Quantum Mechanics 2, Spring 2016

Homework #4, Due 17/5/2016

1. A 1-d harmonic oscillator is acted upon by a uniform electric field (small enough in magnitude to be considered a perturbation), whose time dependence is given by

$$E(t) = \frac{A}{\sqrt{\pi}\tau} e^{-(t/\tau)^2} \quad ,$$

where A is a constant.

   (a) If the oscillator is in the ground state at $t \to -\infty$, find the probability that it is in an excited state at $t \to +\infty$. Plot this probability as a function of $\omega\tau$ by taking suitable values for the other parameters.

   (b) For $\tau \gg 1/\omega$, calculate the total transition probability to excited states using adiabatic approximation. Relate the result to that obtained in (a) in the same limit.

2. Consider a particle that experiences a potential V that is constant $(= V_0)$ for $0 \leq t \leq \tau$ and zero otherwise.

   (a) Using sudden approximation, find the state the particle will be in at $t \gg \tau$, if its initial state was $\sum a_n |n\rangle$. Take $|n\rangle$ to be the eigenstates of H_0 and $|\mu\rangle$ to be the eigenstates of $(H_0 + V_0)$. Find the answer keeping only terms linear in $V_0\tau$.

   (b) Relate this result to that obtained in problem 1(a) in the limit $\tau \ll 1/\omega$. Calculate and comment.

3. Consider an ensemble of particles of mass m in the ground state of an attractive one-dimensional delta function potential: $V_0(x) = -\beta\delta(x)$. These particles may be ionized by applying an additional harmonic potential $V(x,t) = eEx\cos(\omega t)$, where $\hbar\omega > E_0$ (binding energy of the ground state). Estimate the ionization rate through the following steps:

   (a) Find the energy of the ground state $|0\rangle$, and its normalized wavefunction.

   (b) Consider a state $|k\rangle$ with wavenumber k, normalized as $\langle x|k\rangle = e^{ikx}/\sqrt{L}$, where L is an arbitrarily large "box size". Calculate the matrix element $\langle k|V|0\rangle$. Hence determine the probability of transition to the state $|k\rangle$.

   (c) Using the density of states with wavenumber k, estimate the total ionization rate. What is the limit of large box size ?

4. Sakurai, Chapter 5, Problem 29 (1994 Edition: two spin-1/2 objects)
5. Sakurai, Chapter 5, Problem 30 (1994 Edition: two-level system)
6. Sakurai, Chapter 5, Problem 40 (1994 edition: lifetime of 2p level)

Not to be submitted:

1. Sakurai, Chapter 5, Problem 35 (1994 edition: Hydrogen atom in t-dependent potential).
2. Sakurai, Chapter 5, Problem 36 (1994 edition: density of states in 2 dimensions.
3. Sakurai, Chapter 5, Problem 39 (1994 edition). Hence derive (5.7.36), the angular distribution for the photoelectric effect.