Quantum Mechanics, Spring 2004

Midterm #1 , 4/3/2004

1. (4 points) Prove or disprove the following statements:

(a) If two Hermitian operators A and B anticommute, it is not possible to have a simultaneous eigenket of A and B.

(b) Let two observables A_1 and A_2 not involve time explicitly. If $[A_1, H] = 0$ and $[A_2, H] = 0$, then $[A_1, A_2] \neq 0$ only if the energy eigenvalues are degenerate.

2. (4 points)

If $\mathcal{P}$ and $\mathcal{X}$ are the momentum and position operators in one dimension, and the Hamiltonian is

$$H = \frac{\mathcal{P}^2}{2m} + \frac{1}{2} m\omega^2 \mathcal{X}^2 + mC\omega^2 \mathcal{X} \quad ,$$

(a) Determine the equations of motion for the Heisenberg operators $\mathcal{P}(t)$ and $\mathcal{X}(t)$

(b) Calculate $\langle \psi(t)|\mathcal{P}|\psi(t)\rangle$ and $\langle \psi(t)|\mathcal{X}|\psi(t)\rangle$, given $\langle \mathcal{X} \rangle = 0 = \langle \mathcal{P} \rangle$ at $t = 0$.

3. (4 points)

(a) Given a magnetic field $\vec{B} = B\hat{z}$, find a vector potential A satisfying $\nabla \cdot A = 0$. Is the solution unique ?

(b) Expand the Hamiltonian in powers of B. Show that the linear term is proportional to $L \cdot B$ and the quadratic term proportional to $x^2 + y^2$. Determine the proportionality constants.

4. (4 points)

The total angular momentum of a spin-S particle with orbital angular momentum L is $\vec{J} = \vec{L} + \vec{S}$.

(a) Given an interaction term $L \cdot S$ in the Hamiltonian, which of the quantum numbers $j, m_j, \ell, m_\ell, s, m_s$ are conserved ? Show explicitly by computing the relevant commutators.

(You may find the following relations useful:

$2L \cdot S = 2L_z S_z + L_+ S_- + L_- S_+, \quad [J_z, J_\pm] = \pm\hbar J_\pm$)

(b) Calculate the eigenvalues of $L \cdot S$ in terms of the conserved quantum numbers.

5. (4 points)

Consider transitions from the states $|j_i = 3/2, m_i\rangle$ to the states $|j_f = 1/2, m_f\rangle$ mediated by the dipole moment operator $\rho\vec{x}$.

(a) Give all the pairs $(m_i \to m_f)$ for which the transition is forbidden.

(b) Calculate the ratio of intensities of the transitions between $(m_i = 3/2 \to m_f = 1/2)$ and $(m_i = 1/2 \to m_f = -1/2)$. You may leave your answer in terms of Clebsch-Gordan coefficients.