Quantum Mechanics, Spring 2004

Homework #3, Due 26/2/04

1. Consider the motion of an electron in a uniform magnetic field along the z axis. Choose the guage $A = (0, Bx, 0)$.
(a) Argue that the wavefunction may be written in the form

$$\psi(x, y, z) = \exp(ik_z z + ik_y y)\phi(x)$$

(b) Show that the Schrödinger's equation may be written in the form

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}\phi(x) + \frac{e^2B^2}{2mc^2}(x - x_0)^2\phi(x)\right) = E_{xy}\phi(x)$$

where $E_{xy} = E^2 - \hbar^2 k_z^2$. Determine x_0.
(c) Looking at the above equation as a simple harmonic oscillator, determine the eigenvalues of the original Hamiltonian.
(d) Interprete the solution $\psi(x, y, z)$ in terms of the classical motion of an electron in a uniform magnetic field.

2. Consider the Aharonov-Bohm experiment (fig. 2.8 in Sakurai). Electrons with momentum p impinge on the double-slit at a constant rate from one side. Let the distance between the slits be d.
(a) Find the interference pattern measured by detectors kept at a distance $L \gg d$ on the other side of the screen, in the absence of any magnetic field in the solenoid. What is the distance between successive maxima ?
(b) Consider a detector that is situated symmetrically with respect to the two slits at $L \gg d$. Turning on the magnetic field in the solenoid will change the rate of particles detected. Find the values of the magnetic flux for which the rate will be a maximum.

3. (a) Show that, for any operators A and G,

$$\exp(iG\lambda)A\exp(-iG\lambda) = A + i\lambda[G, A] + \left(\frac{i^2\lambda^2}{2!}\right)[G, [G, A]\ ] + ...$$

$$... + \left(\frac{i^n\lambda^n}{n!}\right)[G, [G, [G, ...[G, A]\ ]\ ]\ ] + ...$$

(b) Hence show that

$$\exp(-iJ_y\theta/\hbar)\ J_z\ \exp(iJ_y\theta/\hbar) = J_z\cos\theta + J_x\sin\theta$$

Interprete the result physically.

4. Determine the matrix $\langle jm'|J_y|jm\rangle$ for $j = 3/2$

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Recommended problems (not for submission):

1. Check the $[\Pi_i, \Pi_j]$ commutation relations and obtain the Lorentz force equation

$$m\frac{d^2x}{dt^2} = e\left[\frac{1}{c}\left(\frac{dx}{dt}\times B - B\times\frac{dx}{dt}\right)\right]$$

2. Check that the electromagnetic probability flux

$$j = \left(\frac{\hbar}{m}\right)\mathrm{Im}(\psi^*\nabla\psi) - \left(\frac{e}{mc}\right)A|\psi|^2$$

satisfies the continuity equation.

3. Confirm the commutation relations between $J_z, J_\pm$.

4. Fill in the blanks between the steps (on Sakurai page 195) for calculating the matrix of the reduced Wigner function $d^{(1)}(\beta)$