Quantum Mechanics, Spring 2004

Homework #7, Due 6/4/2004

1. Calculate the effect of a finite nuclear size on the energy of H atom states with $n = 1$ and $n = 2$: Take the H nucleus to be a sphere of radius 1 fm with uniform charge density, and calculate $\Delta E/E$ numerically for all the states.

2. Sakurai Chapter 5, problem 8 (page 347).

3. (a) Estimate the ranges of magnetic field (numerically, in gauss) for which the $L \cdot S$ term, the $(L + 2S) \cdot B$ term and the $|B|^2$ term respectively (from the Hamiltonian for an electron in a Hydrogen atom) dominate over the others.

(b) Consider the 3d levels of an electron in the Hydrogen atom. Show the energy level diagrams in the limits of weak and strong magnetic field (strong magnetic field still not strong enough to take into account the $|B|^2$ term). Indicate the values of all level splittings in terms of

$$a \equiv \frac{e\hbar|B|}{2mc} \quad \text{and} \quad b_j \equiv \frac{1}{2m^2c^2}\left\langle \frac{1}{r}\frac{dV}{dr} \right\rangle_j$$

4. An Hydrogen atom in $n = 2$ state is kept in mutually perpendicular constant electric and magnetic fields. For strong fields (so that the spin-orbit interaction may be neglected), determine the energy level splittings. Draw the energy level diagram.

5. Using the trial function $|\alpha\rangle = e^{-ar^2}$, find an upper bound on the ground state energy of

(a) a simple Harmonic oscillator in one dimension

(b) an electron in an Hydrogen atom

In both the cases, estimate how close the ground state wavefunction has been guessed by computing a lower bound on $|\langle\tilde{\alpha}|0\rangle|^2$ (where $|\tilde{\alpha}\rangle$ is normalized $|\alpha\rangle$). (You may use the knowledge of the exact energy spectra, but do not use the actual forms of $|0\rangle$ even if you know it).

6. For a H_2 molecule, let the interatomic interactions be taken as perturbations, so that the unperturbed ground state is $U_0 = U_{100}(\vec{r_1})U_{100}(\vec{r_2})$. (See Fig. 5.3 in Sakurai). The perturbation may be expanded in powers of r_i/r to get

$$V = \frac{e^2}{r^3}(x_1x_2 + y_1y_2 - 2z_1z_2) + \mathcal{O}\left(\frac{1}{r^4}\right)$$

(You may try to derive this, but no need to show it here.)

(a) Calculate the lowest order correction to the ground state energy. (Assume that the $\mathcal{O}(1/r^4)$ terms give no first order contribution). The answer may be in the form of a summation. Find a lower bound on this summation, and hence on the ground state energy.

(b) Choose the trial function $|\alpha\rangle = U_{100}(\vec{r_1})U_{100}(\vec{r_2})(1+aV)$ to determine an upper bound on the ground state energy.

Recommended:

Problems 1 – 21 from Sakurai chapter 5. An extremely good collection.