Quantum Mechanics, Spring 2004

Homework #0, Due 29/1/04

1. Show that if P is a projection operator ($P^2 = P = P^\dagger$), then (I - 2P) is both Hermitian and unitary. Is the converse true (is every operator that is both Hermitian and unitary of this form) ?

2. Given creation and annihilation operators $a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$ and $a|n\rangle = \sqrt{n}|n-1\rangle$, calculate

(a) $[a, a^\dagger]$ and $\{a, a^\dagger\}$
(b) $\langle 0|aa^\dagger aaa^\dagger a^\dagger|0\rangle$
(c) $\langle m|a^\dagger a^\dagger a^\dagger a|n\rangle$

3. Given a one dimensional square potential well
$[V(x) = -V_0$ for $-a < x < a$, zero otherwise$]$,

(a) Draw qualitatively (no need to solve the Schrödinger equation) the lowest three bound state wavefunctions. Assume that the well is deep enough. A free hand sketch is sufficient.
(b) Draw qualitatively one of the wavefunctions with $E > 0$.

4. Calculate the expectation value of the operator $X = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ in the following three systems:

(a) all particles in the state $|A\rangle = \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]$
(b) Half the particles in the state $|B_1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and half in $|B_2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
(c) all particles in the state $|C\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

5. The Pauli σ matrices, that come handy in many problems involving rotation, are given by

$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

(a) Show that $(\vec{\sigma}.\hat{n})^2 = 1$ for every unit vector $\hat{n}$
(b) Using the above, show that $e^{i\theta\vec{\sigma}.\hat{n}} = \cos\theta + i\vec{\sigma}.\hat{n}\sin\theta$