Quantum Mechanics 2, Spring 2015

Tutorial #1

1. Counting degrees of freedom in a density matrix:
   Consider an enseble of spin-1 systems. How many independent real parameters are needed to characterize it completely ? Is anything needed in addition to $[S_x], [S_y], [S_z]$?

2. Spin precession in density matrix formalism:
   Consider spin-1/2 particles passing through a constant magnetic field in x direction. The initial ensemble is prepared such that a fraction a of the particles are in the state $|+\rangle$ and the remaining particles [fraction $(1-a)$] are in the state $|-\rangle$. (Here $|+\rangle$ and $|-\rangle$ are the eigenstates of S_z)

   (a) Write down the density matrix in the basis of $|+\rangle$ and $|-\rangle$.

   (b) Calculate the evolution of the density matrix, determine the time dependence of $\langle S_z \rangle$, make a rough plot, and give its physical interpretation.

   (c) Check your answer in the limit $a = 1$.

3. Matrix elements of J (Wigner functions):

   (a) Determine the matrix $\langle jm'|J_y|jm\rangle$ for $j = 3/2$.

   (b) Show that $\langle 1m'|(J_y/\hbar)^3|1m\rangle = \langle 1m'|(J_y/\hbar)|1m\rangle$, and hence in this $(j = 1)$ case

$$\exp\left(\frac{-iJ_y\beta}{\hbar}\right) = 1 - \left(\frac{J_y}{\hbar}^2\right)(1 - \cos\beta) - i\left(\frac{J_y}{\hbar}\sin\beta\right)$$

   (c) Using the above, compute the reduced Wigner functions

$$d^{(1)}_{m'm}(\beta) \equiv \langle jm'| \exp\left(\frac{-iJ_y\beta}{\hbar}\right)|jm\rangle$$

4. Addition of angular momentum (Clebsch-Gordan coefficients):
An electron is in the state with quantum numbers $L = 1, L_z = 0, s_z = +1/2$. Find the probability that its total angular momentum is $J = 1/2$. (Calculate all relevant quantities by yourselves – do not use any tables).

5. Application of Wigner-Eckart theorem:
The matrix element for the electromagnetic decay of an atomic d-state ($\ell = 2, m_\ell = m_f$) to a p-state ($\ell = 1, m_\ell = m_i$) is given by the matrix element

$$\mathcal{M} = \alpha\vec{\epsilon} \cdot \langle \ell = 1, m_f | \vec{r} | \ell = 2, m_i \rangle$$

where $\alpha\vec{\epsilon} \cdot \vec{r}$ is the interaction responsible for the decay, and $\vec{\epsilon}$ is the polarization vector of the outgoing photon.

Consider the intensity of right circularly polarized light [$\vec{\epsilon} = (1, i, 0)/\sqrt{2}$] emitted along the z axis. Determine the relative intensities of such light for all 15 combinations of m_i and m_f. (You may use Clebsch-Gordan coefficients from a table).