Quantum Mechanics, Spring 2004

Homework #4, Due 9/3/2004

1. Problem 21, chapter 3 from Sakurai (page 245).

2. An electron is in the state with quantum numbers $L = 1, L_z = 0, s_z = +1/2$. Find the probability that its total angular momentum is $J = 1/2$. (Calculate all relevant quantities by yourselves – do not use any tables).

3. (a) Starting from $[V_i, J_j] = i\epsilon_{ijk}V_k$ (where $\{i,j,k\} = \{x,y,z\}$), find $V_{\pm1,0}$ that satisfy the commulation relations of J with $T_q^{(k)}$ [as in Sakurai (3.10.25)].
(b) Starting from $T_{\pm2}^{(2)} = U_{\pm1}V_{\pm1}$, determine $T_{\pm1}^{(2)}$ and $T_0^{(2)}$ using *only* the commutation relations of J with T, U, V.

4. The matrix element for the electromagnetic decay of an atomic d-state ($\ell = 2, m_\ell = m_i$) to a p-state ($\ell = 1, m_\ell = m_i$) is given by the matrix element

$$\mathcal{M} = \alpha\vec{\epsilon}\cdot\langle \ell = 1, m_f|\vec{r}|\ell = 2, m_i\rangle$$

where $\alpha\vec{\epsilon}\cdot\vec{r}$ is the interaction responsible for the decay, and $\vec{\epsilon}$ is the polarization vector of the outgoing photon.

Consider the intensity of right circularly polarized light [$\vec{\epsilon} = (1, i, 0)/\sqrt{2}$] emitted along the z axis. Determine the relative intensities of such light for all 15 combinations of m_i and m_f. (You may use Clebsch-Gordan coefficients from a table).

5. Problem 28, chapter 3 from Sakurai (page 247).

6. Find the angular distribution of the decay of a spin-1 particle with $s_z = +1$ into:
(a) a spin-1 and a spin-0 particle, (b) two spin-1/2 particles
(c) a spin-2 and a spin-0 particle
(You may use the values of Wigner matrix elements from a table / any reference).