Quantum Mechanics, Spring 2004

Mathematica problem set

1. Consider the one-dimensional Hydrogen atom, such that the electron confined to the x axis experiences an attractive force e^2/r^2.

(a) Write down Schrödinger's equation for the electron wavefunction $\psi(x)$, and bring it to a convenient form by making the substitutions

$$a = \hbar^2/(me^2), \quad E = -\hbar^2/(2ma^2\alpha^2), \quad z = 2x/(\alpha a)$$

(b) Solve the Schrödinger's equation for $\psi(z)$. (You may need Mathematica, symmetry arguments plus some properties of the Confluent Hypergeometric functions from the Mathematical Physics course).

(c) Find the three lowest allowed values of energy and the corresponding bound state wavefunctions. Plot them for suitable parameter values.

2. (a) Consider a free particle in one dimension, which is described at $t = 0$ by a Gaussian wavepacket

$$\langle x|\alpha\rangle = \left[\frac{1}{\pi^{1/4}\sqrt{d}}\right] \exp\left[ikx - \frac{x^2}{2d^2}\right]$$

Plot the probability density $|\langle x|\alpha\rangle|^2$ as a function of time for suitable values of time and animate. Determine the width of the wavepacket $d(t)$ as a function of time.

(b) Consider two wavepackets of the form

$$\langle x|\alpha_i(t)\rangle = \left[\frac{1}{\pi^{1/4}\sqrt{d}}\right] \exp\left[ikx - \frac{(x - v_i t)^2}{2d^2}\right]$$

where v_1 and v_2 are the velocities of the two wavepackets. Using suitable values for the relevant parameters, plot the overlap of the wavefunctions as a function of time. (Assume that the time dependence of d may be neglected).

3. Consider the symmetric double well potential with the total well width $2w$. Take the width of the barrier to be $2a$ and its height to be h.

(a) Find the two lowest wavefunctions $|S\rangle$ and $|A\rangle$ and their energies.

(b) If a particle is put in the state $|R\rangle = (|S\rangle + |A\rangle)/\sqrt{2}$, plot the evolution of this state as a function of time for suitable parameter values. Animate.