Quantum Mechanics, Spring 2004

Homework #2, Due 17/2/04

1. Determine the transmission amplitude $T(E)$ for the 1-d attractive δ-potential $V(x) = -\lambda\delta(x)$. How many bound states and resonances does this potential have ?

2. Consider a particle in a uniform gravitational field with a 'floor', i.e. $V = mgz$ for $z \geq 0$ and infinity otherwise.
(a) Using the semiclassical approximation, find the quantization condition for stationary states. (The relevant connection formulae may be used directly).
(b) Determine the energy spectrum

3. Using the semiclassical approximation, estimate the number of bound states for a "triangular well" potential ($V = -(V_0/a)\ ||x| - a|$ for $|x| < a$, zero otherwise)

4. Determining the potential $V(x)$ given the energy spectrum $E(n)$: Given a potential $V(x)$ that is even in x and is a monotonically increasing function for $x > 0$, use the semiclassical approximation to show that

$$x(V) \approx \frac{\hbar}{\sqrt{2m}} \int_{E_0}^{V} \frac{dE}{(dE/dn)\sqrt{(V-E)}}$$

where $x(V)$ is the inverse function of $V(x)$ (defined in the range $x > 0$) and E_0 is the ground state energy.
(Hint: Evaluate the integral $\int_0^\alpha \frac{dn}{dE} \frac{dE}{\sqrt{\alpha - E}}$ and take the limit $\alpha \to V$)

5. Use semiclassical approximation to calculate the transmission coefficient for electrons from a metal under the action of a large electric field E_x. Take the potential to be $V = -V_0$ for $x < 0$ and $V = -eE_x x$ for $x > 0$.
(a) Find the condition for the semiclassical approximation to be valid.
(b) Calculate the transmission coefficient for $E_{\text{electron}} = -4$ eV and the field $E_x = 10^6, 10^7, 10^8$ V/cm.