Quantum Mechanics 2, Spring 2016

Assignment #1, Due 15/03/2016 in class

1. Find the relation between $\Delta x|\alpha\rangle$ and $\Delta p|\alpha\rangle$ that satisfies the *equality*

$$\langle(\Delta x)^2\rangle\langle(\Delta p)^2\rangle = \hbar^2/4 \ .$$

Solve this differential equation and find $|\alpha\rangle$.

2. K-mesons are created as "strangeness" eigenstates $|K\rangle$ or $|\overline{K}\rangle$. Since the strangeness operator does not commute with the Hamiltonian, these are not the mass eigenstates. The mass eigenstates are

$$|K_1\rangle \equiv \frac{1}{\sqrt{2}}(|K\rangle + |\overline{K}\rangle) \quad \text{and} \quad |K_2\rangle \equiv \frac{1}{\sqrt{2}}(|K\rangle - |\overline{K}\rangle) \ .$$

The Hamiltonian in the mass eigenstate basis is

$$H = \begin{pmatrix} m_1 - i\Gamma_1/2 & 0 \\ 0 & m_2 - i\Gamma_2/2 \end{pmatrix} \ .$$

If a $|K\rangle$ meson is produced at $t = 0$, calculate the probability that a $|\overline{K}\rangle$ meson is observed at time t. Draw a sketch that will bring out the appropriate features. (These are the $K - \overline{K}$ oscillations that led to the first signature of CP violation.)

3. Consider the motion of an electron with energy E in a uniform magnetic field along the z axis. Choose the gauge $\vec{A} = (0, Bx, 0)$.

   (a) Argue that the wavefunction may be written in the form

   $$\langle\vec{x}|\psi\rangle = \exp(ik_z z + ik_y y)\langle x|\phi\rangle$$

   (b) Show that the Schrödinger's equation may be written in the form

   $$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}\langle x|\phi\rangle + \frac{e^2B^2}{2mc^2}(x-x_0)^2\langle x|\phi\rangle\right) = E_{xy}\langle x|\phi\rangle \ .$$

   Determine x_0 and E_{xy}.

   (c) Looking at the above equation as a simple harmonic oscillator, determine the eigenvalues of the original Hamiltonian.

   (d) Interprete the solution $\langle\vec{x}|\psi\rangle$ in terms of the classical motion of an electron in a uniform magnetic field.

4. Density matrix of spin-1/2 particles:

   (a) Show that the density matrix ρ describing a spin-1/2 particle may be written in the form
   $$\rho = (1/2)[1 + \vec{P} \cdot \vec{\sigma}]$$
   where σ_i are Pauli matrices. Calculate the ensemble average $[\vec{\sigma}]$.

   (b) If the system is placed in a constant magnetic field $B\hat{z}$, find the equation of motion for $\vec{P}$. Interprete the result physically.

5. Consider a system that consists of a 50%-50% mixture of $|\alpha_1\rangle = |1\rangle$ and $|\alpha_2\rangle = \sqrt{\frac{1}{2}}\,(|1\rangle + |2\rangle)$. where $|1\rangle$ and $|2\rangle$ are orthonormal states. Calculate the density matrix ρ, and write down its diagonalized form. Using this, propose another mixture of states that would give rise to the same density matrix.