Quantum Mechanics 2, Spring 2016

Assignment #2, Due date 29/03/2015

1. Angular momentum under rotation:

   (a) Show that, for any operators A and G,

   $$\exp(iG\lambda)A\exp(-iG\lambda) = A + i\lambda[G,A] + \left(\frac{i^2\lambda^2}{2!}\right)[G,[G,A]\,] + ...$$

   $$... + \left(\frac{i^n\lambda^n}{n!}\right)[G,[G,[G,...[G,A]\,]\,]\,] + ...$$

   (b) Using the above result, determine how the angular momentum operator J_z transforms under rotation about y-axis by an angle θ. Interprete the result physically.

2. Consider a 3d electron, with $m_l = 0$ and $m_s = +1/2$.

   (a) Find the probability that this electron has an angular momentum quantum number $j = 3/2$.

   (b) This electron (in the $|j = 3/2, m = +1/2\rangle$ state) can jump to a state $|n = 2, j', m'\rangle$ in the presence of an electric field $\vec{E} = E_0\hat{x}$. List the values of $|j', m'\rangle$ to which such a transition is possible.

   (c) Calculate the ratio of intensities of the transitions of this electron (in the $|j = 3/2, m = +1/2\rangle$ state) to states of the form $|n = 2, j' = 3/2, m'\rangle$, for different m' values.

3. Problem 21, chapter 3 from Sakurai (Revised ed. 1994: page 245).

4. Problem 28, chapter 3 from Sakurai (Revised ed. 1994: page 247). (Give the answer by substituting appropriate values of the Clebsch-Gordan coefficirents.)

5. Problem 29, chapter 3 from Sakurai (Revised ed. 1994: page 247).

6. An operator A satisfies the following commutation relations:

   $$[A, J_z] = A\ , \qquad [\,[A, J^2],\ J^2] = 2(AJ^2 + J^2A)\ .$$

   Find the conditions for which transitions from $|jm\rangle$ to $|j'm'\rangle$ are possible when mediated by the operator A.

7. Find the angular distribution of the decay of a spin-1 particle with $s_z = +1$ into:
   (a) two spin-1 particles with different masses
   (b) two photons
   in terms of the relevant helicity amplitudes $A_{\lambda_B\lambda_C}$.