Quantum Mechanics 2, Spring 2015

Assignment #3, Due date 31/03/2015

1. Density matrix of spin-1/2 particles:

   (a) Show that the density matrix ρ describing a spin-1/2 particle may be written in the form

   $$\rho = (1/2)[1 + \vec{P} \cdot \vec{\sigma}]$$

   where σ_i are Pauli matrices. Calculate the ensemble average $[\vec{\sigma}]$.

   (b) If the system is placed in a constant magnetic field $B\hat{z}$, find the equation of motion for $\vec{P}$. Interprete the result physically.

2. Angular momentum under rotation:

   (a) Show that, for any operators A and G,

   $$\exp(iG\lambda)A\exp(-iG\lambda) = A + i\lambda[G,A] + \left(\frac{i^2\lambda^2}{2!}\right)[G,[G,A]\,] + ...$$

   $$... + \left(\frac{i^n\lambda^n}{n!}\right)[G,[G,[G,...[G,A]\,]\,]\,] + ...$$

   (b) Hence show that

   $$\exp(-iJ_y\theta/\hbar)\ J_z\ \exp(iJ_y\theta/\hbar) = J_z\cos\theta + J_x\sin\theta\ .$$

   Interprete the result physically.

   (c) Without the expansion in (a), check the above result by taking the first and second derivatives of the quantity and matching boundary conditions at $\theta = 0$.

3. Problem 21, chapter 3 from Sakurai (page 245).

4. Problem 28, chapter 3 from Sakurai (page 247).

5. Find the angular distribution of the decay of a spin-1 particle with $s_z = +1$ into:
   (a) a spin-1 and a spin-0 particle, (b) two spin-1/2 particles
   (c) a spin-2 and a spin-0 particle
   (You may use the values of Wigner matrix elements from a table / any reference).