Quantum Mechanics 2, Spring 2015

Homework #5, Due 14/5/2015

1. A particle confined to an one-dimensional potential well of size L is in the ground state. The size of the well is suddenly magnified to $2L$.

(a) What is the probability that the particle will be in the n^{th} excited state of the new well ?

(b) What is the expectation value of energy of the particle after the expansion ?

2. A 1-d harmonic oscillator is acted upon by a uniform electric field (small enough in magnitude to be considered a perturbation), whose time dependence is given by

$$E(t) = \frac{A}{\sqrt{\pi}\tau} e^{-(t/\tau)^2} \quad ,$$

where A is a constant.

(a) If the oscillator is in the ground state at $t \to -\infty$, find the probability that it is in an excited state at $t \to +\infty$.

(b) For $\tau \gg 1/\omega$, calculate the total transition probability to excited states using adiabatic approximation. Relate the result to that obtained in (a) in the same limit.

3. Consider a particle that experiences a potential V that is constant $(= V_0)$ for $0 \le t \le \tau$ and zero otherwise.

(a) Using sudden approximation, find the state the particle will be in at $t \gg \tau$, if its initial state was $\sum a_n |n\rangle$. Take $|n\rangle$ to be the eigenstates of H_0 and $|\mu\rangle$ to be the eigenstates of $(H_0 + V_0)$. Find the answer keeping only terms linear in $V_0\tau$.

(b) Relate this result to that obtained in problem 2(a) in the limit $\tau \ll 1/\omega$.

4. Sakurai, Chapter 5, Problem 29

5. Sakurai, Chapter 5, Problem 35

6. Sakurai, Chapter 5, Problem 40

Not to be submitted

1. Calculate the second order contribution $c_n^{(2)}(t)$ to the transition amplitude from the initial state $|i\rangle$ to final state $n\rangle$, when the potential V_0 is turned on suddenly at $t = 0$.

2. For the exponentially increasing potential $V_0 e^{\eta t}$, calculate the value of $(1/c_i)(dc_i/dt)$ where c_i, the amplitude of survival in the initial state $|i\rangle$, is calculated to two orders. Take care of the $\eta \to 0$ limit with caution.

3. Sakurai, Chapter 5, Problem 39. Hence derive (5.7.36), the angular distribution for the photoelectric effect.