Quantum Mechanics, Spring 2004

Homework #1, Due 10/2/04

1. A particle is scattered by a one-dimensional complex potential $V(x) = \mathrm{Re}V(x) + i\mathrm{Im}V(x)$. Calculate $\partial\rho/\partial t + \nabla\cdot J$ in terms of $V(x)$ and ρ. Comment.

2. Find the relation between $\Delta x|\alpha\rangle$ and $\Delta p|\alpha\rangle$ that satisfies the *equality* $\langle(\Delta x)^2\rangle\langle(\Delta p)^2\rangle = \hbar^2/4$. Solve this differential equation and find $|\alpha\rangle$.

3. K-mesons are created as "strangeness" eigenstates $|K\rangle$ or $|\overline{K}\rangle$. Since the strangeness operator does not commute with the Hamiltonian, these are not the mass eigenstates. The mass eigenstates are $|K_1\rangle \equiv (1/\sqrt{2})(|K\rangle + |\overline{K}\rangle)$ and $|K_2\rangle \equiv (1/\sqrt{2})(|K\rangle - |\overline{K}\rangle)$. The Hamiltonian in the mass eigenstate basis is $H = \begin{pmatrix} m_1 - i\Gamma_1/2 & 0 \\ 0 & m_2 - i\Gamma_2/2 \end{pmatrix}$.

If a $|K\rangle$ meson is produced at $t = 0$, calculate the probability that a $|\overline{K}\rangle$ meson is observed at time t. (These are the $K - \overline{K}$ oscillations that led to the first signature of CP violation.)

4. The benzene ring of six carbon atoms has radius R.

(a) Find the energy spectrum of an electron confined to such a circular ring.

(b) Each carbon atom in the ring contributes one electron. Given that maximum 2 electrons (one with spin up, one with spin down) can occupy each energy level, find the lowest sum of electron energies, E_{ring}.

(c) Imagine breaking the ring while keeping its length unchanged. Find the new energy spectrum.

(d) Find the lowest sum of electron energies, E_{linear}.

(e) Calculate the "π-bond energy", $E_{bond} \equiv E_{ring} - E_{linear}$, given $R = 1.53A^\circ$.

5. Calculate the average force $\langle dV/dx\rangle$ on one side of an infinite potential well ($V = 0$ for $|x| \leq a$, infinity otherwise) by a particle inside the well in terms of its energy. (Hint: You may have to start with a finite well and take the limit as the well becomes infinitely deep.)

(b) Consider a classical particle with speed v trapped inside the well. Calculate the average force on the wall and compare with the quantum mechanical answer in (a).

6. Consider an α particle with $E > 0$ in a nucleus, under the influence of a short-distance attractive potential V_0 and the Coulomb repulsion: $V(x) = -V_0\,\Theta(R-x) + Z_1 Z_2 e^2/x$. Calculate the probability for it to tunnel out as a function of its energy in the limit $E \ll Z_1 Z_2 e^2/R$.

The mean lifetime for α decay is $\tau = (2R/v_i)|T(E)|^{-2}$ where $v \sim 10^9$ cm/s is the typical speed of the particle inside the nucleus. Use $R \sim 10^{-12}$ cm and estimate the lifetime (in years) of a $Z_1 = 90$ nucleus that decays with $E_\alpha = 5$ MeV. (An order of magnitude estimation is fine.)