M_V^2}\right)^{-2}$$

$$G_M^p(q^2) = (1 + \mu_p) G_E^p(q^2), \quad G_M^n(q^2) = \mu_n G_E^p(q^2)$$

$$G_E^n(q^2) = \left(\frac{q^2}{4M^2}\right) \mu_n G_E^p(q^2) \xi_n, \quad \xi_n = \frac{1}{1 - \lambda_n \frac{q^2}{4M^2}}$$

$$F_A(Q^2) = F_A(0) \left(1 - \frac{q^2}{M_V^2}\right)^{-2}$$

Deviations from the dipole behaviour have been discussed recently

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W. M. Alberico et al., arXiv:0812.3539 [hep-ph]

Local Density Approximation

The neutrino scatters from a neutron moving in the finite nucleus of neutron density $\rho_n(r)$, with a local occupation number $n_n(\mathbf{p}, \mathbf{r})$, and σ is given by

$$\sigma_{Nucleus} = \int \rho_n(r) d^3r [\sigma_{FreeNucleon}]$$

$$\rho_n(r) = 2 \int d\mathbf{p}_n \frac{1}{(2\pi)^3} n_n(\mathbf{p}, \mathbf{r})$$

$$\sigma(E_e, |\vec{k}'|) = \int 2d\mathbf{r} d\mathbf{p} \frac{1}{(2\pi)^3} n_n(\mathbf{p}, \mathbf{r}) \sigma_0(E_e, k')$$

$$\begin{aligned} \sigma_0(E_e, |\vec{k}'|) = & G_F^2 \cos^2 \theta_c \frac{|\vec{k}'|^2}{8\pi E_{\nu_e} E_e} \frac{M_n M_p}{E_n E_p} \\ & \bar{\Sigma} \Sigma |T|^2 \delta[q_0 + E_n - E_p] \end{aligned}$$

We take into account:

(i) Pauli blocking and Fermi motion of the nucleons

(ii) Q-value of the reaction

(iii) Coulomb distortion of the charged leptons in an effective momentum approximation

(iv) Medium polarization effects in an Random Phase Approximation(RPA) which includes the particle hole and Δ -hole degrees of freedom

We will focus on the inclusive nuclear reaction driven by the electroweak CC



The double differential cross section in the Laboratory frame by

$$\frac{d^2\sigma_{\nu l}}{d\Omega(\hat{k}')dE'_l} = \frac{|\vec{k}'|}{|\vec{k}|} \frac{G^2}{4\pi^2} L_{\mu\sigma} W^{\mu\sigma}$$

$$L_{\mu\sigma} = L_{\mu\sigma}^s + iL_{\mu\sigma}^a = k'_\mu k_\sigma + k'_\sigma k_\mu - g_{\mu\sigma} k \cdot k' + i\epsilon_{\mu\sigma\alpha\beta} k'^\alpha k^\beta$$

The hadronic tensor is given in terms of six independent structure functions $W_i(q^2)$,

$$\begin{aligned} \frac{W^{\mu\sigma}}{2M_i} &= -g^{\mu\sigma} W_1 + \frac{P^\mu P^\sigma}{M_i^2} W_2 \\ &+ i \frac{\epsilon^{\mu\sigma\gamma\delta} P_\gamma q_\delta}{2M_i^2} W_3 + \frac{q^\mu q^\sigma}{M_i^2} W_4 \\ &+ \frac{P^\mu q^\sigma + P^\sigma q^\mu}{2M_i^2} W_5 + i \frac{P^\mu q^\sigma - P^\sigma q^\mu}{2M_i^2} W_6 \end{aligned}$$

Taking \vec{q} in the z direction and $P^\mu = (M_i, \vec{0})$, the six structure functions can be written in terms of $W^{00}, W^{xx} = W^{yy}, W^{zz}, W^{xy}$ and W^{0z} components of the hadronic tensor

These relations read

$$W_1 = \frac{W^{xx}}{2M_i} \qquad W_3 = -i \frac{W^{xy}}{|\vec{q}|}$$

$$W_2 = \frac{1}{2M_i} \left(W^{00} + W^{xx} + \frac{(q^0)^2}{|\vec{q}|^2} (W^{zz} - W^{xx}) - 2 \frac{q^0}{|\vec{q}|} \text{Re } W^{0z} \right)$$

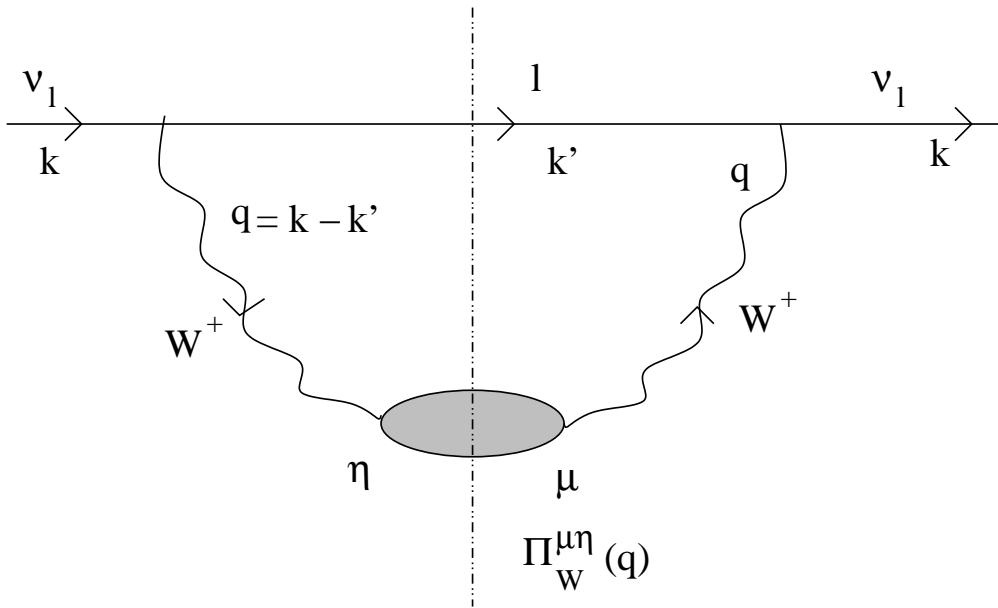
$$W_4 = \frac{M_i}{2|\vec{q}|^2} (W^{zz} - W^{xx}) \qquad W_6 = \frac{\text{Im } W^{0z}}{|\vec{q}|}$$

$$W_5 = \frac{1}{|\vec{q}|} \left(\text{Re } W^{0z} - \frac{q^0}{|\vec{q}|} (W^{zz} - W^{xx}) \right)$$

$$\begin{aligned} \frac{d^2\sigma_{\nu l}}{d\Omega(\hat{k}')dE'_l} &= \frac{|\vec{k}'|E'_l M_i G^2}{\pi^2} \left\{ 2W_1 \sin^2 \frac{\theta'}{2} \right. \\ &+ W_2 \cos^2 \frac{\theta'}{2} - W_3 \frac{E_\nu + E'_l}{M_i} \sin^2 \frac{\theta'}{2} \\ &+ \frac{m_l^2}{E'_l(E'_l + |\vec{k}'|)} \left[W_1 \cos \theta' - \frac{W_2}{2} \cos \theta' \right. \\ &+ \frac{W_3}{2} \left(\frac{E'_l + |\vec{k}'|}{M_i} - \frac{E_\nu + E'_l}{M_i} \cos \theta' \right) \\ &+ \frac{W_4}{2} \left(\frac{m_l^2}{M_i^2} \cos \theta' + \frac{2E'_l(E'_l + |\vec{k}'|)}{M_i^2} \sin^2 \theta' \right) \\ &\left. - W_5 \frac{E'_l + |\vec{k}'|}{2M_i} \right\} \end{aligned}$$

In our MBF, the hadronic tensor is determined by the W^+ -boson selfenergy, $\Pi_W^{\mu\rho}(q)$, in the nuclear medium. We evaluate the selfenergy, $\Sigma_\nu^r(k; \rho)$, of a neutrino, with four-momentum k and helicity r , moving in infinite nuclear matter of density ρ .

$$\begin{aligned}
 -i\Sigma_\nu^r(k; \rho) &= \int \frac{d^4q}{(2\pi)^4} \bar{u}_r(k) \left(-i\frac{g}{2\sqrt{2}}\gamma_L^\mu \right. \\
 &\quad \left. iD_{\mu\alpha}(q) \left(-i\Pi_W^{\alpha\beta}(q; \rho) \right) iD_{\beta\sigma}(q) i\frac{k' + m_l}{k'^2 - m_l^2 + i\epsilon} \right. \\
 &\quad \left. \left(-i\frac{g}{2\sqrt{2}} \right) \gamma^\sigma (1 - \gamma_5) \right) u_r(k)
 \end{aligned}$$



$$D_{\mu\alpha}(q) = (-g_{\mu\alpha} + q_\mu q_\alpha / M_W^2) / (q^2 - M_W^2 + i\epsilon)$$

$\Pi_W^{\mu\eta}(q; \rho)$ is the virtual W^+ selfenergy in the medium

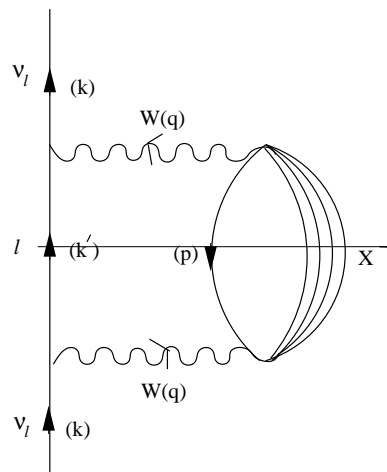
The sum over helicities leads to traces in the Dirac's space

$$\Sigma_\nu(k; \rho) = \frac{8iG}{\sqrt{2}M_W^2} \int \frac{d^4q}{(2\pi)^4} \frac{L_{\eta\mu} \Pi_W^{\mu\eta}(q; \rho)}{k'^2 - m_l^2 + i\epsilon}$$

The neutrino disappears from the elastic flux, by inducing 1p1h, 2p2h excitations, $\Delta(1232)$ -hole (Δh) excitations etc. given by

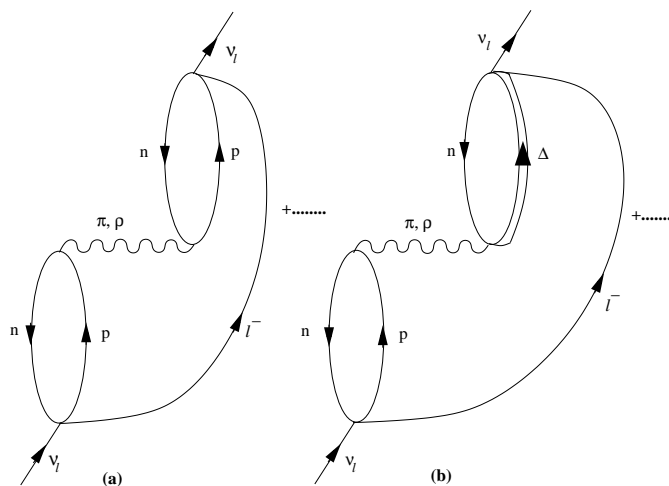
$$\Gamma(k; \rho) = -\frac{1}{k^0} \text{Im} \Sigma_\nu(k; \rho)$$

The virtual W^+ can be absorbed by one nucleon leading to the QE contribution of the nuclear response function. Such a contribution corresponds to a 1p1h nuclear excitation.



Diagrammatic representation of the neutrino self-energy diagram corresponding to the ph-excitation leading to $\nu_l + n \rightarrow l^- + p$ in nuclei.

We perform a many body expansion, where the relevant gauge boson absorption modes would taken into account: absorption by one nucleon, or a pair of nucleons or even three nucleon mechanisms, real and virtual meson (π , ρ , \dots) production, excitation of Δ of higher resonance degrees of freedom, etc.



Many body Feynman diagrams (drawn in the limit $M_W \rightarrow \infty$) accounting for the medium polarization effects contributing to the process $\nu_l + n \rightarrow l^- + p$

We get the imaginary part of Σ_ν by using the Cutkosky's rules. We cut with a straight vertical line, through the intermediate lepton state and those implied by the W -boson polarization (shaded region)

Those states are then placed on shell by taking the imaginary part of the propagator, selfenergy, etc.

Thus, we obtain for $k^0 > 0$

$$\text{Im } \Sigma_\nu(k) = \frac{8G}{\sqrt{2}M_W^2} \int \frac{d^3k'}{(2\pi)^3} \frac{\Theta(q^0)}{2E'_l} \text{Im} \{ \Pi_W^{\mu\eta}(q; \rho) L_{\eta\mu} \}$$

Since $\Gamma dt dS$ provides a probability times a differential of area, which is a contribution to the (ν_l, l) cross section, we have

$$\begin{aligned} d\sigma &= \Gamma(k; \rho) dt dS = -\frac{1}{k^0} \text{Im} \Sigma_\nu(k; \rho) dt dS \\ &= -\frac{1}{|\vec{k}|} \text{Im} \Sigma_\nu(k; \rho) d^3r \end{aligned}$$

The nuclear cross section is then given by

$$\sigma = -\frac{1}{|\vec{k}|} \int \text{Im} \Sigma_\nu(k; \rho(r)) d^3r$$

Thus the differential scattering cross section is written as

$$\begin{aligned} \frac{d^2\sigma_{\nu l}}{d\Omega(\hat{k}')dk'^0} &= -\frac{|\vec{k}'|}{|\vec{k}|} \frac{G^2}{4\pi^2} \left(\frac{2\sqrt{2}}{g}\right)^2 \int \frac{d^3r}{2\pi} \\ &\times \{L_{\mu\eta}^s \text{Im}(\Pi_W^{\mu\eta} + \Pi_W^{\eta\mu}) - L_{\mu\eta}^a \text{Re}(\Pi_W^{\mu\eta} - \Pi_W^{\eta\mu})\} \\ &\times \Theta(q^0) \end{aligned}$$

and the hadronic tensor reads

$$W_s^{\mu\sigma} = -\Theta(q^0) \left(\frac{2\sqrt{2}}{g}\right)^2 \int \frac{d^3r}{2\pi} \text{Im} [\Pi_W^{\mu\sigma} + \Pi_W^{\sigma\mu}] (q; \rho)$$

$$W_a^{\mu\sigma} = -\Theta(q^0) \left(\frac{2\sqrt{2}}{g}\right)^2 \int \frac{d^3r}{2\pi} \text{Re} [\Pi_W^{\mu\sigma} - \Pi_W^{\sigma\mu}] (q; \rho)$$

$$\begin{aligned} W^{\mu\nu}(q^0, \vec{q}) &= -\frac{\cos^2\theta_C}{2M^2} \int_0^\infty dr r^2 \\ &\times \left(2\Theta(q^0) \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(\vec{p})} \frac{M}{E(\vec{p} + \vec{q})} \Theta(k_F^n(r) - |\vec{p}|) \right. \\ &\times \Theta(|\vec{p} + \vec{q}| - k_F^p(r)) (-\pi) \\ &\times \left. \delta(q^0 + E(\vec{p}) - E(\vec{p} + \vec{q})) A^{\nu\mu}(p, q)|_{p^0=E(\vec{p})} \right) \end{aligned}$$

The relativistic Lindhard function is defined as

$$\bar{U}_R(q, k_F^n, k_F^p) = 2 \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(\vec{p})} \frac{M}{E(\vec{p} + \vec{q})} \times \frac{\Theta(k_F^n - |\vec{p}|) \Theta(|\vec{p} + \vec{q}| - k_F^p)}{q^0 + E(\vec{p}) - E(\vec{p} + \vec{q}) + i\epsilon}$$

Imaginary part of the Lindhard function is given by

$$\begin{aligned} \text{Im}\bar{U}_R(q, k_F^n, k_F^p) &= \int d^3p \mathcal{F}_R(q, \vec{p}, k_F^n, k_F^p) \\ &= -M^2 \frac{\Theta(q^0) \Theta(-q^2)}{2\pi |\vec{q}|} \\ &\quad \Theta(E_F^n - E_F^p + q^0) \Theta(E_F^n - \mathcal{E}_R^p) (E_F^n - \mathcal{E}_R^p) \end{aligned}$$

with

$$\begin{aligned} \mathcal{F}_R(q, \vec{p}, k_F^n, k_F^p) &= -\frac{M^2}{4\pi^2} \frac{\Theta(q^0) \delta(q^0 + E(\vec{p}) - E(\vec{p} + \vec{q}))}{E(\vec{p}) E(\vec{p} + \vec{q})} \\ &\quad \Theta(k_F^n - |\vec{p}|) \Theta(|\vec{p} + \vec{q}| - k_F^p) \\ \mathcal{E}_R^p &= \text{Max} \left\{ M, E_F^p - q^0, \frac{-q^0 + |\vec{q}| \sqrt{1 - 4M^2/q^2}}{2} \right\}, \end{aligned}$$

being Max(...) the maximum of the quantities included in the bracket.

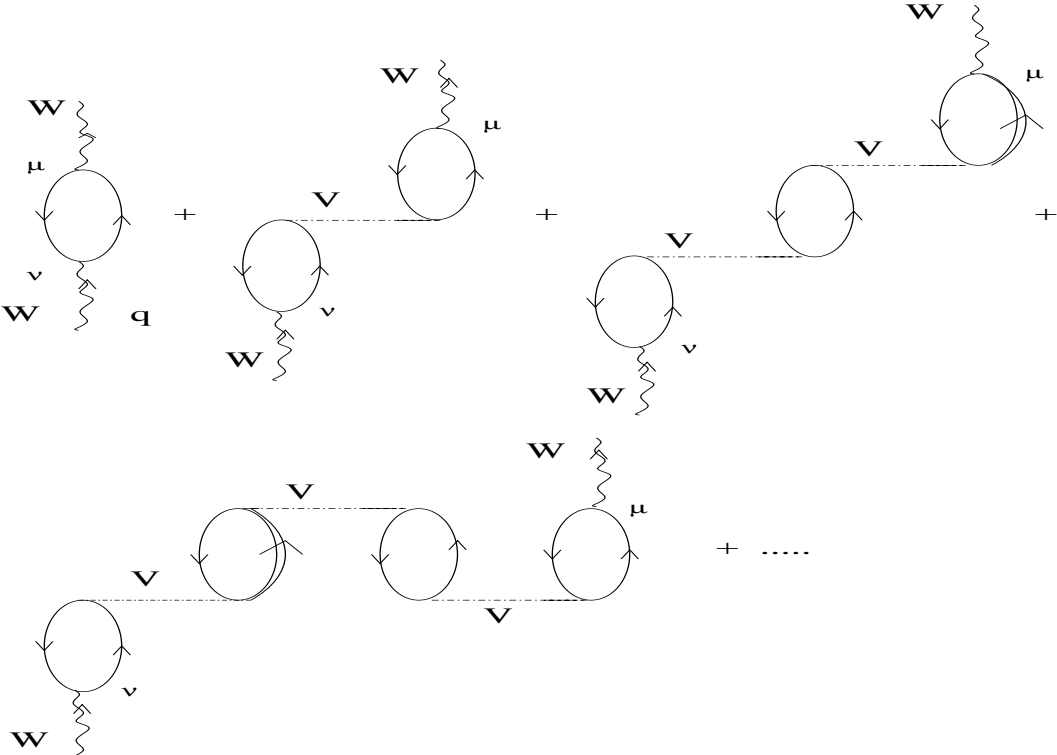
$A^{\mu\nu}$ is given by

$$\begin{aligned}
A^{\mu\nu}(p, q) &= 16(F_1^V)^2 \left\{ (p+q)^\mu p^\nu + (p+q)^\nu p^\mu + \frac{q^2}{2} g^{\mu\nu} \right\} \\
&+ 2q^2(\mu_V F_2^V)^2 \left\{ 4g^{\mu\nu} - 4\frac{p^\mu p^\nu}{M^2} \right. \\
&- \left. 2\frac{p^\mu q^\nu + q^\mu p^\nu}{M^2} - q^\mu q^\nu \left(\frac{4}{q^2} + \frac{1}{M^2} \right) \right\} \\
&- 16F_1^V \mu_V F_2^V (q^\mu q^\nu - q^2 g^{\mu\nu}) \\
&+ 4G_A^2 \left\{ 2p^\mu p^\nu + q^\mu p^\nu + p^\mu q^\nu + g^{\mu\nu} \left(\frac{q^2}{2} - 2M^2 \right) \right. \\
&- \left. \frac{2M^2(2m_\pi^2 - q^2)}{(m_\pi^2 - q^2)^2} q^\mu q^\nu \right\} \\
&- 16iG_A (\mu_V F_2^V + F_1^V) \epsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta
\end{aligned}$$

As we see, the basic object is the selfenergy of the Gauge Boson (W^\pm) inside of the nuclear medium.

Furthermore, nuclear effects such as RPA or Short Range Correlations should also be taken into account.

In the nucleus the strength of the electroweak coupling may change from their free nucleon values due to the presence of strongly interacting nucleons. Conservation of Vector Current (CVC) forbids any change in the charge coupling while magnetic and axial vector couplings are likely to change from their free nucleon values. These changes are calculated by considering the interaction of ph excitations in the nuclear medium in Random Phase Approximation (RPA)



RPA effects in the $1p1h$ contribution to the W self energy

$$\begin{aligned}
\frac{A_{\text{RPA}}^{00}}{4M^2} &= 8(F_1^V)^2 \left\{ \underline{\mathbf{C}}_{\text{N}} \left(\frac{E(\vec{p})}{M} \right)^2 + \frac{q^2/4 + q^0 E(\vec{p})}{M^2} \right\} \\
&- 2 \frac{q^2}{M^2} (\mu_V F_2^V)^2 \left\{ \frac{\vec{p}^2 + q^0 E(\vec{p}) + (q^0)^2/4}{M^2} + \frac{(q^0)^2}{q^2} \right\} \\
&- 4 \underline{\mathbf{C}}_{\text{N}} F_1^V \mu_V F_2^V \frac{\vec{q}^2}{M^2} + 2G_A^2 \left\{ \frac{q^0 E(\vec{p}) + q^2/4 + \vec{p}^2}{M^2} \right. \\
&\left. - \frac{\underline{\mathbf{C}}_{\text{L}}}{m_\pi^2 - q^2} \left(\frac{q^2}{m_\pi^2 - q^2} + 2 \right) \right\}
\end{aligned}$$

$$\frac{A_{\text{RPA}}^{xy}}{4M^2} = 4iG_A(F_1^V + \mu_V F_2^V) \left(\frac{q^0 p_z}{M^2} - \underline{\mathbf{C}}_{\text{T}} \frac{|\vec{q}| E(\vec{p})}{M^2} \right)$$

The polarization coefficients are defined as

$$\underline{\mathbf{C}}_{\text{N}}(\rho) = \frac{1}{|1 - c_0 f'(\rho) U_{\text{N}}(q, k_{\text{F}})|^2}$$

$$\underline{\mathbf{C}}_{\text{T}}(\rho) = \frac{1}{|1 - U(q, k_{\text{F}}) V_{\text{t}}(q)|^2}$$

$$\underline{\mathbf{C}}_{\text{L}}(\rho) = \frac{1}{|1 - U(q, k_{\text{F}}) V_{\text{l}}(q)|^2}$$

Thus, in presence of nuclear medium effects $\sigma(E_\nu)$ is given by

$$\begin{aligned} \sigma^{FF(MEMMA)}(E_\nu) &= -\frac{2G_F^2 \cos^2 \theta_c}{\pi} \int_{r_{min}}^{r_{max}} r^2 dr \\ &\times \int_{p_e^{min}}^{p_e^{max}} p_e^2 dp_e \int_{-1}^1 d(\cos\theta) \\ &\times \frac{1}{E_\nu E_e} L_{\mu\nu} J_{RPA}^{\mu\nu} ImU_N^{(MEMMA)}. \end{aligned}$$

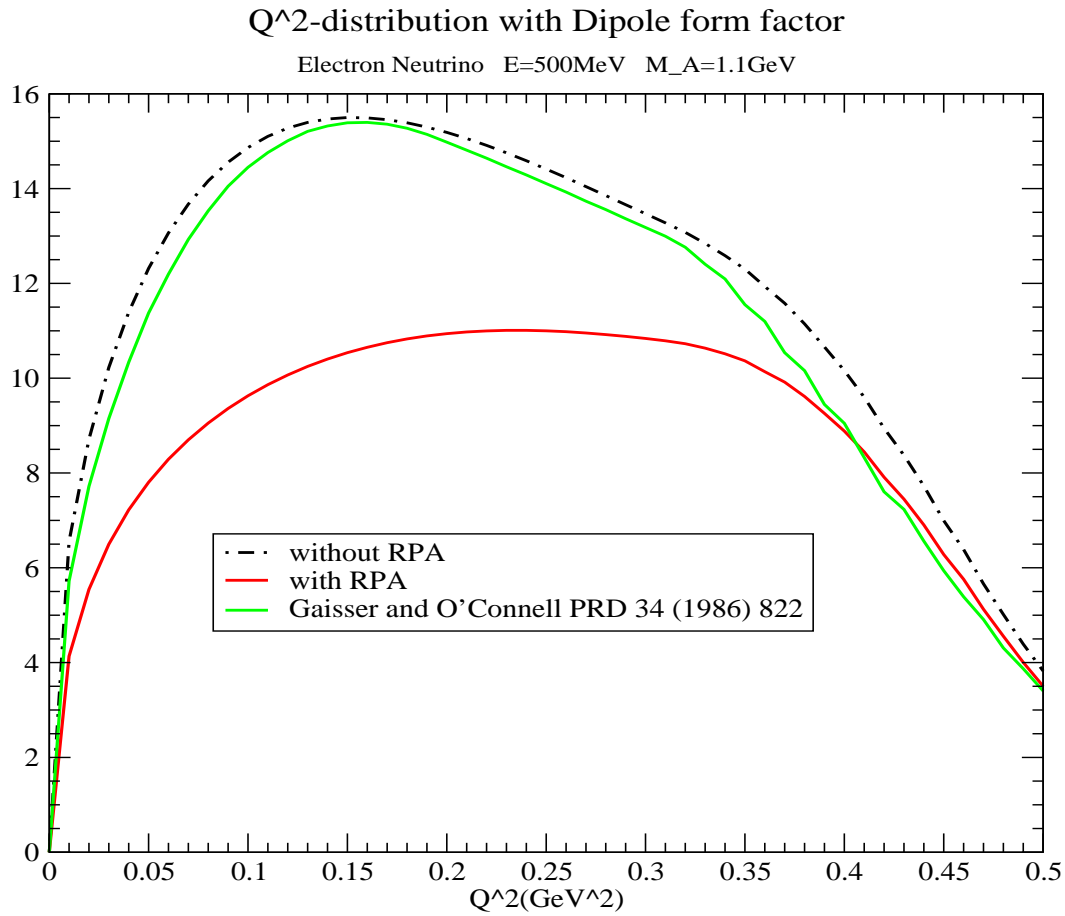
where

$$ImU_N^{MEMMA} = ImU_N[E_{\nu_e} - E_e - Q - V_c(r), \vec{q}]$$

Results

Intermediate Energy ν -A Reaction

$\frac{d\sigma}{dQ^2}$ vs Q^2 in $10^{-38} \frac{cm^2}{GeV^2}$ in $\nu_e + {}^{16}O$ scattering



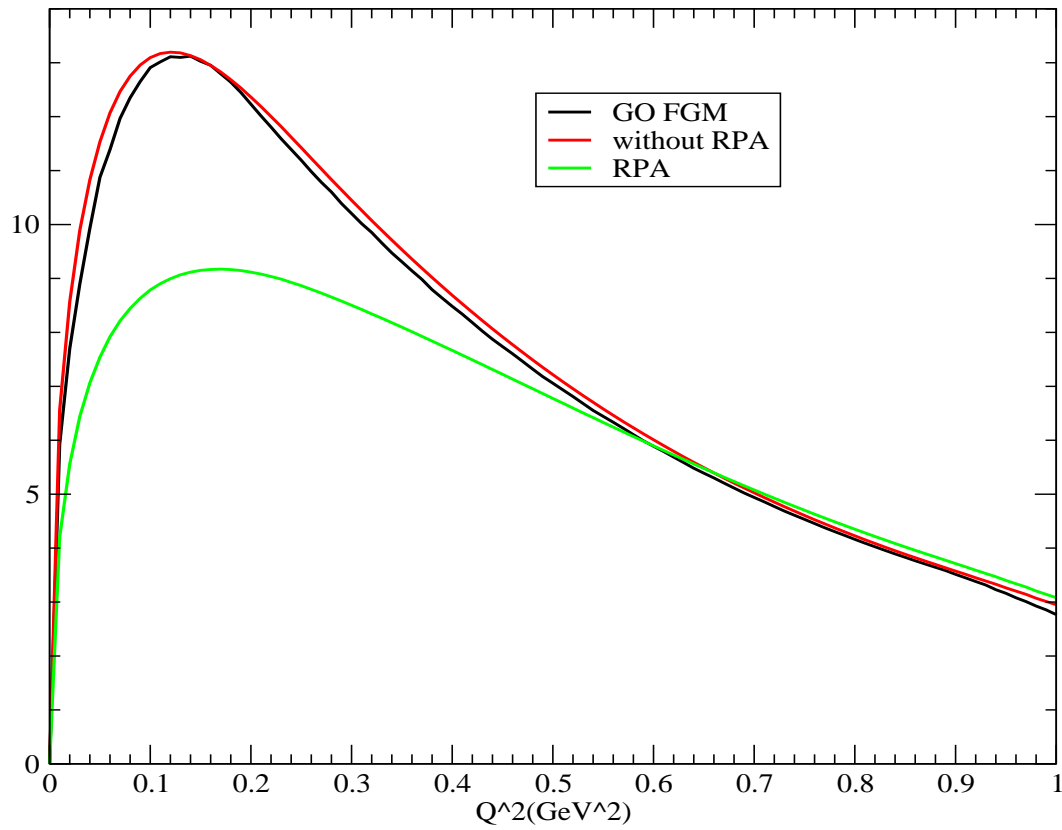
$Q^2(\text{GeV}^2)$	with RPA
0.1	70
0.15	32
0.2	28
0.3	19
0.5	8

% reduction in the Q^2 distribution when RPA is incorporated

$\frac{d\sigma}{dQ^2}$ vs Q^2 in $10^{-38} \frac{cm^2}{GeV^2}$ in $\nu_e + {}^{16}O$ scattering

Q^2 distribution at $E=1GeV$

Electron Neutrino Dipole F.F. $M_A=1.1GeV$

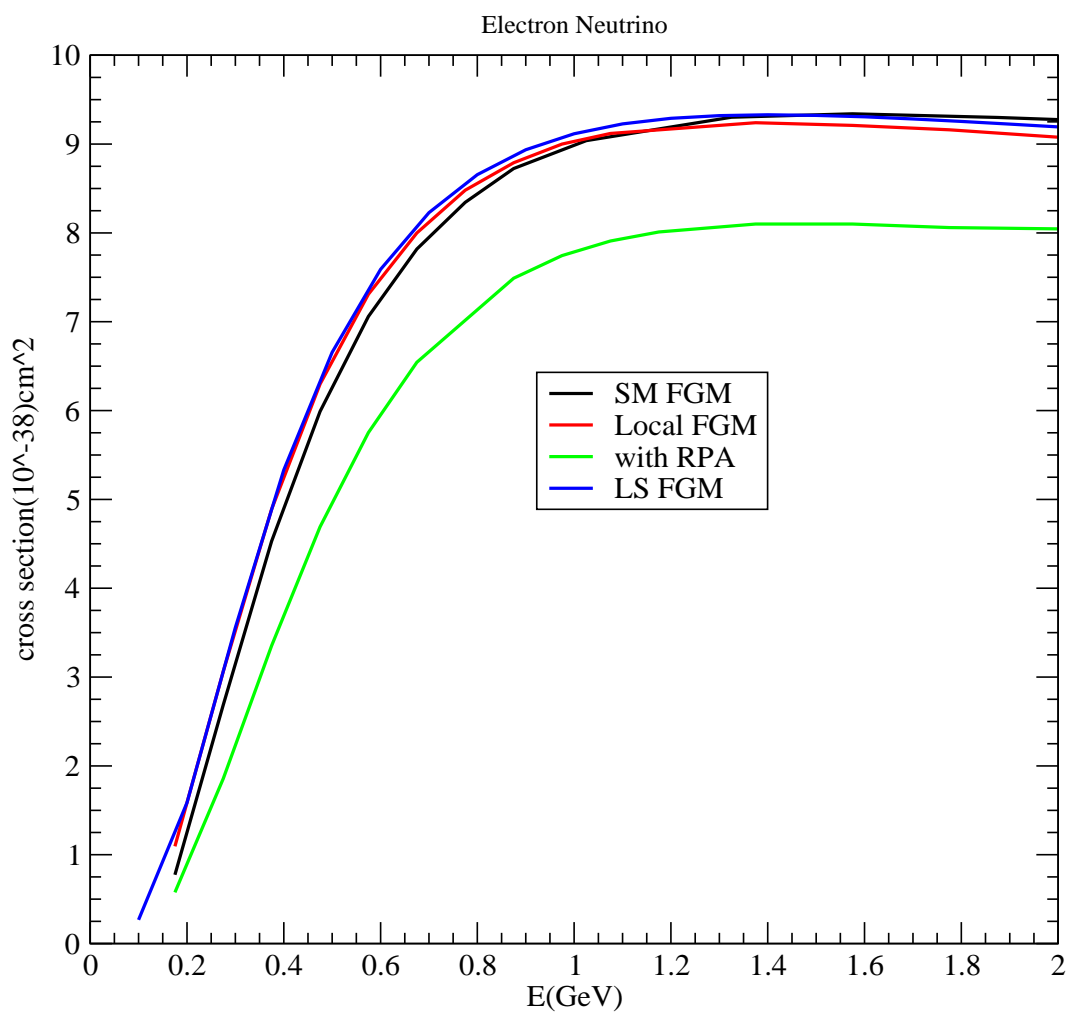


$Q^2(GeV^2)$	with RPA
0.1	33
0.15	30
0.2	26
0.3	18
0.5	6

% reduction in the Q^2 distribution when RPA is incorporated

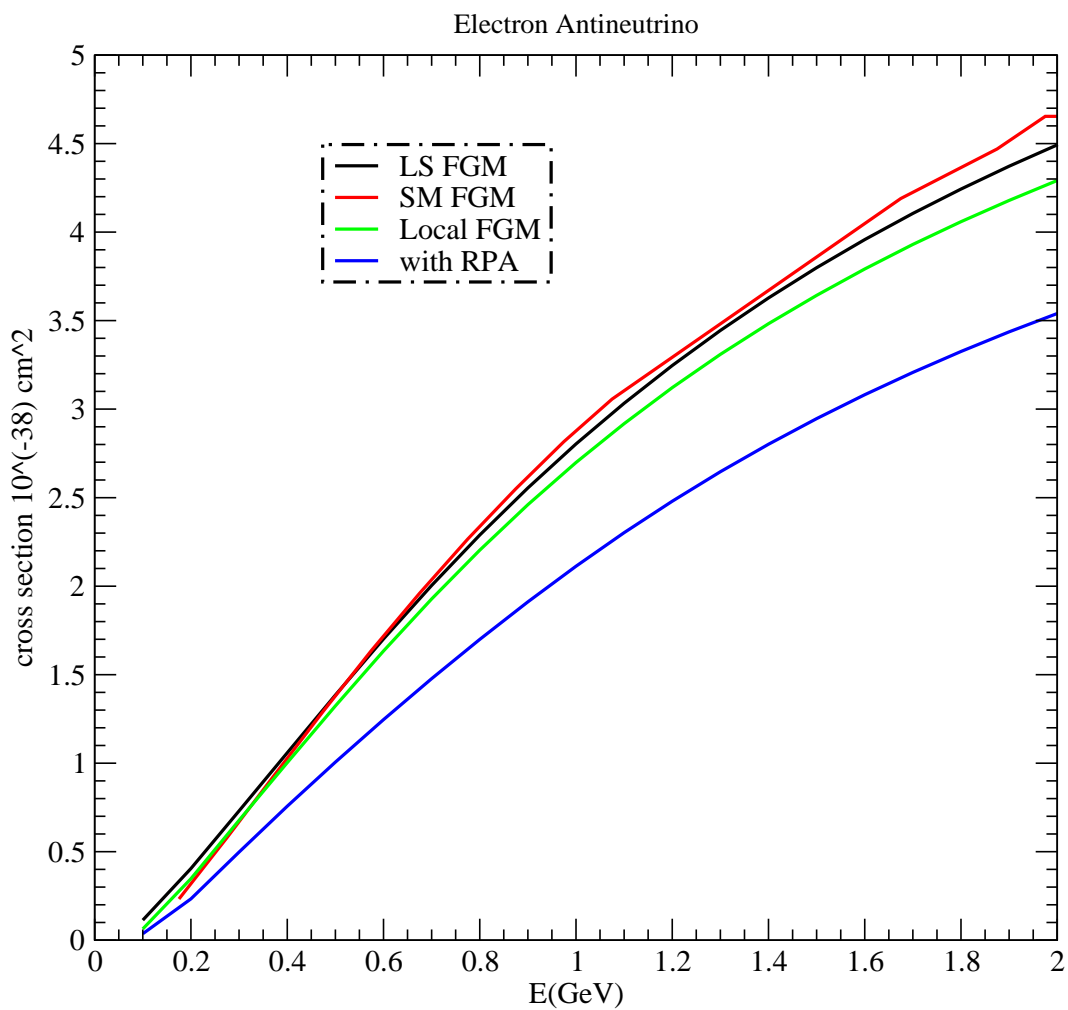
σ vs E_{ν_e} in 10^{-38}cm^2 in $\nu_e + {}^{16}\text{O}$ scattering

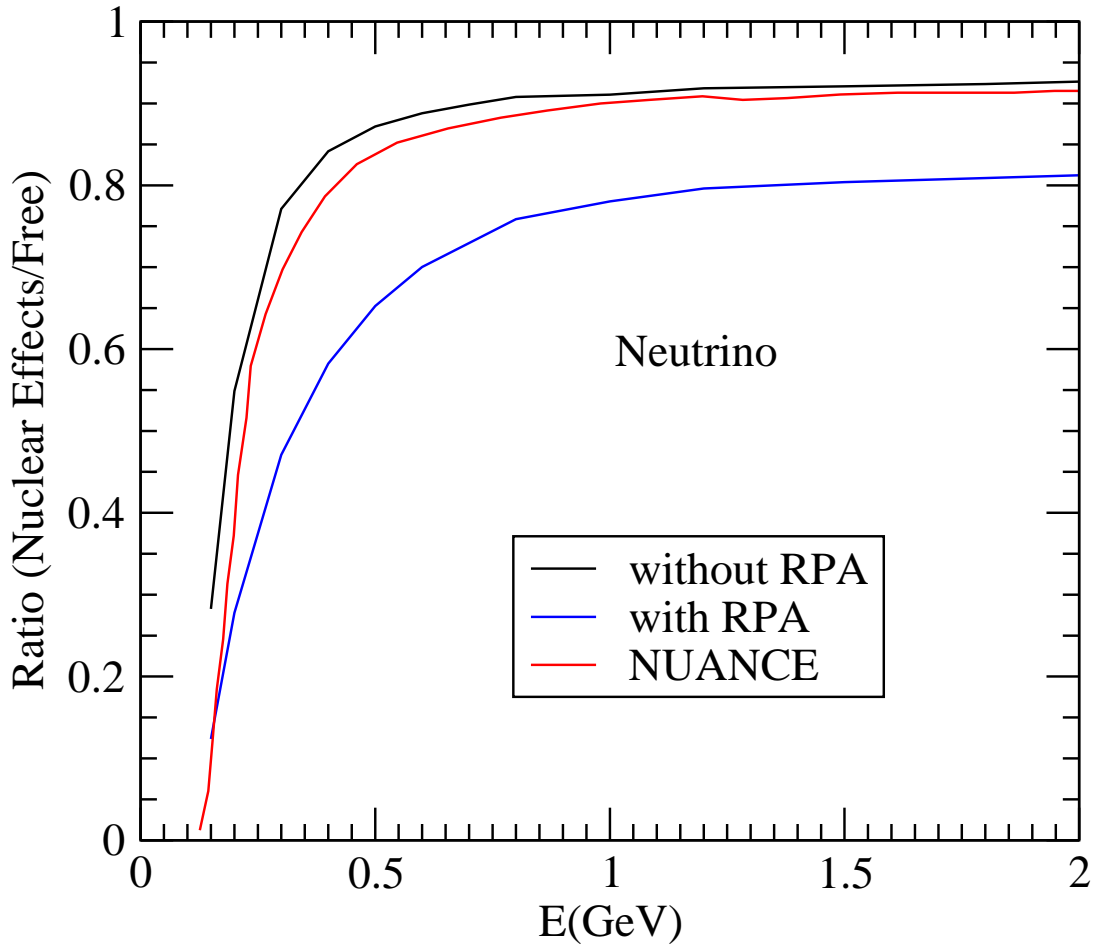
Total Cross Section in Oxygen



σ vs $E_{\bar{\nu}_e}$ in 10^{-38}cm^2 in $\bar{\nu}_e + {}^{16}\text{O}$ scattering

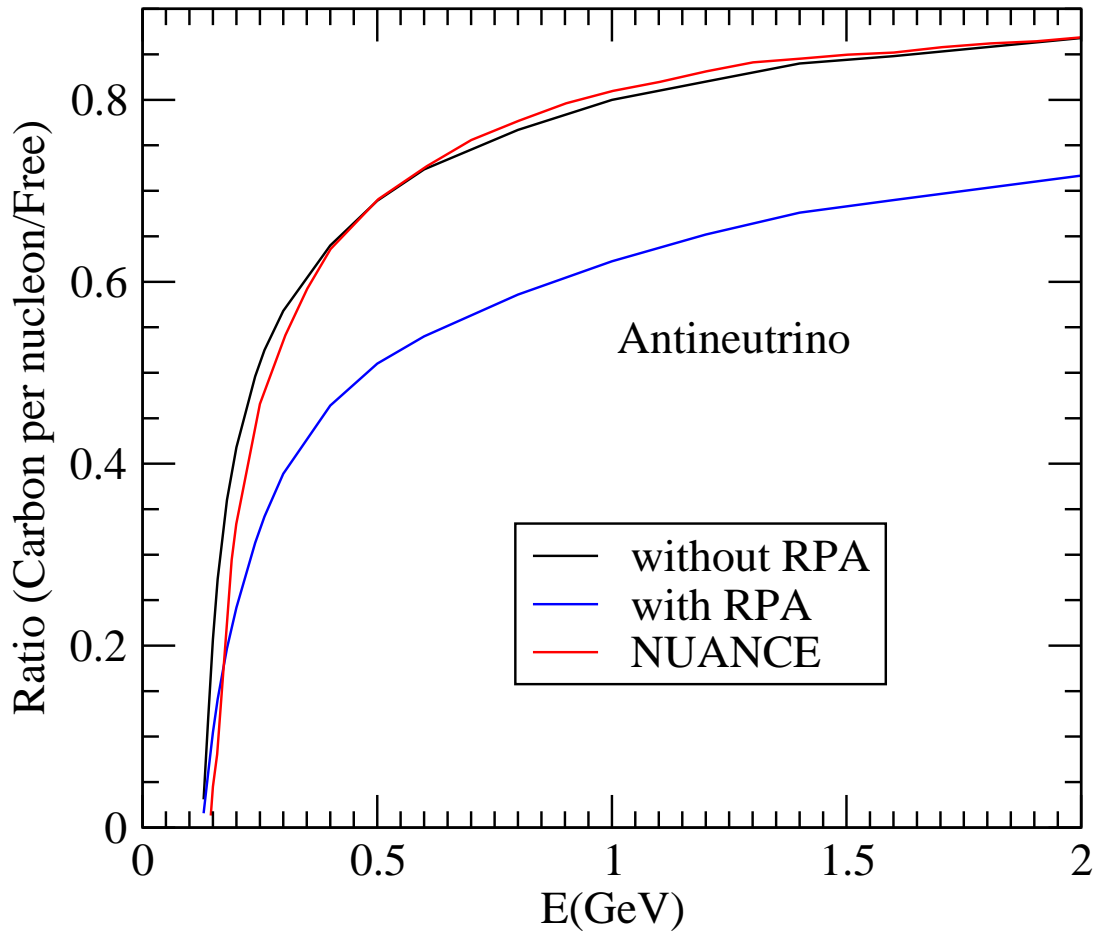
Total Cross Section in Oxygen





% reduction in the total cross section σ

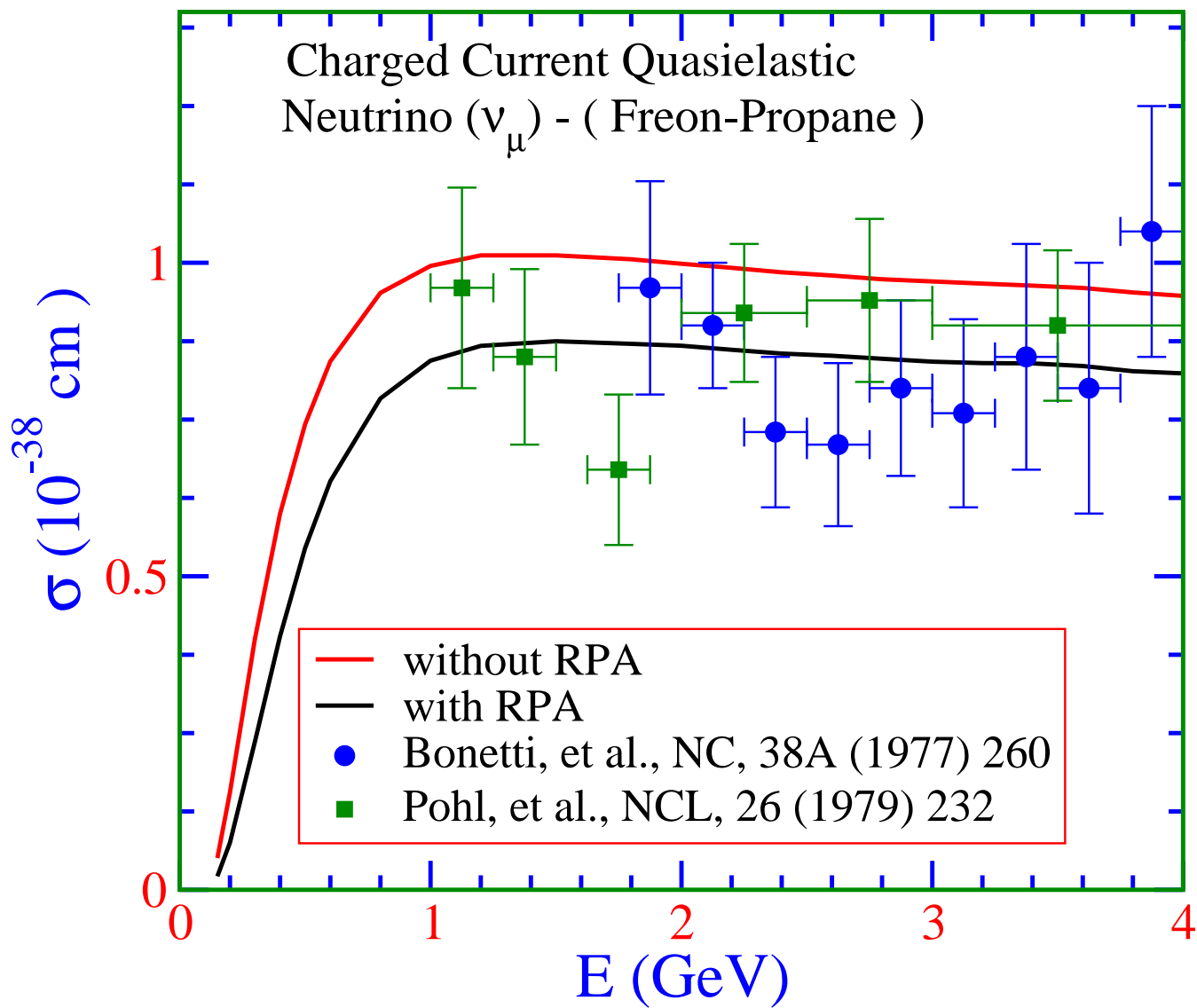
E_ν (MeV)	FGM	with RPA
200	45	72
400	16	42
1000	9	22
1500	8	20
2000	7	18

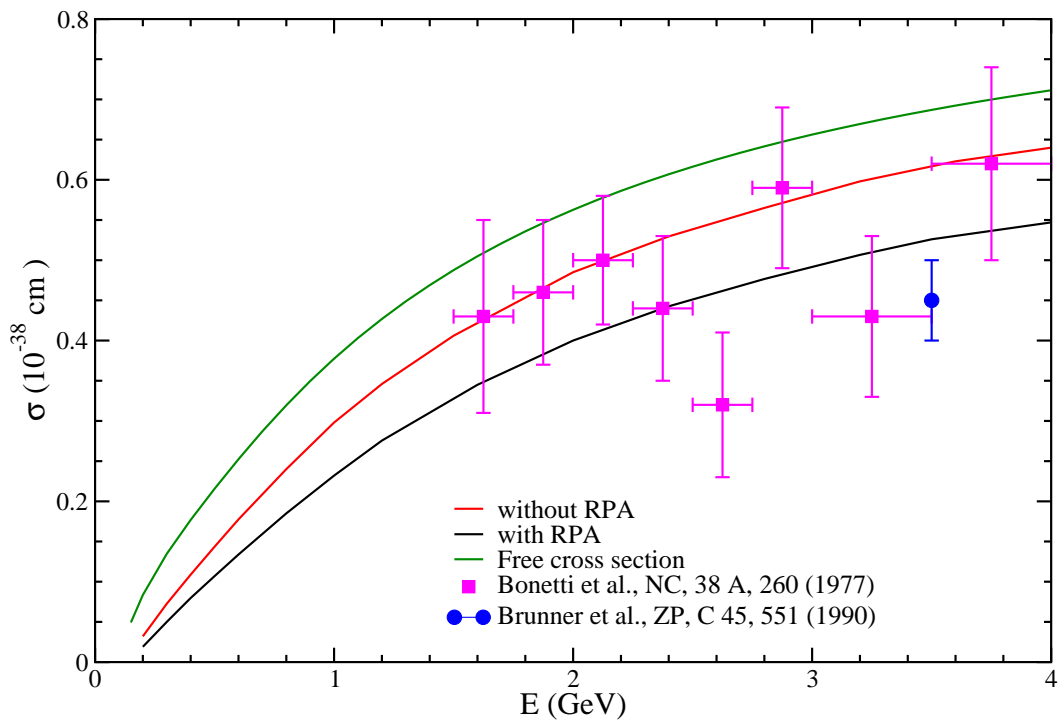


% reduction in the total cross section σ

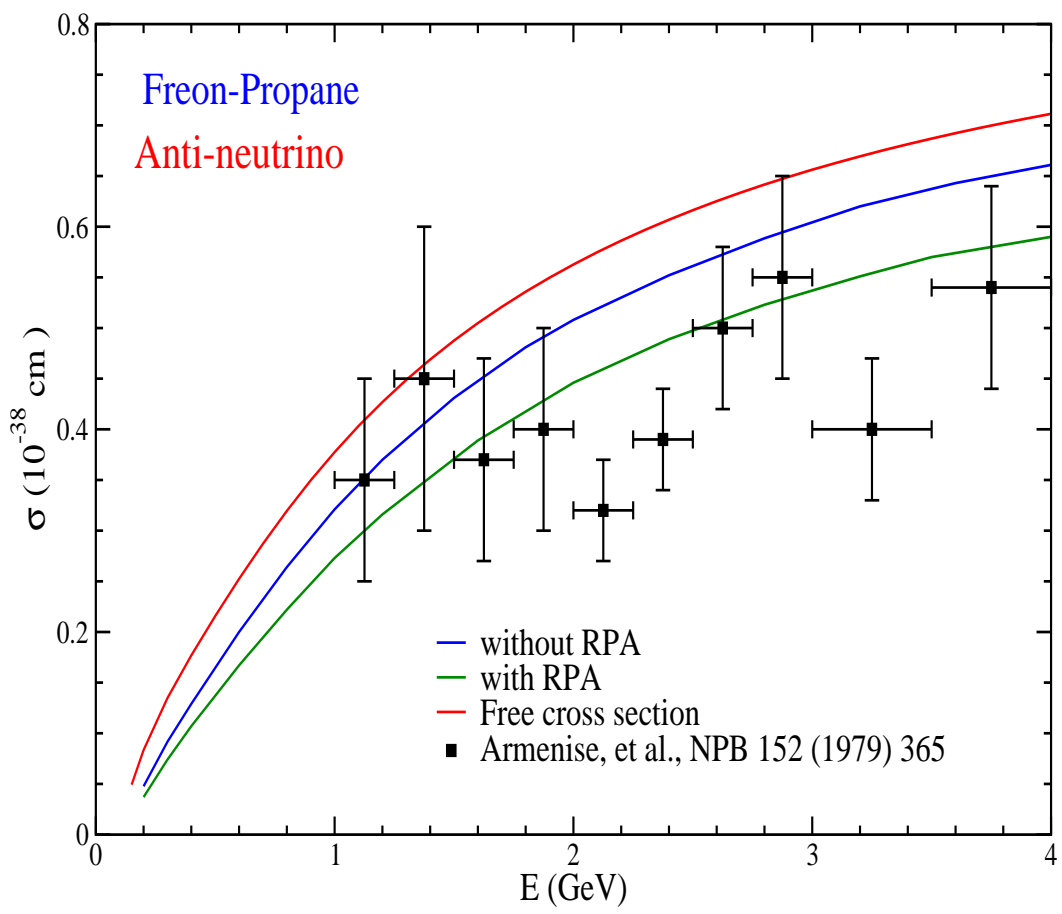
E_ν (MeV) without RPA with RPA

200	60	75
400	36	54
1000	20	38
1500	16	32
2000	12	28





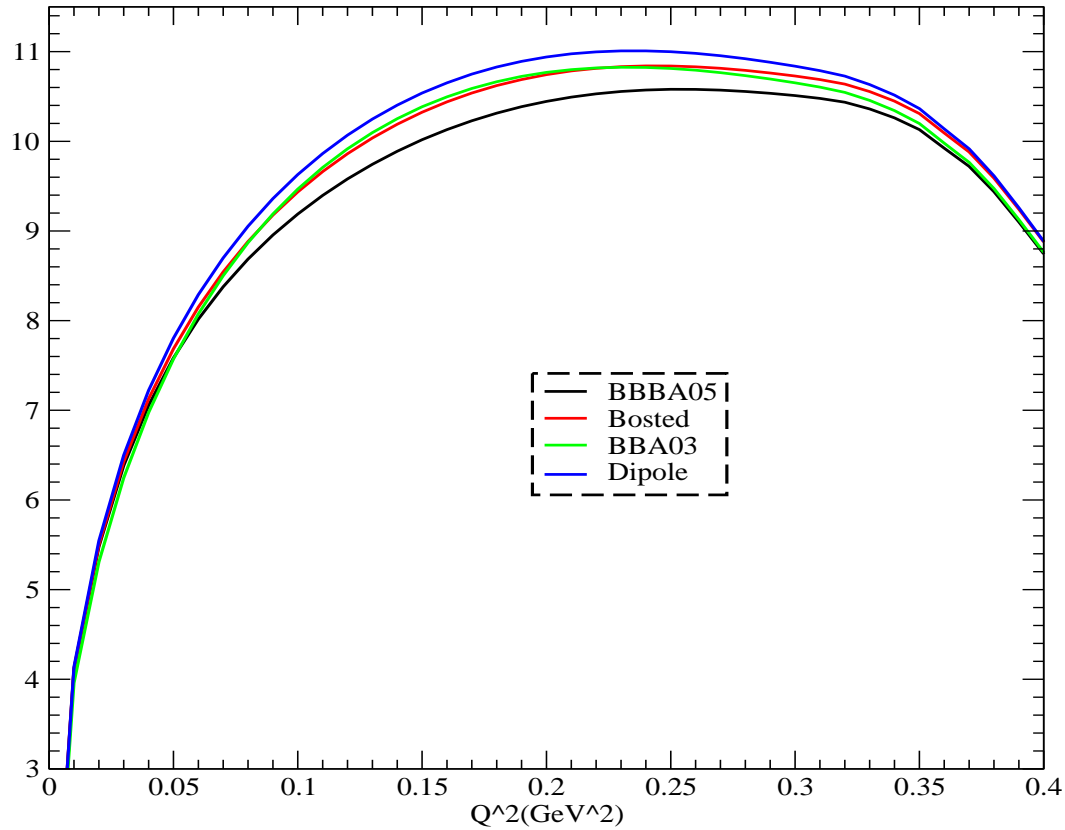
Antineutrino reaction cross section on Freon



$\frac{d\sigma}{dQ^2}$ vs Q^2 in $10^{-38} \frac{cm^2}{GeV^2}$ in $\nu_e + {}^{16}O$ scattering

Q² Distribution M_A=1.1GeV

ELECTRON NEUTRINO E=500MeV



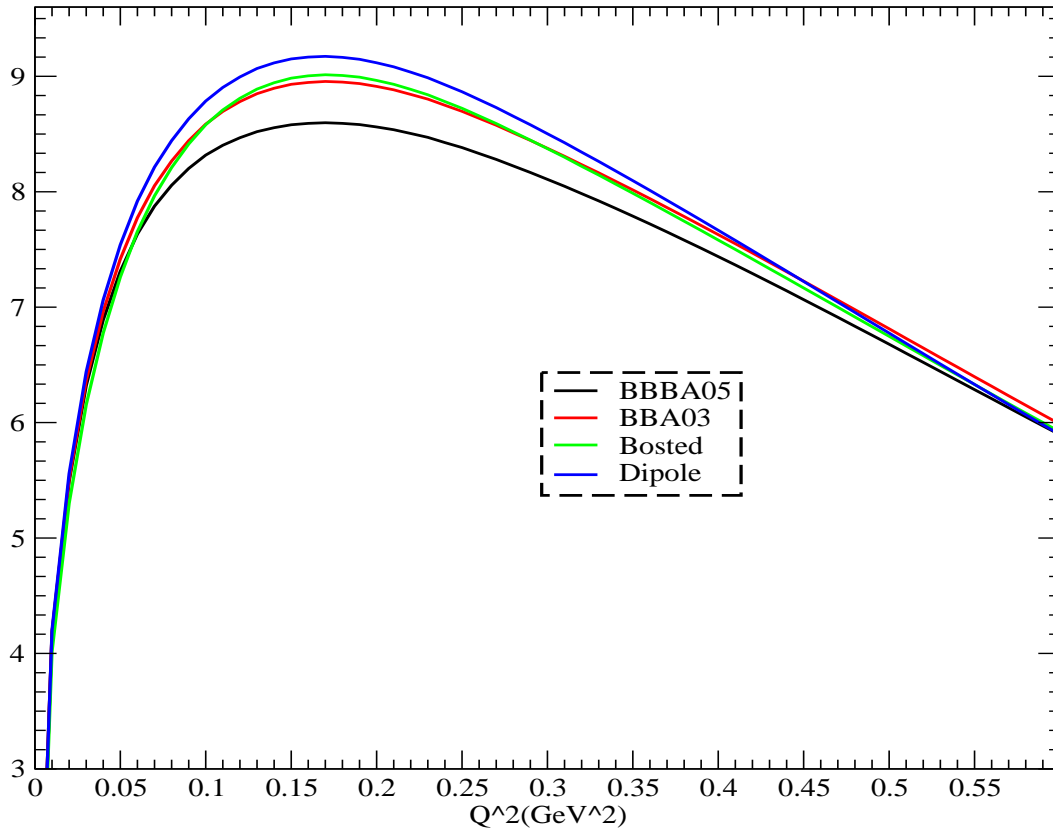
$Q^2(\text{GeV}^2)$	BBBA05	BBA03	Bosted
0.05	2.8	1.5	2.9
0.25	3.8	1.4	1.7
0.5	0.34	—	0.68

% reduction in the Q^2 distribution as compared to Dipole

$\frac{d\sigma}{dQ^2}$ vs Q^2 in $10^{-38} \frac{cm^2}{GeV^2}$ in $\nu_e + {}^{16}O$ scattering

Q² Distribution M_A=1.1GeV

ELECTRON NEUTRINO E=1GeV



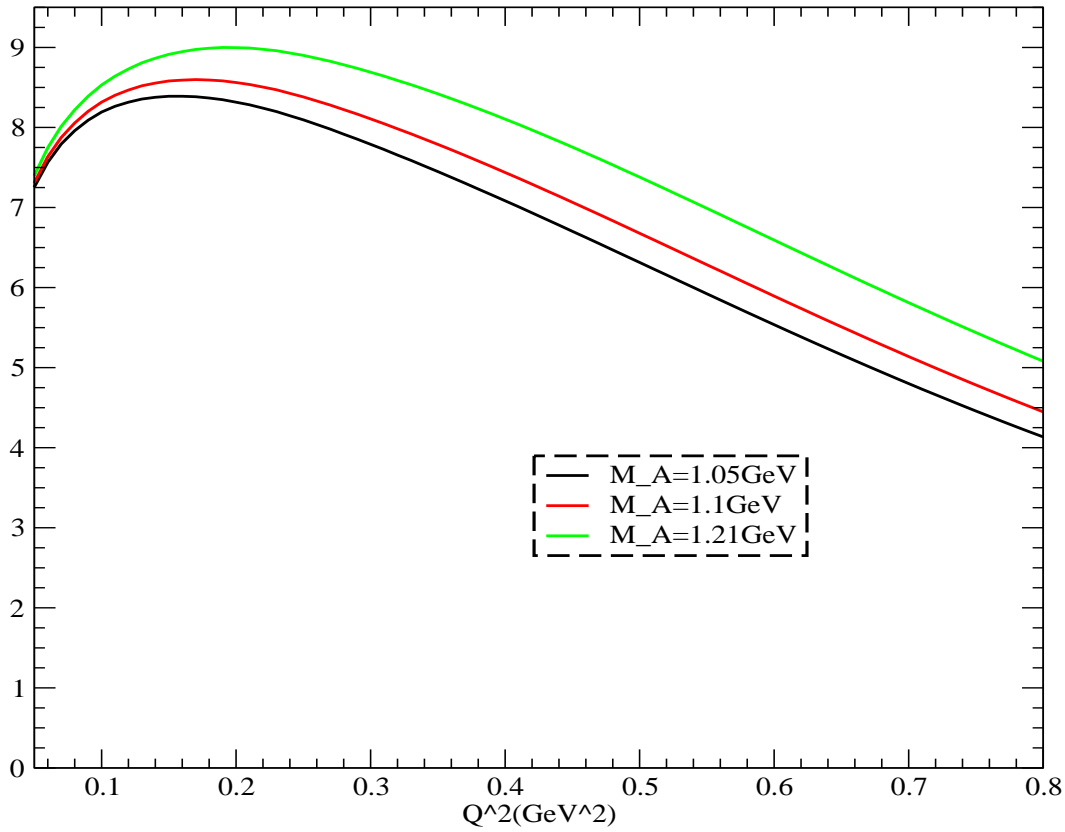
$Q^2(\text{GeV}^2)$	BBBA05	BBA03	Bosted
0.05	3	1.5	<0.5
0.17	6.2	2.2	1.7
0.4	3	<0.5	1

% reduction in the Q^2 distribution as compared to Dipole

$\frac{d\sigma}{dQ^2}$ vs Q^2 in $10^{-38} \frac{cm^2}{GeV^2}$ in $\nu_e + {}^{16}O$ scattering

Q² Distribution BBBA05

Electron Neutrino E=1GeV



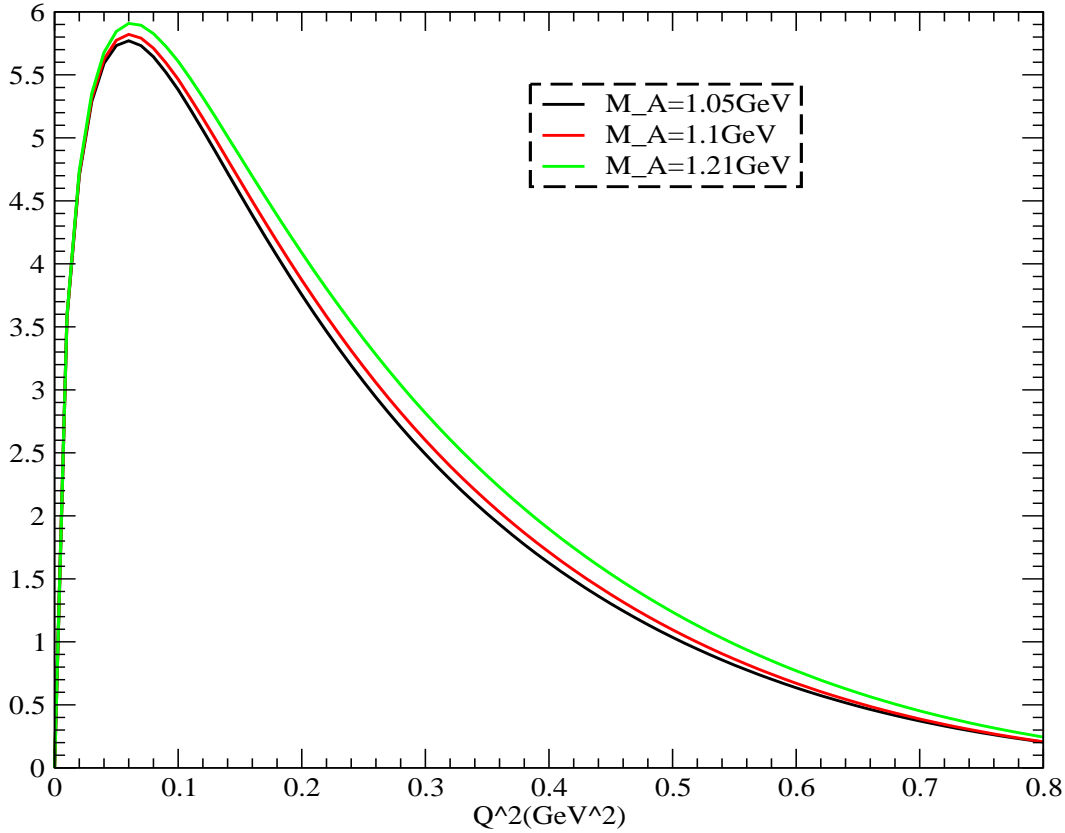
$Q^2(\text{GeV}^2)$	$M_A=1.05$	$M_A=1.21\text{GeV}$
0.1	1.5	2.5
0.2	2.9	5.0
0.3	7.1	7.0
0.5	5.4	10.5
0.7	6.6	13.0

% change in the Q^2 distribution as compared to $M_A=1.1\text{GeV}$

$\frac{d\sigma}{dQ^2}$ vs Q^2 in $10^{-38} \frac{cm^2}{GeV^2}$ in $\bar{\nu}_e + {}^{16}O$ scattering

Q² Distribution BBBA05

Electron Antineutrino E=1GeV

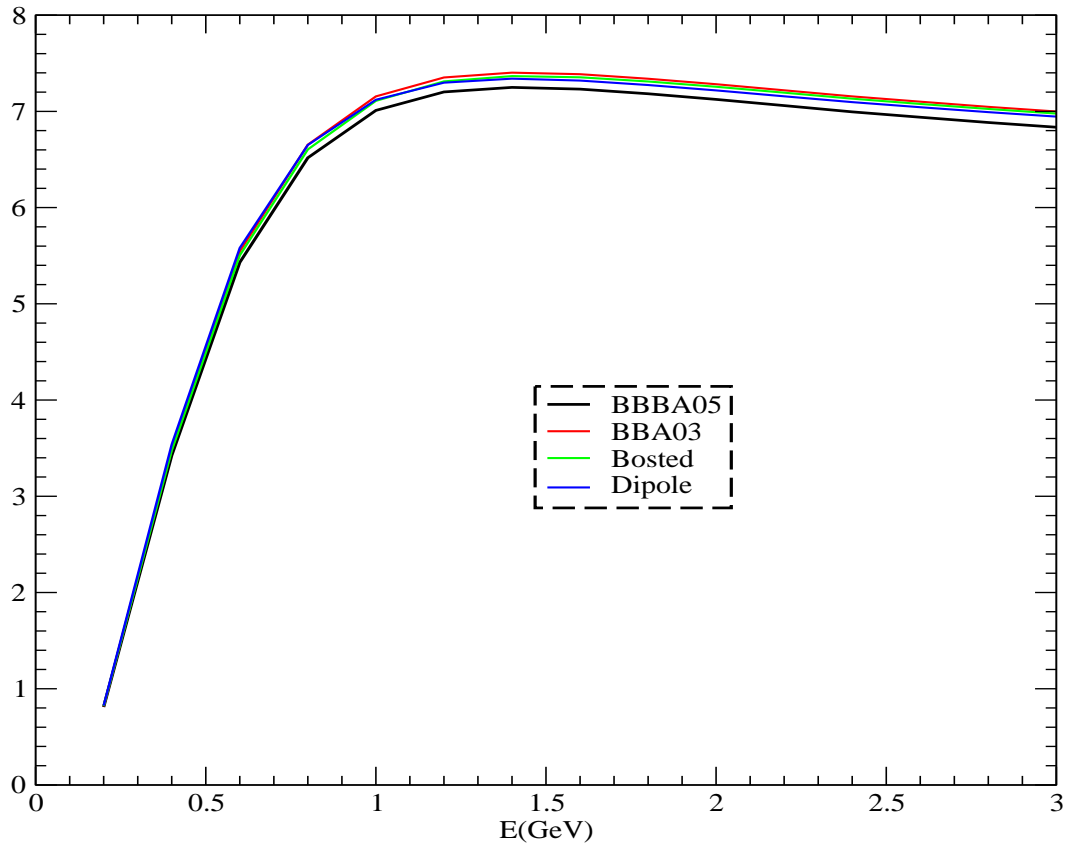


$Q^2(\text{GeV}^2)$	$M_A=1.05$	$M_A=1.21\text{GeV}$
0.05	0.7	1.2
0.1	1.5	2.6
0.2	3.0	5.5
0.3	4.3	8.3

% change in the Q^2 distribution as compared to $M_A=1.1\text{GeV}$

Total Cross Section

Electron Neutrino

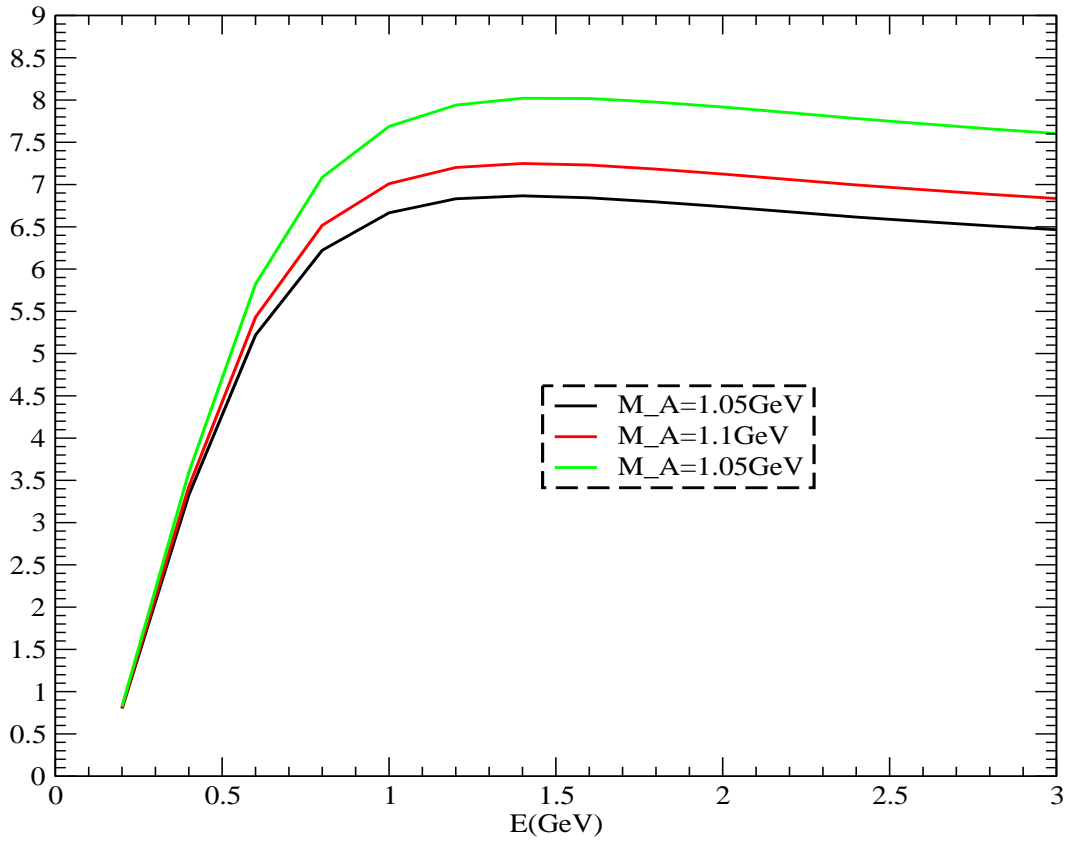


E(GeV)	BBBA05	BBA03	Bosted
0.6	2.7	<0.5	<0.5
1.0	1.5	—	—
2.0	1.3	—	—

% reduction in σ as compared to Dipole

Total Cross section BBBA05

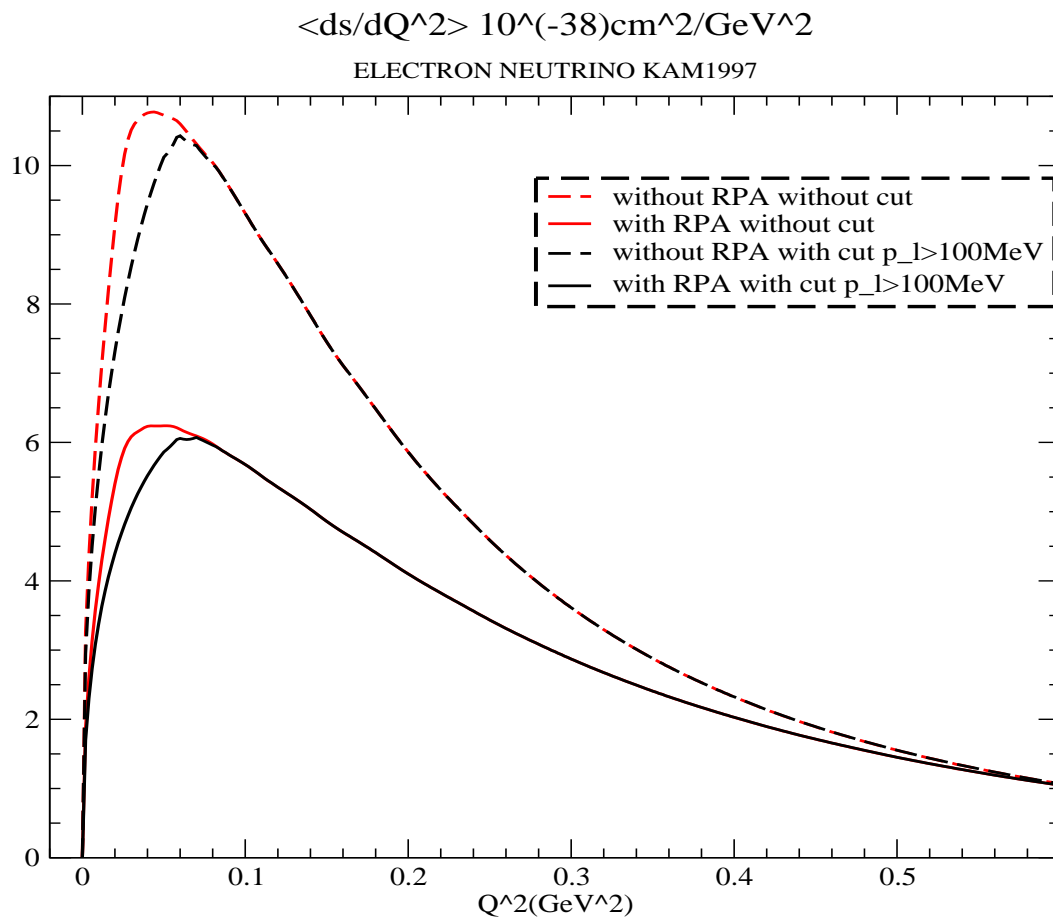
Electron Neutrino



$E(\text{GeV})$	$M_A=1.05$	$M_A=1.21\text{GeV}$
0.6	4.0	7.3
1.0	5.0	9.6
2.0	5.3	11.0
3.0	5.4	11.2

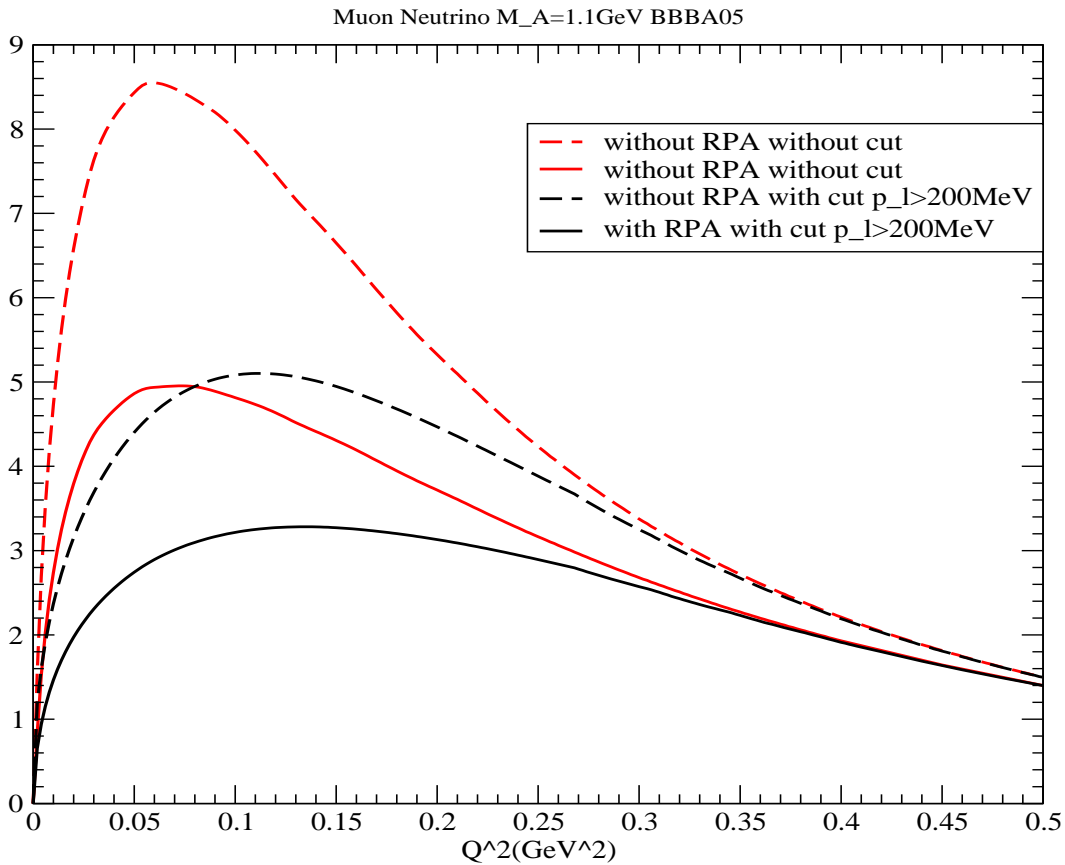
% change in σ as compared to $M_A=1.1\text{GeV}$

$\langle \frac{d\sigma}{dQ^2} \rangle$ vs Q^2 in $10^{-38} \frac{\text{cm}^2}{\text{GeV}^2}$ in $\nu_e + {}^{16}\text{O}$ scattering averaged over Kamioka 1997 flux given by Honda et al.



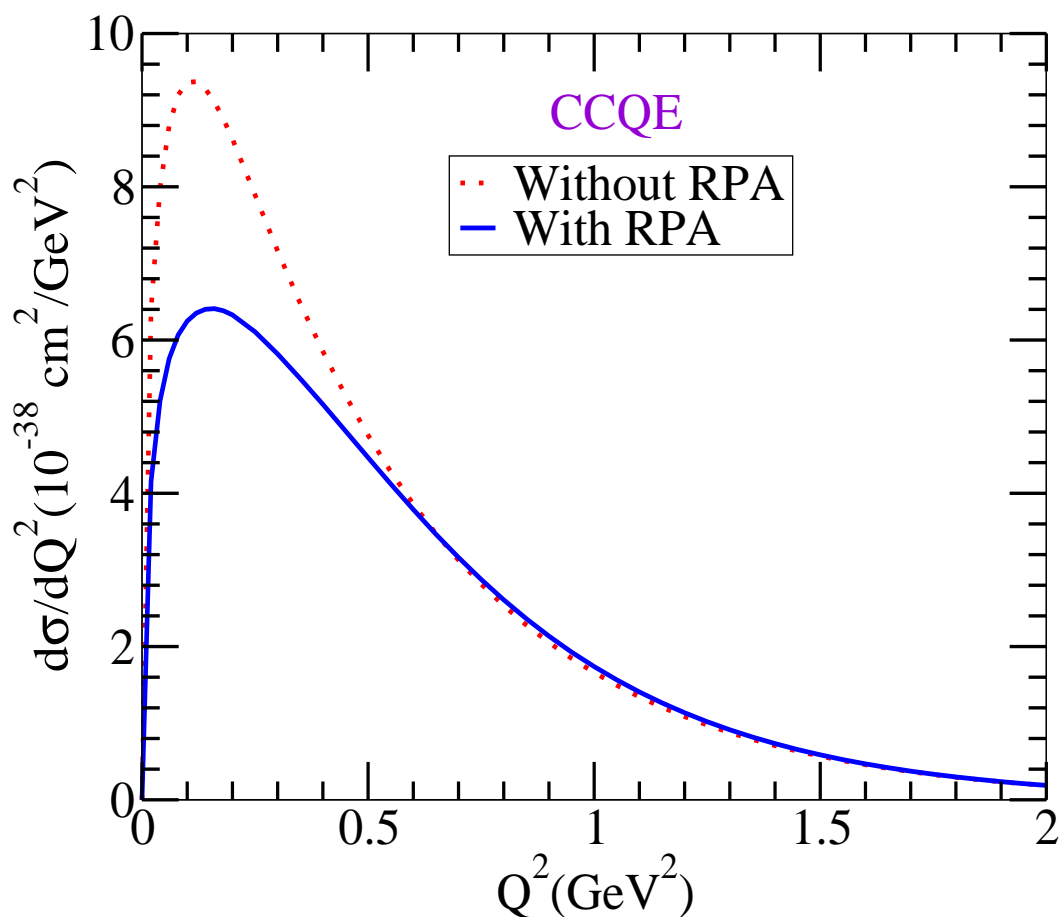
$\langle \frac{d\sigma}{dQ^2} \rangle$ vs Q^2 in $10^{-38} \frac{cm^2}{GeV^2}$ in $\nu_\mu + {}^{16}O$ scattering averaged over Kamioka 1997 flux given by Honda et al.

Q² distribution averaged over Kam1997 Flux



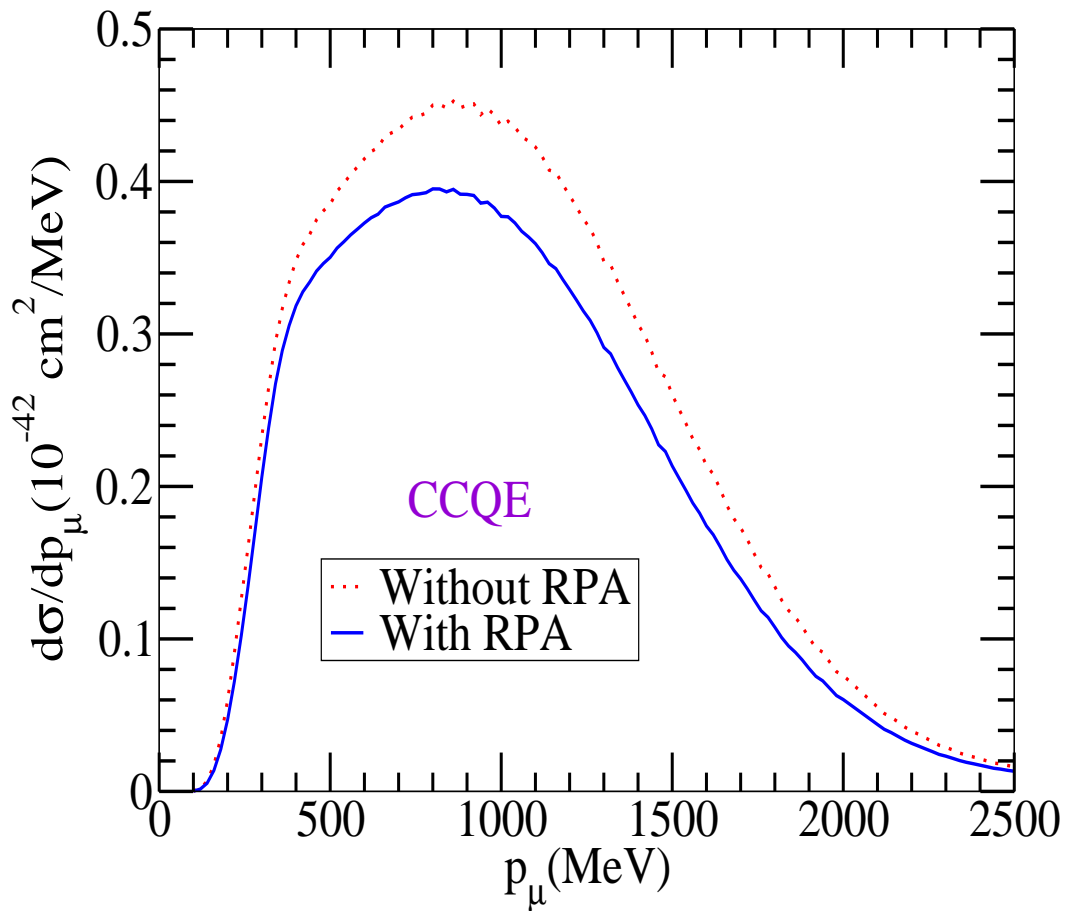
$Q^2(\text{GeV}^2)$	with RPA	with RPA with cut	Diff. with cut
0.05	42	38	42
0.2	30	30	15

$\langle \frac{d\sigma}{dQ^2} \rangle$ vs Q^2 in $10^{-38} \frac{cm^2}{GeV^2}$ in $\nu_\mu + {}^{12}C$ scattering
 averaged over K2K flux



$Q^2(\text{GeV}^2)$	with RPA
0.02	35
0.12	32
0.2	25

$\langle \frac{d\sigma}{dp_l} \rangle$ vs p_l in $10^{-40} \frac{cm^2}{MeV}$ in $\nu_\mu + {}^{12}C$ scattering
 averaged over K2K flux



CCQE

⋯ Without RPA
— With RPA

p_μ (MeV)	with RPA
860	12
980	14

Muon Events				
	Process	Free Case	FGM	FGM With RPA
I	$\nu_{\mu}n \rightarrow \mu^{-}p(\text{in } ^{16}\text{O})$	3332	2472	1894
II	$\bar{\nu}_{\mu}p \rightarrow \mu^{+}n(\text{in } ^{16}\text{O})$	966	620	461
III	$\bar{\nu}_{\mu}p \rightarrow \mu^{+}n(\text{on free p})$	241	241 [†]	241 [†]
	$\nu_{\mu} + \bar{\nu}_{\mu}$	4539	3232	2596
Electron Events				
	Process	Free Case	FGM	FGM With RPA
IV	$\nu_{e}n \rightarrow e^{-}p(\text{in } ^{16}\text{O})$	2332	1754	1278
V	$\bar{\nu}_{e}p \rightarrow e^{+}n(\text{in } ^{16}\text{O})$	609	358	266
VI	$\bar{\nu}_{e}p \rightarrow e^{+}n(\text{on free p due to } H_2)$	152	152 [†]	152 [†]
	$\nu_{e} + \bar{\nu}_{e}$	3093	2264	1696

Total number of lepton events for a quasielastic process.

†: For reaction on free protons the events would be the same in all the three columns.

Inelastic Scattering Cross Section

In the intermediate energy region of about 1GeV the pion production from nucleons is dominated by Δ excitation

$$\nu_l(k) + p(p) \rightarrow l^-(k') + \Delta^{++}(p') \searrow p + \pi^+$$

$$\nu_l(k) + n(p) \rightarrow l^-(k') + \Delta^+(p') \begin{matrix} \searrow n + \pi^+ \\ \searrow p + \pi^0 \end{matrix}$$

In this model of Δ dominance the neutrino induced charged current one pion production is calculated using the Lagrangian

$$L = \frac{G_F}{\sqrt{2}} l_\mu(x) J^{\mu\dagger}(x) + h.c., \text{ where}$$

$$l_\mu(x) = \bar{\psi}(k') \gamma_\mu (1 - \gamma_5) \psi(k)$$

$$J^\mu(x) = \cos \theta_c (V^\mu(x) + A^\mu(x))$$

θ_c being the Cabibbo angle.

The matrix element of the vector current V^μ and axial vector current A^μ of the hadronic current J^μ for the Δ excitation from proton target is written as:

Matrix elements:

$$\begin{aligned} \langle \Delta^{++} | V^\mu | p \rangle = & \sqrt{3} \bar{\psi}_\alpha(p') \left[\frac{C_3^V(q^2)}{M} (g^{\alpha\mu} \not{q} - q^\alpha \gamma^\mu) \right. \\ & + \frac{C_4^V(q^2)}{M^2} (g^{\alpha\mu} q \cdot p' - q^\alpha p'^\mu) + \frac{C_5^V(q^2)}{M^2} (g^{\alpha\mu} q \cdot p - q^\alpha p^\mu) \\ & \left. + \frac{C_6^V(q^2)}{M^2} q^\alpha q^\mu \right] \gamma_5 u(p) \end{aligned}$$

$$\begin{aligned} \langle \Delta^{++} | A^\mu | p \rangle = & \sqrt{3} \bar{\psi}_\alpha(p') \left[\frac{C_3^A(q^2)}{M} (g^{\alpha\mu} \not{q} - q^\alpha \gamma^\mu) \right. \\ & + \frac{C_4^A(q^2)}{M^2} (g^{\alpha\mu} q \cdot p' - q^\alpha p'^\mu) \\ & \left. + C_5^A(q^2) g^{\alpha\mu} + \frac{C_6^A(q^2)}{M^2} q^\alpha q^\mu \right] u(p) \end{aligned}$$

C_i^V (i=3-6) are vector and C_i^A (i=3-6) are axial vector transition form factors.

For our numerical calculations we have taken the transition form factors of

[1] P. A. Schreiner and von Hippel, NPB 58, 333 (1973)

[2] E. A. Paschos, et al., PRD 69, 014013 (2004)

[3] O. Lalakulich, et al., PRD 74, 014009 (2006)

In the nucleus, the neutrino interacts with the nucleon moving inside the nucleus of density $\rho(r)$ with its corresponding momentum \vec{p} constrained to be below its Fermi momentum.

The total scattering cross section is given by

$$\frac{1}{(4\pi)^5} \int_{r_{min}}^{r_{max}} (Z\rho_p(r) + N\rho_n(r)) d\vec{r} \int_{Q_{min}^2}^{Q_{max}^2} dQ^2 \int_{-1}^{+1} d(\cos\theta_{\pi q}) \times$$

$$\frac{\pi |\vec{k}'| |\vec{k}_\pi|}{M E_\nu^2 E_l} \frac{1}{E'_p + E_\pi \left(1 - \frac{|\vec{q}|}{|\vec{k}_\pi|} \cos(\theta_\pi)\right)} |\mathcal{M}_{fi}|^2$$

The transition matrix element \mathcal{M}_{fi} is given by

$$\mathcal{M}_{fi} = \frac{G_{Fa}}{\sqrt{2}} \frac{f_{\pi N\Delta}}{m_\pi} \bar{\Psi}(\mathbf{P}) k_\pi^\sigma \mathcal{P}_{\sigma\lambda} \mathcal{O}^{\lambda\alpha} L_\alpha u(\mathbf{p})$$

$$L_{\mu\nu} = 8(k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu} k \cdot k' + i\epsilon_{\mu\nu\alpha\beta} k^\alpha k'^\beta),$$

and

$$J^{\mu\nu} = \bar{\Sigma} \Sigma J^{\mu\dagger} J^\nu$$

which is calculated with the use of spin $\frac{3}{2}$ projection operator $P^{\mu\nu}$ defined as

$$P^{\mu\nu} = \sum_{spins} \psi^\mu \bar{\psi}^\nu$$

and is given by:

$$P^{\mu\nu} = -\frac{\not{p}' + M_\Delta}{2M_\Delta} \left(g^{\mu\nu} - \frac{2p'^\mu p'^\nu}{3M'^2} + \frac{1}{3} \frac{p'^\mu \gamma^\nu - p'^\nu \gamma^\mu}{M'} - \frac{1}{3} \gamma^\mu \gamma^\nu \right)$$

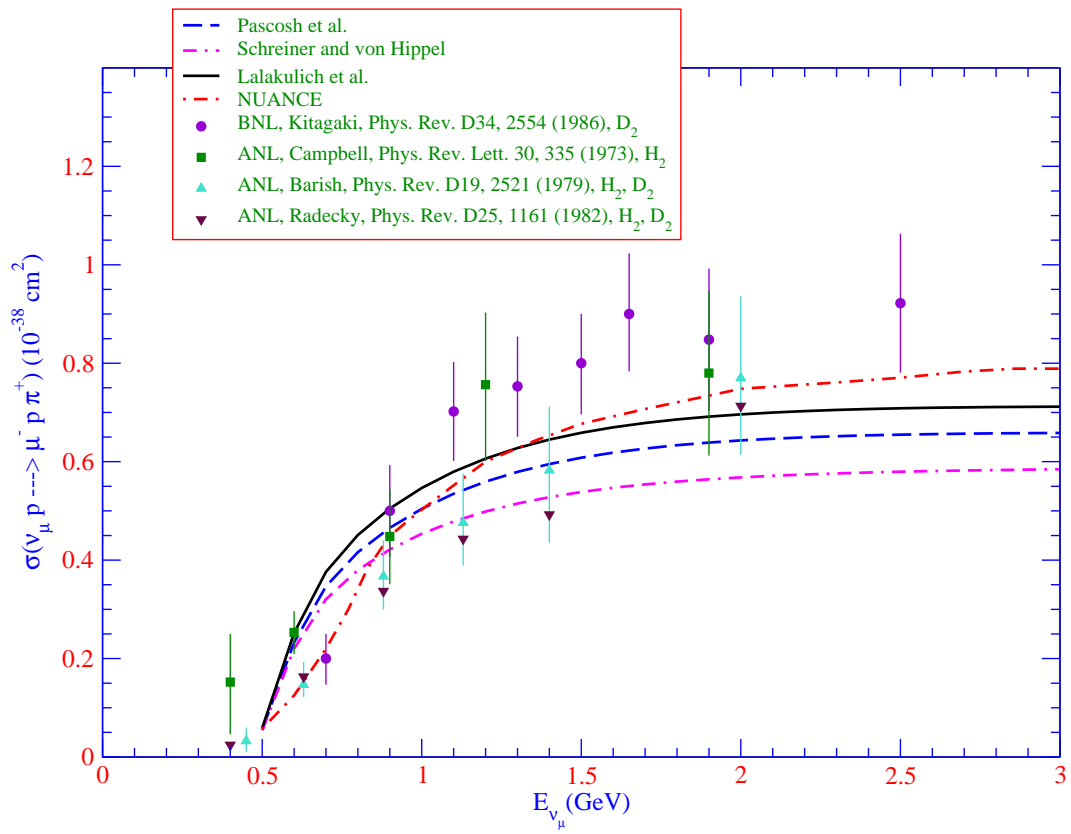
In nuclear medium the properties of Δ like its mass and decay width Γ are modified due to nuclear effects.

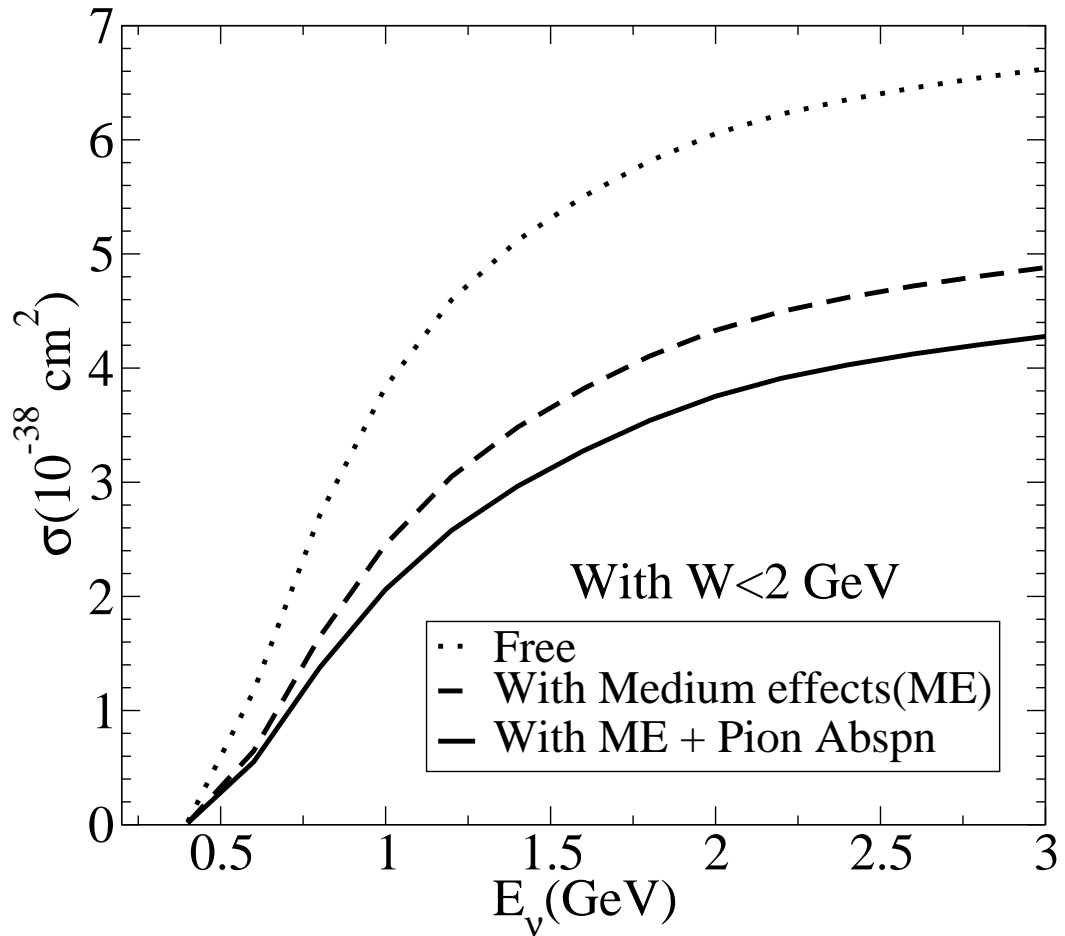
These are mainly due to following processes.

(i) In the nuclear medium Δ s decay mainly through $\Delta \rightarrow N\pi$ channel. The final nucleons have to be above the Fermi momentum k_F of the nucleon in the nucleus thus inhibiting the decay. This leads to a modification in the delta decay width

$$\tilde{\Gamma} = \Gamma \times F(k_F, E_\Delta, k_\Delta)$$

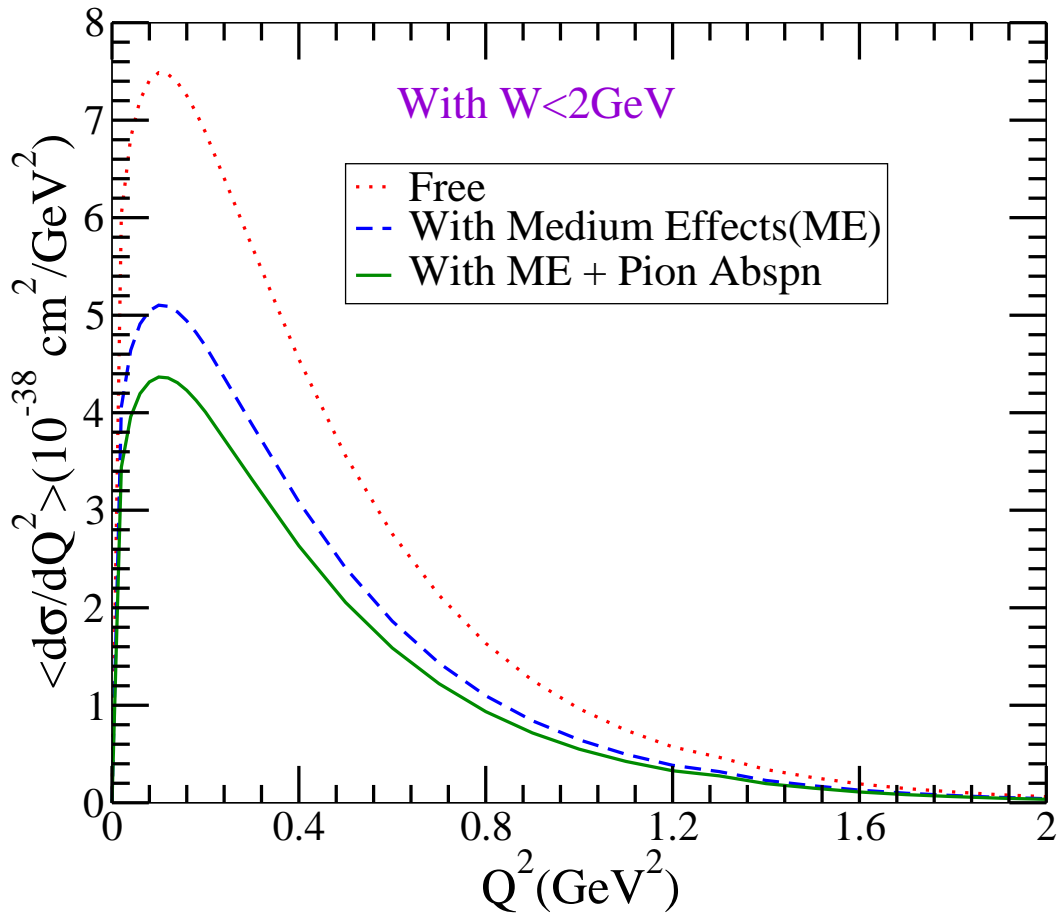
(ii) In the nuclear medium there are additional decay channels open due to two body and three body absorption processes like $\Delta N \rightarrow NN$ and $\Delta NN \rightarrow NNN$ through which Δ 's disappear in the nuclear medium without producing a pion while a two body Δ absorption process like $\Delta N \rightarrow \pi NN$ gives rise to some more pions. Due to these changes $\tilde{\Gamma}$ and M_Δ modify.





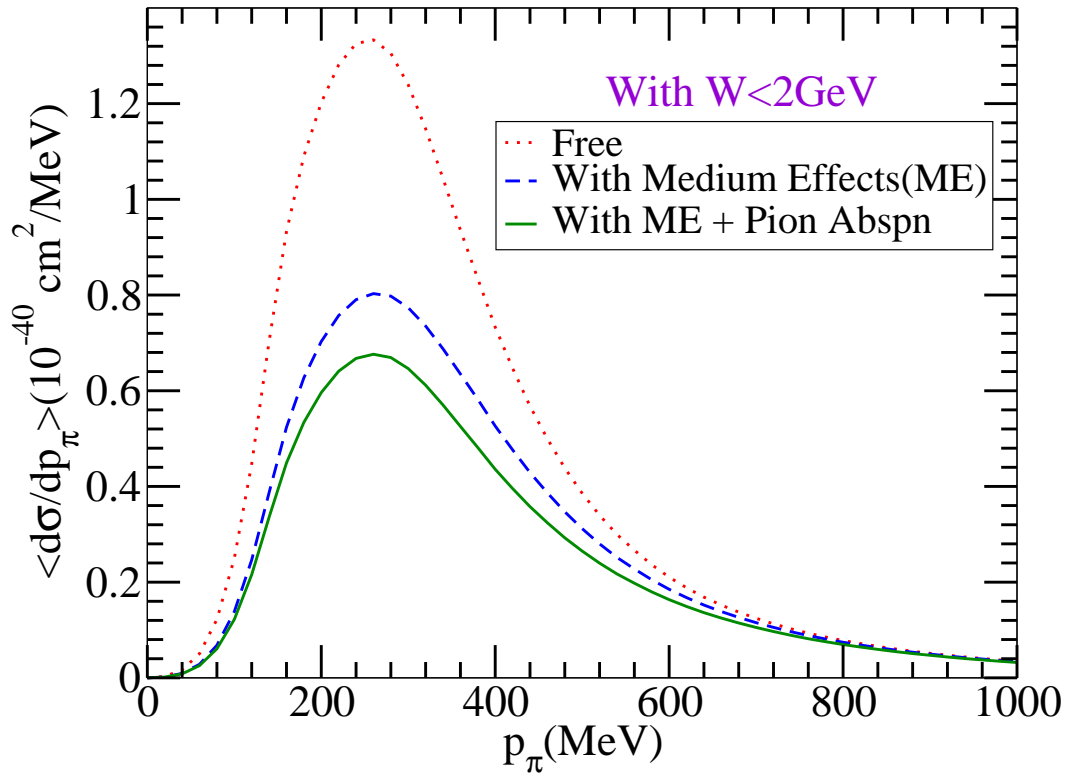
Total cross section for the charged current one π^+ process on ^{12}C target.

E_ν (MeV)	without PA	with PA
800	40	16
1400	30	14
2000	28	13
3000	25	12



$\langle \frac{d\sigma}{dQ^2} \rangle$ averaged over K2K spectrum for the charged current one π^+ process on ^{12}C target.

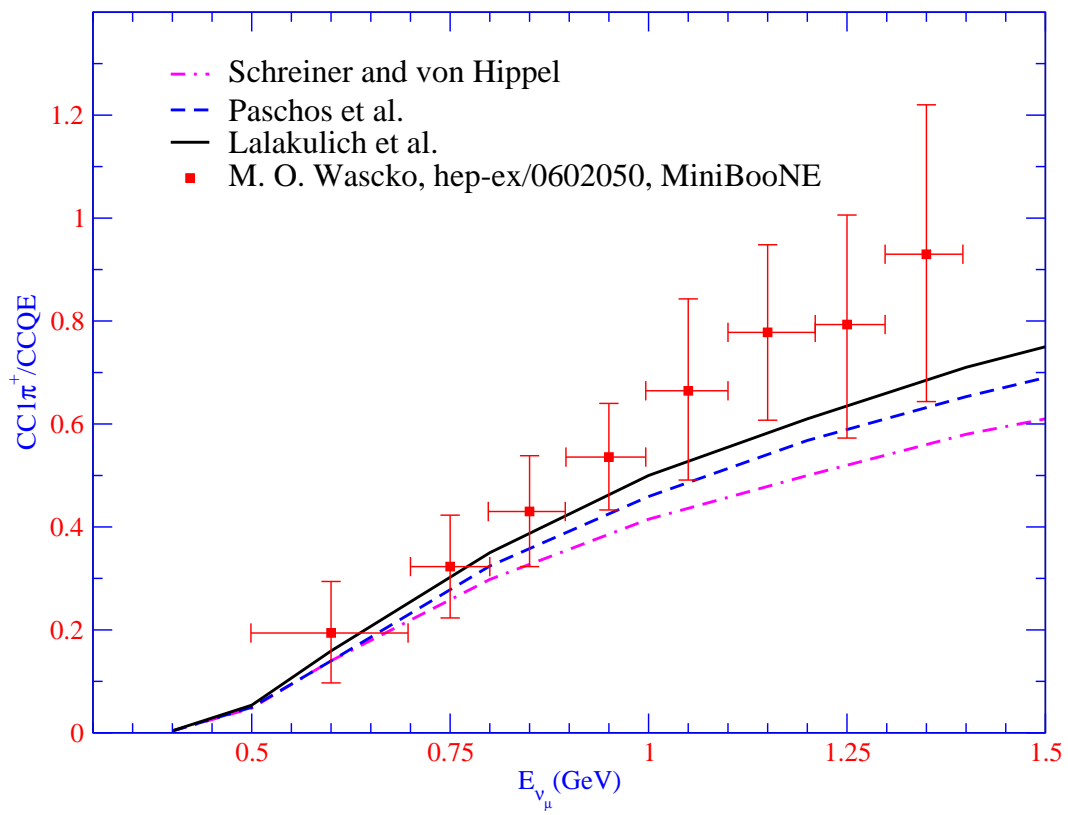
$Q^2 (\text{GeV}^2)$	without PA	with PA
0.02	32	15
0.1	30	14



$\langle \frac{d\sigma}{dp_\pi} \rangle$ averaged over K2K spectrum for the charged current one π^+ process on ^{12}C target.

dp_π (MeV)	without PA	with PA
200	40	15
300	36	16

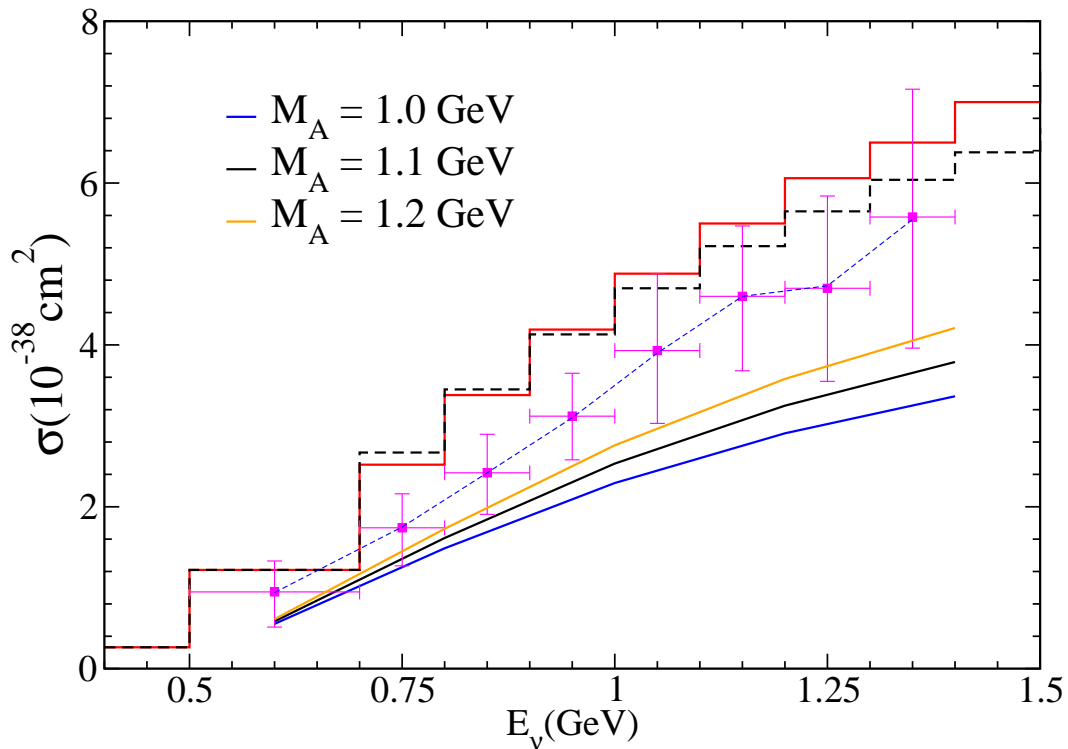
Ratio of C.C. $1\pi^+$ /CCQE



The experimental points are

$$\sigma(\text{CC}1\pi^+) =$$

$$\left[\frac{\sigma(\text{CC}1\pi^+)}{\sigma(\text{CCQE})} \right]_{\text{MiniBooNE}} \times \sigma(\text{CCQE}) \text{ MC}$$



The dashed(solid) stairs are the cross sections from NEUGEN(NUANCE) MC generators. Experimental points are the MiniBooNE results.

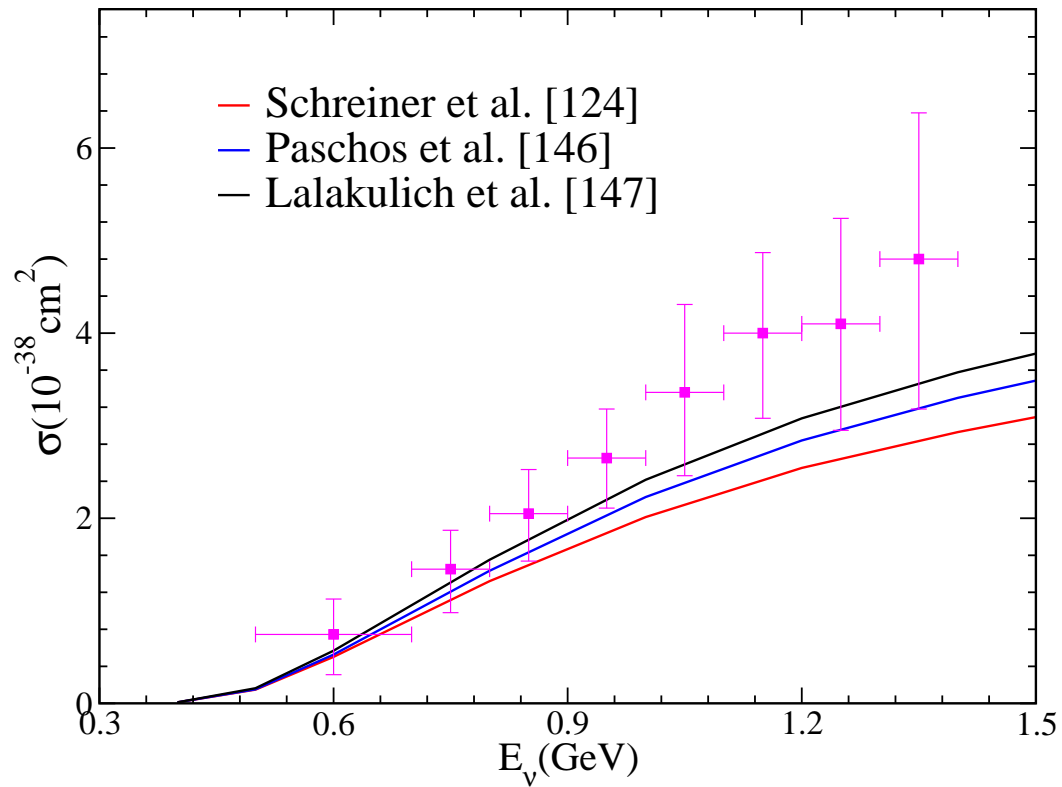
The theoretical curves show the $\text{CC}1\pi^+$ cross section using Lalakulich et al. $N-\Delta$ transition form factors.

The dashed line is

$$\sigma(\text{CC}1\pi^+) =$$

$$\left[\frac{\sigma(\text{CC}1\pi^+)}{\sigma(\text{CCQE})} \right]_{\text{MiniBooNE}} \times \sigma(\text{CCQE}) \text{ without RPA}$$

with $M_A = 1.05 \text{ GeV}$



CC1 π^+ cross section for ν_μ induced reaction in ^{12}C .

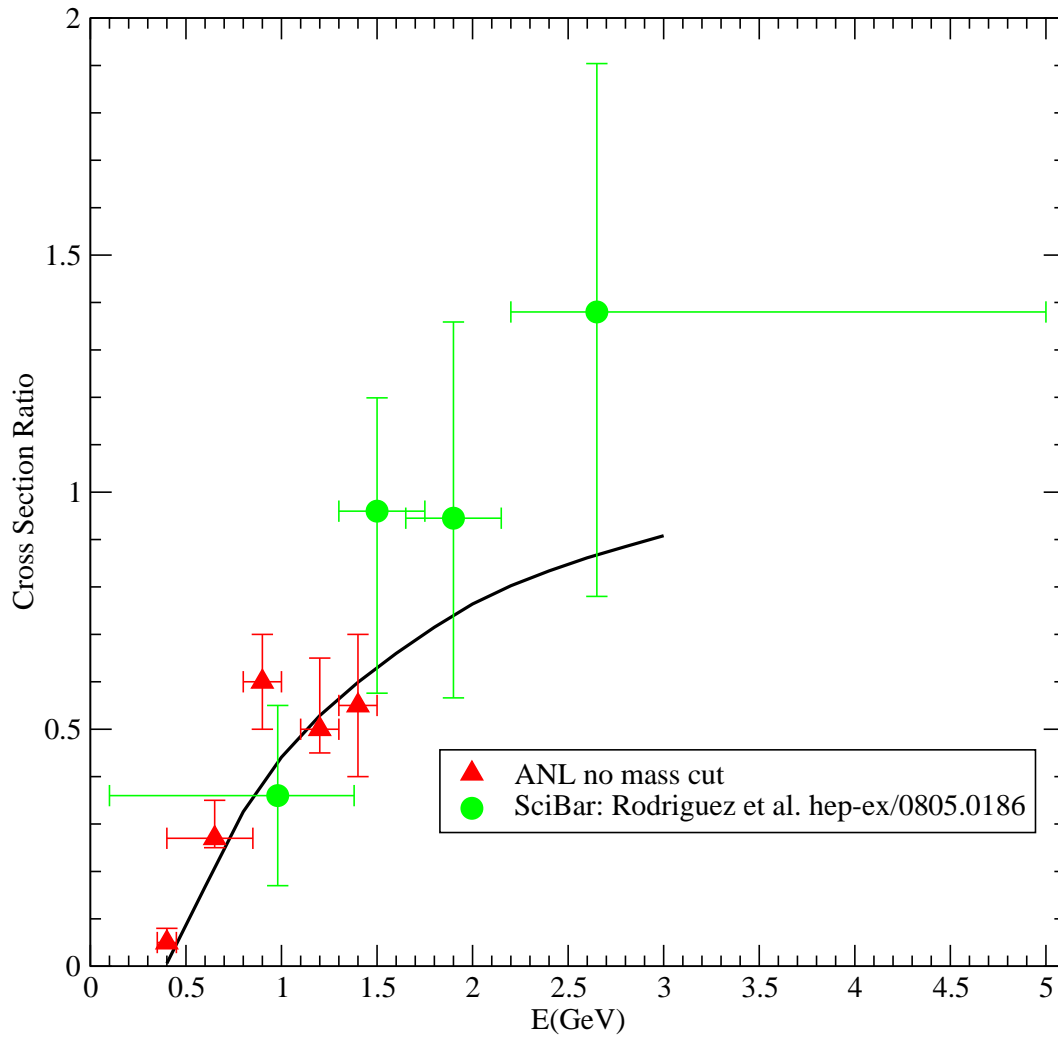
The experimental points show

$$\sigma(\text{CC1}\pi^+) =$$

$$\left[\frac{\sigma(\text{CC1}\pi^+)}{\sigma(\text{CCQE})} \right]_{\text{MiniBooNE}} \times \sigma(\text{CCQE}) \text{ with RPA}$$

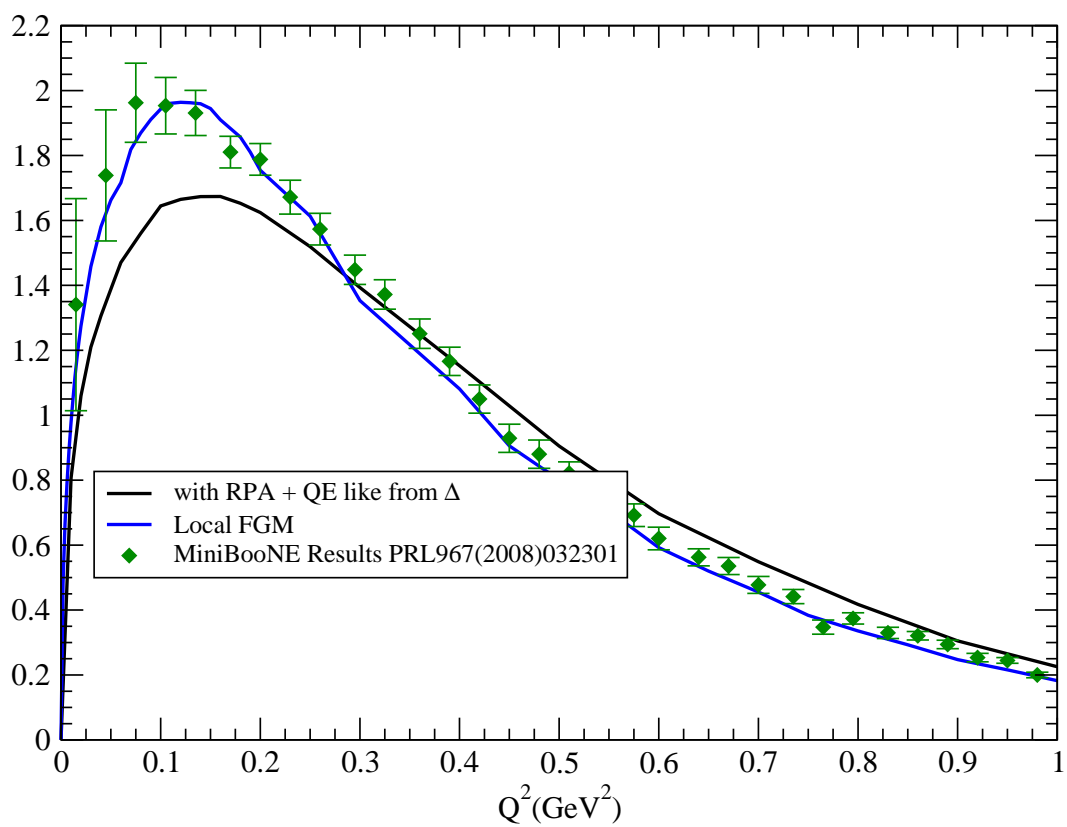
with $M_A = 1.05 \text{ GeV}$

$\frac{\sigma_{CC1\pi^+}}{\sigma_{CCQE}}$ for ν_μ induced reaction in Polystyrene (C_8H_8).

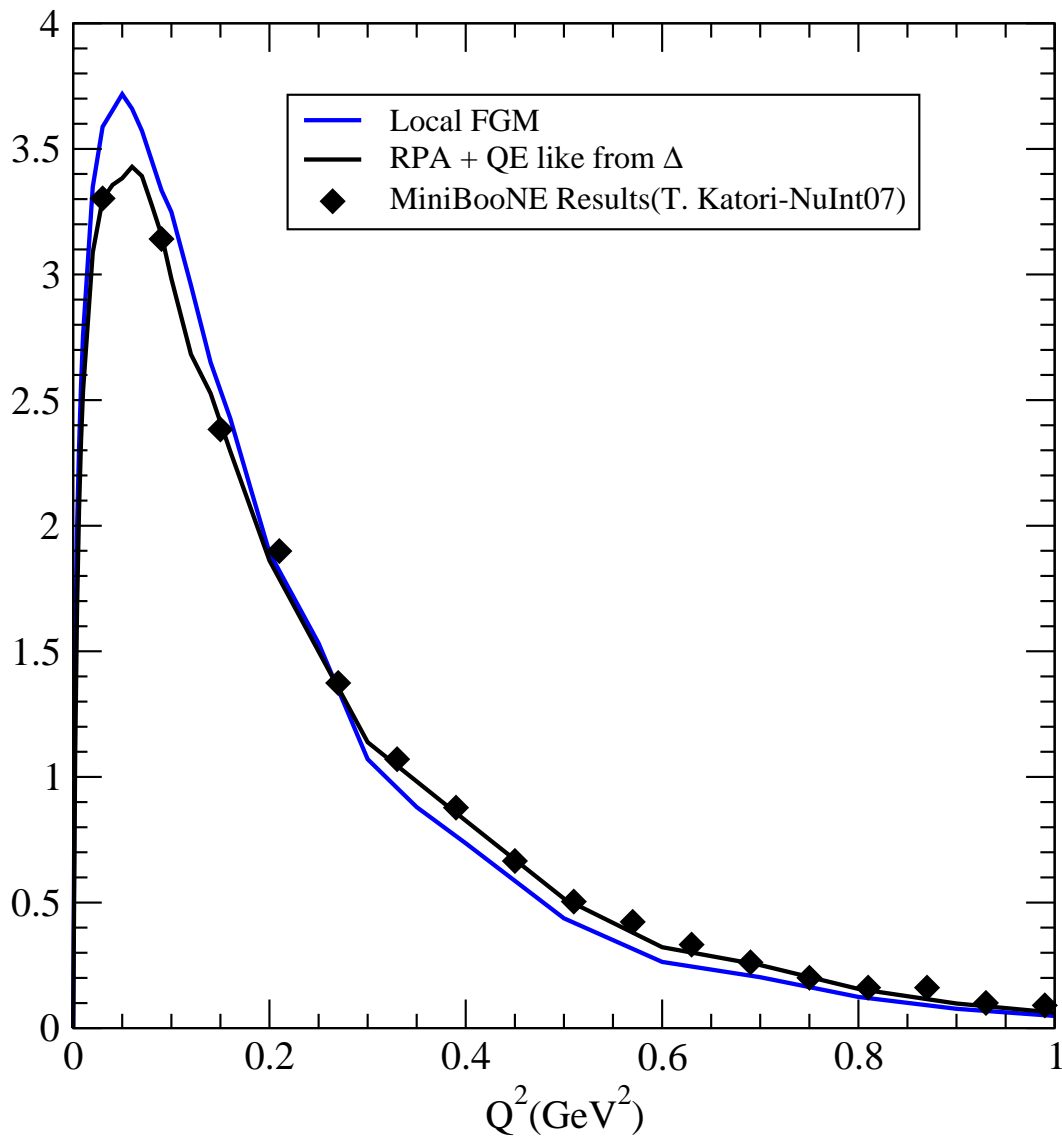


Results from the K2K Collabn. for $\frac{CC1\pi^+}{CCQE}$

ν_μ Differential Cross Section



Antineutrino Differential Cross Section



T. Katori, AIP Conf. Proc. 967 (2007) 123.

Coherent Weak Pion Production

The coherent pion production is the process in which the nucleus remains in the ground state. We calculate the coherent pion production induced by charged current interaction i.e.

$$\nu + \frac{A}{Z} X \rightarrow l^- + \pi^+ + \frac{A}{Z} X$$

The calculations are done in a local density approximation using Δ dominance:

Matrix Element

$$\mathcal{A} = \frac{G_F}{\sqrt{2}} \left[\bar{u}(k') \gamma^\mu (1 - \gamma_5) u(k) \right] \times \left[(J_s^\mu + J_u^\mu) F(\mathbf{q} - \mathbf{p}_\pi) \right]$$

$$J_s^\mu = \sqrt{3} \frac{G_F}{\sqrt{2}} \cos \theta_C \frac{f_{\pi N \Delta}}{m_\pi} p_\sigma^\pi \sum_s \bar{\Psi}^s(p') \Delta^{\sigma\lambda} O_{\lambda\mu} \Psi^s(p)$$

$$J_u^\mu = \sqrt{3} \frac{G_F}{\sqrt{2}} \cos \theta_C \frac{f_{\pi N \Delta}}{m_\pi} \sum_s \bar{\Psi}^s(p') p_\sigma^\pi O^{\sigma\lambda} \Delta_{\lambda\mu} \Psi^s(p)$$

$$F(\mathbf{q} - \mathbf{p}_\pi) = \int d^3r \left[\rho_p(\mathbf{r}) + \frac{1}{3} \rho_n(\mathbf{r}) \right] e^{i(\mathbf{q} - \mathbf{p}_\pi) \cdot \mathbf{r}}$$

the nuclear form factor $\mathcal{F}(\mathbf{q} - \mathbf{k}_\pi)$, in impact parameter representation may be written as

$$F(\mathbf{q} - \mathbf{k}_\pi) = \int d^2b dz \rho(\mathbf{b}, z) e^{i(\mathbf{q} - \mathbf{k}_\pi) \cdot (\mathbf{b} + \hat{q}z)}$$

Finally, the equation takes the form

$$F(\mathbf{q} - \mathbf{k}_\pi) = 2\pi \int_0^\infty b db \int_{-\infty}^\infty dz \rho(\mathbf{b}, z) J_0(k_\pi^t b) e^{i(|\mathbf{q}| - k_\pi^l)z}$$

When the pion absorption effect is taken into account the nuclear form factor $\mathcal{F}(\mathbf{q} - \mathbf{k}_\pi)$ modifies to $\tilde{\mathcal{F}}(\mathbf{q} - \mathbf{k}_\pi)$ given as

$$\tilde{\mathcal{F}}(\mathbf{q} - \mathbf{k}_\pi) = 2\pi \int_0^\infty b db \int_{-\infty}^\infty dz \rho(\mathbf{b}, z) J_0(k_\pi^t b) e^{i(|\mathbf{q}| - k_\pi^t)z} e^{-if(\mathbf{b}, z)}$$

where

$$f(\mathbf{b}, z) = \int_z^\infty \frac{1}{2|\mathbf{k}_\pi|} \Pi(\rho(\mathbf{b}, z')) dz'$$

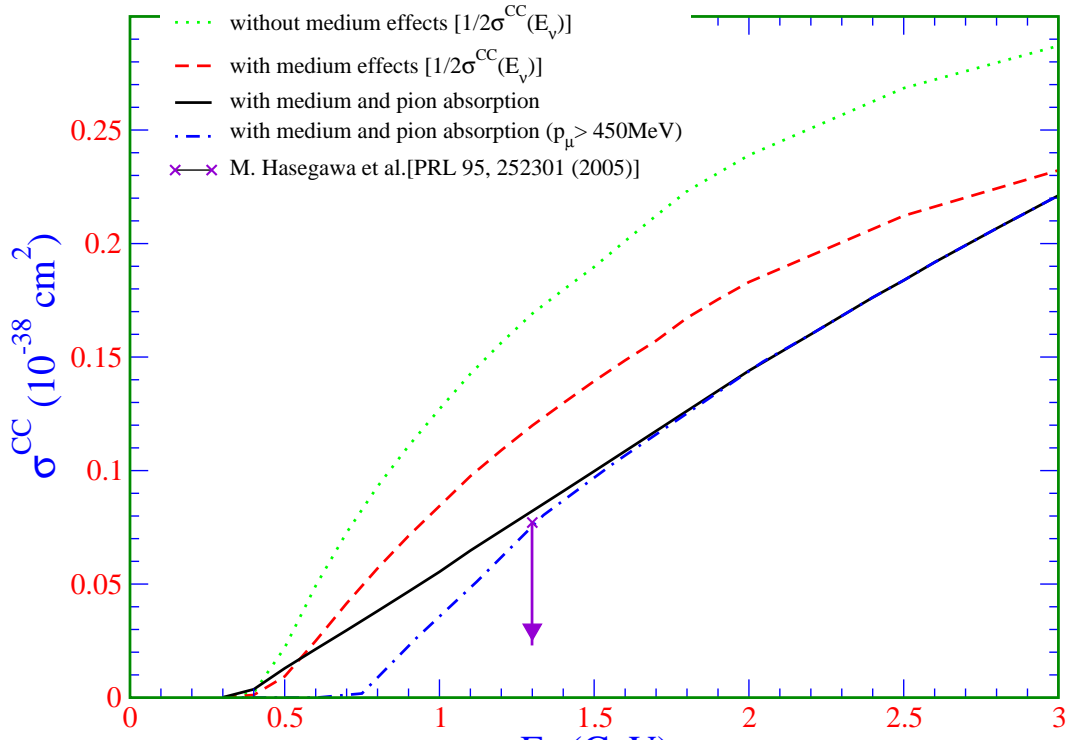
and the pion self-energy Π

Using these expressions the following form of the double differential cross section for pion production is obtained

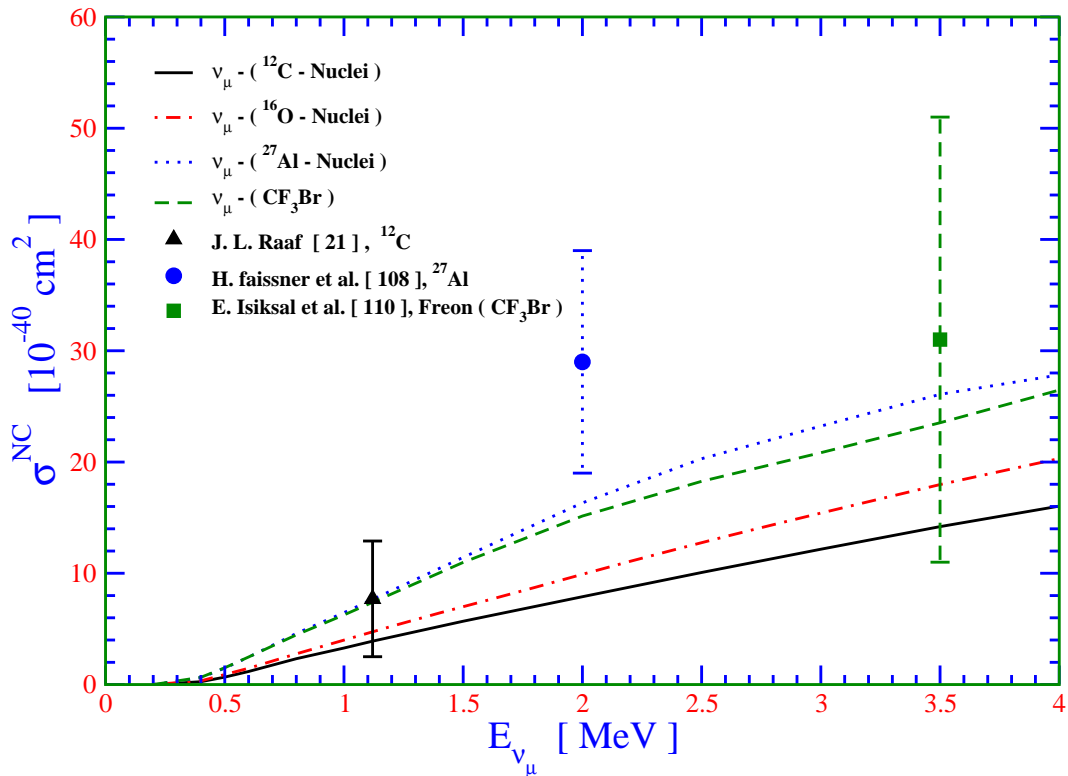
$$\left[\frac{d^5\sigma}{dE_\pi d\Omega_\pi d\Omega_{l'}}$$

$$\mathcal{R} = \left[(E_p + E_{l'} - E_l \cos \theta_{l'}) - \frac{|\mathbf{k}_\pi|}{|\mathbf{q}|} (E_{l'} - E_l \cos \theta_{l'}) \cos \theta_{\pi q} \right]$$

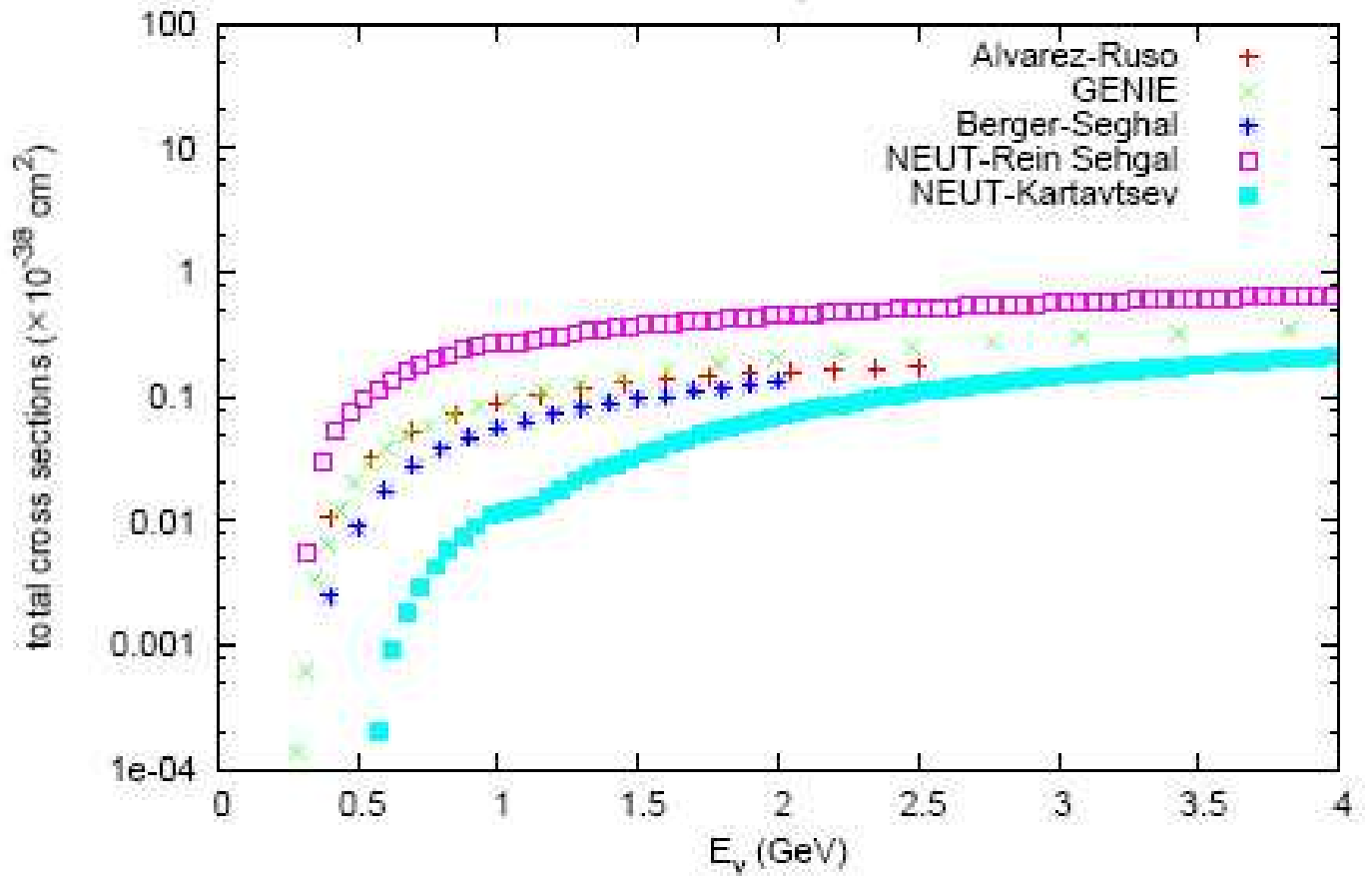
C.C. $1\pi^+$ Coherent pion production ^{12}C



N.C. Coherent Pion production



total CC coherent - ν_μ $^{12}\text{C} \rightarrow ^{12}\text{C} \mu^- \pi^+$

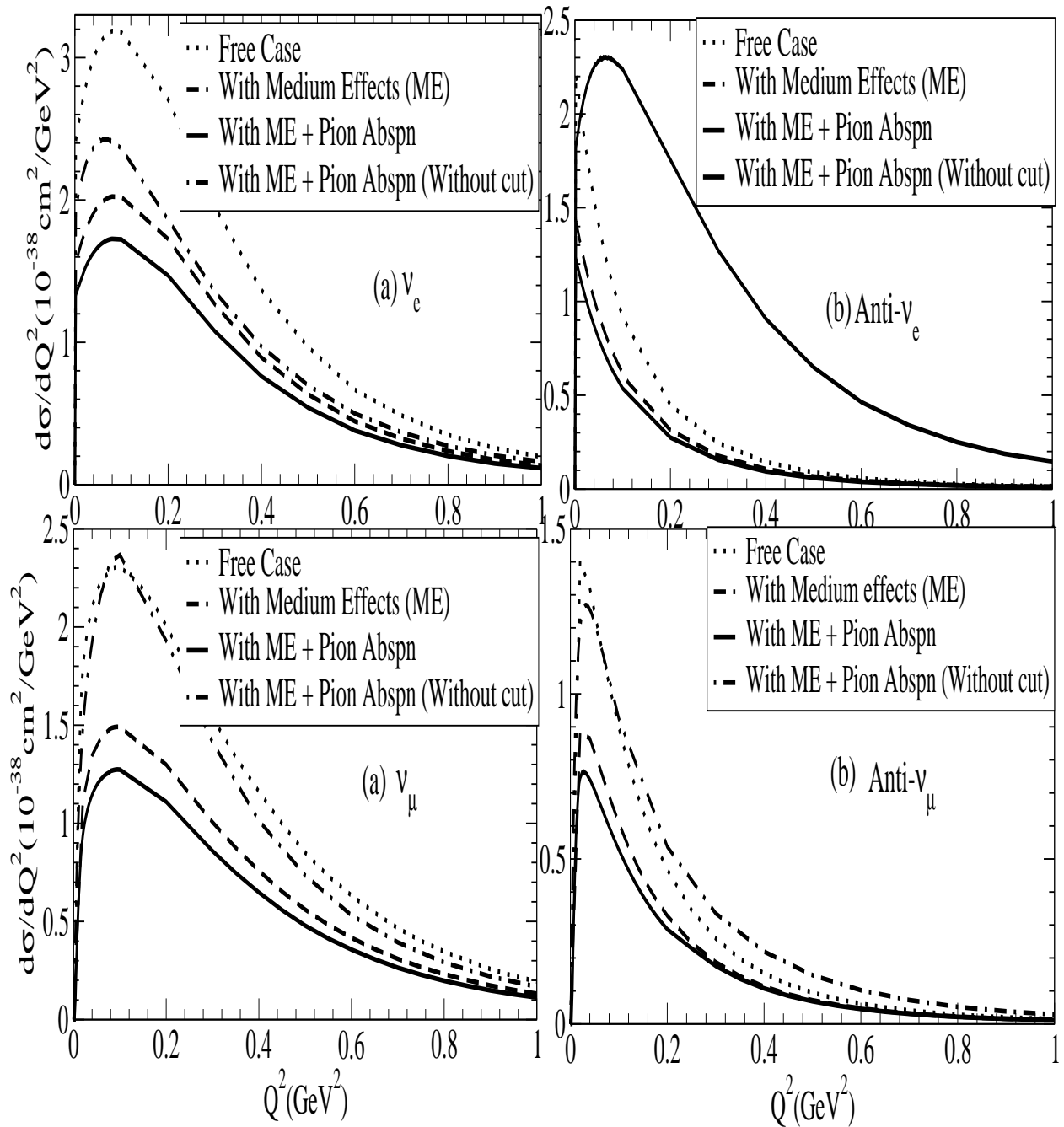


Muon Events		
Inelastic Process	Free Case	Medium effects with Pion abspn
$\nu_\mu p \rightarrow \mu^- \Delta^{++}$ (on free p)	154	154
$\bar{\nu}_\mu p \rightarrow \mu^+ \Delta^0$ (on free p due to H_2)	12	12
$\nu_\mu^{16} O$ (μ^- with π^0)	171	92
$\bar{\nu}_\mu^{16} O$ (μ^+ with π^0)	40	23
$\nu_\mu^{16} O$ (μ^- with π^+)	756	409
$\bar{\nu}_\mu^{16} O$ (μ^+ with π^-)	179	102
$\nu_\mu + \bar{\nu}_\mu$ (Coherent)	233	30
$\nu_\mu + \bar{\nu}_\mu$ (q.e. like events)	-	344
$\nu_\mu + \bar{\nu}_\mu$	1545	1166
Electron Events		
Inelastic Process	Free case	Medium Effects with Pion Abspn
$\nu_e p \rightarrow e^- \Delta^{++}$ (on free p due to H_2)	98	98
$\bar{\nu}_e p \rightarrow e^+ \Delta^0$ (on free p due to H_2)	6	6
$\nu_e^{16} O$ (e^- with π^0)	99	53
$\bar{\nu}_e^{16} O$ (e^+ with π^0)	21	12
$\nu_e^{16} O$ (e^- with π^+)	501	269
$\bar{\nu}_e^{16} O$ (e^+ with π^-)	91	52
$\nu_e + \bar{\nu}_e$ (Coherent)	148	19
$\nu_e + \bar{\nu}_e$ (q.e. like events)	-	200
$\nu_e + \bar{\nu}_e$	964	709

Total number of lepton events for inelastic
process.

Process	$\nu_e + \bar{\nu}_e$	$\nu_\mu + \bar{\nu}_\mu$
Free case(QE+Inelastic)	4057	6084
FGM without RPA+Inelastic with nuclear medium and final state interaction effects	2973	4499
FGM with RPA +Inelastic with nuclear medium and final state interaction effects	2405	3762
Monte Carlo events	2533.9	3979.7
Reported by experiments	3353	3227

Total number of lepton events calculated in our model and its comparison with the observed lepton events by SuperK collaboration and the Monte carlo number used by them



$\langle \frac{d\sigma}{dQ^2} \rangle$ vs Q^2 for the incoherent process induced by electron type (upper panels) and muon type (lower panels) (a) neutrino and (b) antineutrino.

