

Quantum Mechanics 2, Spring 2016

Quiz 0, 12/02/2016

1. If P is a projection operator ($P^2 = P = P^\dagger$), Is $(I - 2P)$ Hermitian ? Is it unitary ?

2. Given creation and annihilation operators

$$a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle \quad \text{and} \quad a|n\rangle = \sqrt{n}|n-1\rangle ,$$

calculate

- (a) $[a, a^\dagger]$ and $\{a, a^\dagger\}$
- (b) $\langle 0|aa^\dagger aaa^\dagger a^\dagger|0\rangle$
- (c) $\langle m|a^\dagger a^\dagger a^\dagger a|n\rangle$

3. Given a particle of mass m in an one dimensional potential well

$$V(x) = \begin{cases} +\infty & \text{for } x < 0 \\ V_0(x-a) & \text{for } 0 \leq x < a \\ 0 & \text{for } x \geq a \end{cases} ,$$

- (a) Draw qualitatively (no need to solve the Schrödinger equation) the lowest two bound state wavefunctions. Assume that the well is deep enough. A free hand sketch is sufficient.
- (b) Draw qualitatively two wavefunctions with $E > 0$, one with $E \ll |V_0|$, one with $E \gg |V_0|$.
- (c) Write down the quantization condition for a bound state with large value of n , using WKB approximation.

4. Calculate the expectation value of the operator $X = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ in the following three systems:

- (a) all particles in the state $|A\rangle = \frac{1}{\sqrt{2}} \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right]$

- (b) Half the particles in the state $|B_1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and half in $|B_2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

- (c) all particles in the state $|C\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$