Quantum Mechanics, Spring 2004

Homework #5, Due 18/3/2004

1. Consider neutrinos travelling from the Sun to the Earth. The relative phase difference accumulated by ν_1 and ν_2 during their travel is $\Delta\phi = \Delta m^2 L/(2E)$ (in natural units). Given that Δm^2 involved in solar neutrino oscillations is $\sim 10^{-4}$ eV^2 (natural units), estimate

(a) The energy resolution $\Delta E/E$ required for observing an oscillation pattern as a function of energy.

(b) For neutrinos with a fixed energy, how small should the production region inside the sun be so that the oscillations as a function of L do not get averaged out ? Compare it with the size of the sun.

(Take the average energy of neutrinos to be 10 MeV, and find your own sources for the values of all relevant astrophysical parameters.)

2. Spin precession in density matrix formalism:

Consider spin-1/2 particles passing through a constant magnetic field in x direction. The initial ensemble is prepared such that a fraction a of the particles are in the state $|+\rangle$ and the remaining particles [fraction $(1-a)$] are in the state $|-\rangle$. (Here $|+\rangle$ and $|-\rangle$ are the eigenstates of S_z)

(a) Write down the density matrix in the basis of $|+\rangle$ and $|-\rangle$.

(b) Calculate the evolution of the density matrix, determine the time dependence of $\langle S_z\rangle$, make a rough plot, and give its physical interpretation.

(c) Check your answer in the limit $a = 1$.

3. (a) Show that the density matrix ρ describing a spin-1/2 particle may be written in the form

$$\rho = (1/2)[1 + \vec{P}\cdot\vec{\sigma}]$$

where σ_i are Pauli matrices. Calculate the ensemble average $[\vec{\sigma}]$.

(b) If the system is placed in a constant magnetic field $B\hat{z}$, find the equation of motion for $\vec{P}$. Interprete the result physically.

4. Sakurai Chapter 3, problem 11 (page 243)

5. A 2×2 scattering matrix S for the scattering of spin-1/2 particles by a spin-zero target is defined such that $|f\rangle = S|i\rangle$, where $|f\rangle$ and $|i\rangle$ are the final and initial states respectively. Let S be written as

$$S = a + b(\hat{n}\cdot\vec{\sigma})$$

where $|a|^2 + |b|^2 = 1$.

If the initial state is unpolarized, find the polarization $[\vec{\sigma}]$ after scattering.