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B physics and CP violation: an introduction

Amol Dighe
Tata Institute of Fundamental Research,
Homi Bhabha Road, Mumbai 400005, INDIA

Abstract

These are preliminary lecture notes for the set of 4 lectures in *"B Physics"* at the SERC school held in IIT Bombay in Feb 2008.

Contents

1 Introduction

Some of the earlier lectures in this SERC school have dealt with Higgs and supersymmetry (SUSY). While we have not observed the Higgs (we do have strong reasons to think that it exists) or any of the superpartners of the standard model particles, more than 10^9 B mesons have already been observed at the collider experiments like CDF/D0, and at the "B factories" BaBar and Belle. The soon-to-be-operational LHC hosts a dedicated B physics experiment, LHCb, and copious amounts of B mesons will also be produced and examined at two other LHC experiments, ATLAS and CMS.

The area of B physics forms a part of the more general field of *flavor physics*, which deals with the six flavors of quarks: the origin of their masses, their electroweak interactions, mixing between them, and phenomena like charge-parity (CP) violation that are observed through their decays. Flavor physics has now entered the era of precision measurements, and B decays in particular are going to be instrumental in indirect searches of physics beyond the standard model.

The notes are only expected to serve as a reminder of the logical progression in the lectures. It is hoped that the students, through their own class notes and the references given at the end [1, 2, 3, 4, 5], are able to reconstruct the arguments given in the lectures. I have cited some "classic" papers for their historical significance, but the references have been chosen more for their padagogical value rather than their claim on original results.

1.1 A historical review

The standard model (SM) consists of three families of quarks and leptons. The quark content may be written as

$$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}, \tag{1}$$

where the quarks in the upper row ("up-type") have electric charge $+2/3$ and those in the lower row ("down-type") have electric charge $-1/3$ in the units of proton charge. The lepton content of the SM is

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}. \tag{2}$$

The particles in the first family are enough to account for most of the objects we observe: atoms and their nuclei do not require anything in the higher families for their description. Indeed, in 1937, when the muon was discovered, a mere copy of an electron with a larger mass did not seem to serve any purpose. The question asked was "who ordered muon ?" The second family of the particles was thus completely unexpected when it was discovered.

The third family, on the other hand, was predicted long before any particle from this family was discovered, by the requirement that the CP violation observed is through the Cabibbo-Kobayashi-Maskawa mechanism (which we shall study in detail in this set of lectures). Let us see this historical development in some detail, as it will offer us insight into the development of flavor physics in general.

1.1.1 Cabibbo angle and GIM mechanism

In 1970, three quarks (u, d, s) and four leptons (e, μ and their associated neutrinos) were known. The idea of quarks and leptons behaving similarly had not taken root yet. An important observation by Cabibbo was that the coupling constants of the following three flavor-changing decay modes were related:

- (i) muon decay $\mu^- \to \nu_\mu e^- \bar{\nu}_e$: coupling constant $g_{e\mu}$,
- (ii) neutron decay $n \to p e^- \bar{\nu}_e$ ($d \to u e^- \bar{\nu}_e$): coupling constant g_{ud}, and

- (ii) kaon decay $K^- \to \pi^0 e^- \bar{\nu}_e$ ($s \to u e^- \bar{\nu}_e$): coupling constant g_{us}.

Measurements of the decay rates were consistent with $|g_{e\mu}|^2 = |g_{ud}|^2 + |g_{us}|^2$, which gave rise to the idea of "universality" that there is only one coupling constant $g \equiv g_{e\mu}$, and the u quark simply couples to one particular combination of d and s, given by $d' \equiv \cos\theta_c \cdot d + \sin\theta_c \cdot s$. The angle θ_c here is the Cabibbo angle [6], which was the first quark mixing angle to be measured.

Cabibbo angle was not enough to account for the suppression of $K_L \to \mu^+\mu^-$ as compared to $K^+ \to \mu^+\nu_\mu$: indeed, it predicted a decay rate that was orders of magnitude above the measured upper bound. This suppression of flavor-changing neutral current (FCNC) could be explained if another "c" quark with charge $+2/3$ were postulated, which couples to the combination $s' \equiv -\sin\theta_c \cdot d + \cos\theta_c \cdot s$. The $s \to u \to d$ contribution to the $s \to d$ amplitude would be cancelled by the $s \to c \to d$ contribution, thus causing the FCNC suppression. This was termed the Glashow-Iliopoulos-Maiani (GIM) mechanism [7], which predicted the existence of the "charmed" quark. The observation of $J/\psi(c\bar{c})$ in 1974 vindicated the mechanism.

Problem 1. *If the c quark were absent, calculate the ratio of rates of $K_L \to \mu^+\mu^-$ and $K^+ \to \mu^+\nu_\mu$. Compare this with the measured value this ratio from the Review of Particle Physics [5].*

1.1.2 CP violation and "prediction" of third generation

Cronin *et al.* discovered the CP violation in the kaon system in 1964 [8]. Several attempts were made to explain it, including the postulation of extra "superweak" interaction. However, the one that turned out to be the most promising was the mechanism proposed by Kobayashi and Maskawa [9] in 1972, which showed that with three generations of quarks, a complex quark mixing matrix arises naturally. In a sense, this was the "prediction" of third generation, which was confirmed by the discovery of τ in 1976, $\Upsilon(b\bar{b})$ in 1977, top quark in 1993, and ν_τ in 2001.

The era of B physics thus began about 30 years ago. Since B mesons (those including a b quark) are much heavier than the earlier K (with an s quark) and D (with a c quark), they can decay through more number of channels. This allows one to have more consistency checks as well as more control over theoretical uncertainties, since one can now take ratios of quantities that are relatively more immune to these uncertainties. Moreover, the large mass (≈ 5 GeV) of b quark makes the quantity Λ_{QCD}/m_b small, so that a systematic expansion in this quantity can be carried out which goes under the name of *Heavy Quark Effective Theory* (HQET).

1.2 Recent results and current excitement in B physics

The study of B decays has led to a much better understanding of the flavor sector of the SM, and of the phenomenon of CP violation in particular. Some of the recent important results obtained are the following:

- The measurement of time dependent asymmetry in $B_d \to J/\psi K_S$ demonstrated cleanly that the CP violation in the SM is large. This implied that CP is not an approximate symmetry of the SM.

- The asymmetry in $\overline{B^0} \to K^\pm \pi^\mp$ demonstrated the "direct" CP violation (more appropriately called "CP violation through decay").

- The asymmetries measured in various decay channels (e.g. $K^+K^-K_S$, $D^{*+}D^{*-}$, $\eta' K_S$, $f_0 K_S$, $\rho^\pm \pi^\mp$) have overconstrained the quark mixing matrix.

- Measurements of radiative B decays ($b \to s\gamma$) as well as limits on the rates of super-rare leptonic decays ($B \to \mu^+\mu^-$) have constrained new physics models like SUSY, leptoquarks, etc.

- The measurements of mass differences in the B–$\overline{B}$ system have led to the right prediction of the top quark mass.

One expects that future experiments in B decays will lead us to a better understanding of flavor physics in the SM, and perhaps even give an indirect signal for physics beyond SM.

2 Mixing between two neutral mesons

In this section, we shall develop a general formalism to deal with mixing of two neutral pseudoscalar mesons P and $\overline{P}$. This formalism will be applicable to K–$\overline{K}$, D–$\overline{D}$ as well as B–$\overline{B}$ system. We shall derive results that can be used later to express our ideas compactly.

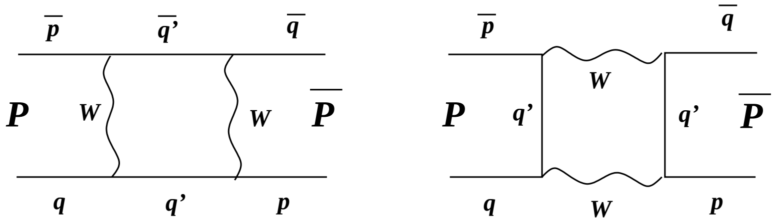

Figure 1: The mixing between $P(\overline{p}q)$ and $\overline{P}(p\overline{q})$. Here q' is the quark whose charge differs by 1 from the charge of p and q. These are the "box" diagrams that give rise to M_{12} and Γ_{12}.

2.1 Hamiltonian, eigenvalues and eigenstates

Let us work in the basis $(P, \overline{P})$, i.e. any superposition state $aP + b\overline{P}$ may be represented as the column vector $(a\ b)^T$. In this basis, the effective Hamiltonian is a 2×2 matrix. Since the P and $\overline{P}$ mesons decay, the evolution is not unitary, and hence the Hamiltonian H is not Hermitian. Indeed, it can be written as the sum of a Hermitian part M and an anti-Hermitian part, conventionally written as $-i\Gamma/2$ where Γ is a Hermitian matrix.

$$H = M - \frac{i}{2}\Gamma = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix} \tag{3}$$

The Hermiticity of H and Γ imply

$$M_{21} = M_{12}^* \qquad \text{and} \qquad \Gamma_{21} = \Gamma_{12}^* \ . \tag{4}$$

In addition, the CPT theorem gives two more constraints:

$$M_{11} = M_{22} \qquad \text{and} \qquad \Gamma_{11} = \Gamma_{22} \ . \tag{5}$$

Note that M_{12} and Γ_{12}, the components of the Hamiltonian that mix P and $\overline{P}$, are essentially the dispersive and absorptive parts of the P–$\overline{P}$ mixing box diagrams. The box diagrams for the $P(p\overline{q})$–$\overline{P}(p\overline{q})$ system is shown in Fig. 1.

Using (4) and (5), one gets the eigenvalues of the Hamiltonian H to be

$$\begin{aligned} \mu_H &= M_{11} - \frac{i}{2}\Gamma_{11} + \frac{1}{2}\left(\Delta m - \frac{i}{2}\Delta\Gamma\right) \ , \\ \mu_L &= M_{11} - \frac{i}{2}\Gamma_{11} - \frac{1}{2}\left(\Delta m - \frac{i}{2}\Delta\Gamma\right) \ , \end{aligned} \tag{6}$$

where Δm and $\Delta\Gamma$ are the solutions of

$$\begin{aligned}(\Delta m)^2 - \left(\frac{\Delta\Gamma}{2}\right)^2 &= 4|M_{12}|^2 - |\Gamma_{12}|^2 \ , \\ \Delta m \cdot \Delta\Gamma &= 4\mathrm{Re}(M_{12}\Gamma_{12}^*) \ . \end{aligned} \tag{7}$$

The labels H and L stand for "heavy" and "light" respectively, by convention. Eq. (6) implies that $\Delta m > 0$ by definition. The sign of $\Delta\Gamma$ depends on the dynamics.

The normalized eigenstates of the Hamiltonian turn out to be

$$\begin{aligned}|P_L\rangle &= p|P\rangle + q|\overline{P}\rangle \ , \\ |P_H\rangle &= p|P\rangle - q|\overline{P}\rangle \ , \end{aligned} \tag{8}$$

where

$$|p|^2 + |q|^2 = 1 \qquad \text{and} \qquad \left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}} \ . \tag{9}$$

Problem 2. *Derive the above results about eigenvalues and eigenstates. What problems would one face if the CPT conditions did not hold ?*

2.2 Phase invariant quantities

We would like to study processes where P or $\overline{P}$ decay to a final state f or its CP-conjugate state $\overline{f}$. The phases of $P, \overline{P}$ as well as $f, \overline{f}$ are arbitrary, so that the CP-conjugation relations can be written in the most general form as

$$CP|P\rangle = e^{i\zeta_P}|\overline{P}\rangle \ , \qquad CP|\overline{P}\rangle = e^{-i\zeta_P}|P\rangle \ , \tag{10}$$

$$CP|f\rangle = e^{i\zeta_f}|\overline{f}\rangle \ , \qquad CP|\overline{f}\rangle = e^{-i\zeta_f}|f\rangle \ . \tag{11}$$

We represent the decay rates for the relevant processes by

$$A_f \equiv \langle f|H|P\rangle \ , \qquad \overline{A}_f \equiv \langle f|H|\overline{P}\rangle \ , \tag{12}$$

$$A_{\overline{f}} \equiv \langle \overline{f}|H|P\rangle \ , \qquad \overline{A}_{\overline{f}} \equiv \langle \overline{f}|H|\overline{P}\rangle \ . \tag{13}$$

The phases ζ_P and ζ_f are unphysical, so that the observable quantities should be independent of these phases. Three such quantities that can be constructed will be relevant for CP violation:

$$\left|\frac{\overline{A}_{\overline{f}}}{A_f}\right| \ , \qquad \left|\frac{q}{p}\right| \ , \qquad \lambda_f \equiv \frac{q}{p}\frac{\overline{A}_f}{A_f} \ . \tag{14}$$

Interchanging f and $\overline{f}$ can give more phase invariant quantities, but that is just a matter of redefinition of f and $\overline{f}$.

2.3 Time Evolution

Let us study the time evolution of an initial flavor eigenstate $|P\rangle$. Since the eigenstates $|P_H\rangle$ and $|P_L\rangle$ evolve independently without mixing, it is easier to write the evolution in terms of these states. At $t = 0$,

$$|P(0)\rangle = \frac{1}{2p}(|P_L\rangle + |P_H\rangle) \ . \tag{15}$$

At time t, one gets

$$\begin{aligned} |P(t)\rangle &= \frac{1}{2p}(e^{-im_L t - \Gamma_L t/2}|P_L\rangle + e^{-im_H t - \Gamma_H t/2}|P_H\rangle \\ &= g_+(t)|P\rangle - \frac{q}{p} g_-(t)|\overline{P}\rangle \ , \end{aligned} \tag{16}$$

where we have defined

$$g_\pm \equiv \frac{1}{2}\left(e^{-im_H t - \Gamma_H t/2} \pm e^{-im_L t - \Gamma_L t/2}\right) \tag{17}$$

for convenience. In terms of $g_\pm$, the evolution of an initial $|\overline{P}\rangle$ can be written simply as

$$|\overline{P}(t)\rangle = g_+(t)|\overline{P}\rangle - \frac{p}{q} g_-(t)|P\rangle \ . \tag{18}$$

Now we are ready to calculate the rate of $P/\overline{P}$ to $f/\overline{f}$ as a function of time. One obtains, after a straightforward algebra,

$$\begin{aligned} \frac{\frac{d\Gamma}{dt}[P(t) \to f]}{e^{-\Gamma t} N_f |A_f|^2} &= (1 + |\lambda_f|^2)\cosh(\Delta\Gamma t/2) + (1 - |\lambda_f|^2)\cos(\Delta m t) \\ &\quad + 2\mathrm{Re}(\lambda_f)\sinh(\Delta\Gamma t/2) + 2\mathrm{Im}(\lambda_f)\sin(\Delta m t) \ , \quad (19) \\ \frac{\frac{d\Gamma}{dt}[\overline{P}(t) \to f]}{e^{-\Gamma t} N_f |A_f|^2 |p/q|^2} &= (1 + |\lambda_f|^2)\cosh(\Delta\Gamma t/2) - (1 - |\lambda_f|^2)\cos(\Delta m t) \\ &\quad + 2\mathrm{Re}(\lambda_f)\sinh(\Delta\Gamma t/2) - 2\mathrm{Im}(\lambda_f)\sin(\Delta m t) \ . \quad (20) \end{aligned}$$

where $\Gamma \equiv (\Gamma_H + \Gamma_L)/2$ and N_f is a common normalization factor. Note that all the observables have been written in terms of phase invariant quantities given in (14). Terms in (19) and (20) that do not involve λ_f are the ones that occur without any P–$\overline{P}$ oscillations, those involving λ_f are associated with decays following an effective oscillation.

In literature one often finds the parameters x and y, defined such that

$$x \equiv \frac{\Delta m}{\Gamma} \; , \quad y \equiv \frac{\Delta\Gamma}{2\Gamma} \; . \tag{21}$$

Problem 3. *Calculate* $\frac{d\Gamma}{dt}[P(t) \to \overline{f}]$ *and* $\frac{d\Gamma}{dt}[\overline{P}(t) \to \overline{f}]$, *especially keeping track of the signs of various terms.*

2.3.1 Tagged and untagged decays

Since there are many states that both P and $\overline{P}$ can decay to, it is often not possible to deduce if the decaying particle was a P or a $\overline{P}$. One can get around this obstacle partly if one looks at P and $\overline{P}$ that are produced coherently from the decay of a resonance (e.g. $\phi \to K\overline{K}, \Upsilon(4s) \to B\overline{B}$). In such a case, if one of these particles decays through a flavor-specific mode (i.e. mode which allows us to identify a P or a $\overline{P}$) at time t, one knows the identity of the other particle at that time t, and its time evolution can then be studied. This process is called "tagging".

More generally, one may consider the "double time evolution" of the coherently produced P and $\overline{P}$ that decay to the final state $f_1 f_2$. One gets

$$\begin{aligned} \frac{\frac{d\Gamma}{dt}(P\overline{P} \to f_1 f_2)}{e^{-\Gamma|\Delta t|} N_{f_1 f_2}} &= (|a_+|^2 + |a_-|^2)\cosh(\Delta\Gamma t/2)(|a_+|^2 - |a_-|^2)\cos(\Delta m t) \\ &- 2\mathrm{Re}(a_+^* a_-)\sinh(\Delta\Gamma t/2) + 2\mathrm{Im}(a_+^* a_-)\sin(\Delta m t) \; , \end{aligned} \tag{22}$$

where

$$\begin{aligned} a_+ &\equiv \overline{A}_{f_1} A_{f_2} - A_{f_1} \overline{A}_{f_2} \\ a_- &\equiv \frac{p}{q} A_{f_1} A_{f_2} - \frac{q}{p} \overline{A}_{f_1} \overline{A}_{f_2} \end{aligned} \tag{23}$$

Problem 4. *Prove the above "double probability distribution". Show that in the limits* $\{A_{f_1} = 0, \overline{A}_{f_1} = 1\}$ *and* $\{\overline{A}_{f_1} = 0, A_{f_1} = 1\}$, *it reduces to the time evolution in (19) and (20).*

Problem 5. *Untagged decays are those where the identity of the decaying particle,* P *or* $\overline{P}$, *is unknown. Calculate the untagged time evolution,* $\frac{1}{2}\left(\frac{d\Gamma}{dt}[P(t) \to f] + \frac{d\Gamma}{dt}[\overline{P}(t) \to f]\right)$, *when* $|p| = |q|$.

2.4 Types of CP violation

The three phase invariant quantities given in (14) can be used to classify CP violation into three types.

2.4.1 CP violation in decay only

In charged meson decays, no mixing is involved. In that case, CP is violated iff

$$|\overline{A}_{\overline{f}}/A_f| \neq 1\ . \tag{24}$$

In such a situation, an observable CP violating quantity is

$$\mathcal{A}_{f^\pm} \equiv \frac{\Gamma(P^- \to f^-) - \Gamma(P^+ \to f^+)}{\Gamma(P^- \to f^-) + \Gamma(P^+ \to f^+)} = \frac{|\overline{A}_{f^-}/A_{f^+}|^2 - 1}{|\overline{A}_{f^-}/A_{f^+}|^2 + 1}\ . \tag{25}$$

A nonvanishing $A_{f^\pm}$ is often termed "direct" CP violation.

2.4.2 CP violation in mixing only

Even when $|\overline{A}_{\overline{f}}| = |A_f|$, it is possible to have observable CP violation in neutral P decays if

$$|q/p| \neq 1\ . \tag{26}$$

An example of such a process would be semileptonic decays, where $P \to \ell^+ X$ and $\overline{P} \to \ell^- X$ have the same amplitude, while the amplitudes of $P \to \ell^- X$ and $\overline{P} \to \ell^+ X$ vanish. If one observes the "wrong sign" leptons, one can measure the CP asymmetry

$$\mathcal{A}_{SL}(t) \equiv \frac{\frac{d\Gamma}{dt}(\overline{P}(t) \to \ell^+ X) - \frac{d\Gamma}{dt}(P(t) \to \ell^- X)}{\frac{d\Gamma}{dt}(\overline{P}(t) \to \ell^+ X) + \frac{d\Gamma}{dt}(P(t) \to \ell^- X)} = \frac{1 - |q/p|^4}{1 + |q/p|^4}\ . \tag{27}$$

Note that, although this asymmetry is defined as a function of time, it turns out to be a constant.

2.4.3 CP violation through mixing-decay interference

This is the CP violation that is governed by the complex quantity λ_f. Consider a final state f_{CP} that both P and $\overline{P}$ decay to. One can then define a CP violating observable

$$\mathcal{A}_{f_{CP}}(t) \equiv \frac{\frac{d\Gamma}{dt}(\overline{P}(t) \to f_{CP}) - \frac{d\Gamma}{dt}(P(t) \to f_{CP})}{\frac{d\Gamma}{dt}(\overline{P}(t) \to f_{CP}) + \frac{d\Gamma}{dt}(P(t) \to f_{CP})}\ . \tag{28}$$

If we neglect the lifetime difference $\Delta\Gamma$, and also take $|q/p| = 1$ to start with (both of which are good approximations for the B_d system), eqs. (19) and (20) give

$$\mathcal{A}_{f_{CP}}(t) = S_{f_{CP}} \sin(\Delta m t) + C_{f_{CP}} \cos(\Delta m t) \; , \tag{29}$$

where

$$S_{f_{CP}} \equiv \frac{2\mathrm{Im}(\lambda_{f_{CP}})}{1 + |\lambda_{f_{CP}}|^2} \; , \qquad C_{f_{CP}} \equiv \frac{1 - |\lambda_{f_{CP}}|^2}{1 + |\lambda_{f_{CP}}|^2} \; . \tag{30}$$

Even when $|\lambda_{f_{CP}}| = 1$, so that there is no CP violation in decay alone ($|\overline{A}_{f_{CP}}| = |A_{f_{CP}}|$), neither is there CP violation in mixing alone ($|q| = |p|$), there can still be CP violation through their interference, as long as

$$\mathrm{Im}(\lambda_{f_{CP}}) \neq 0 \; . \tag{31}$$

We shall return to specific examples of processes involving the above three types of CP violation after we introduce the Cabibbo-Kobayashi-Maskawa mechanism in the next section.

3 The CKM paradigm

An extension of the Cabibbo mechanism discussed in Sec. 1.1.1, the Kobayashi-Maskawa mechanism not only parametrizes the quark mixing in three generations, it also shows that CP violation in three generations is a natural consequence of quark mixing. Furthermore, it goes on to predict that all the observed CP violation can be explained by a single source. This prediction has been borne out by all the experiments till now, and is a major success of the SM.

3.1 Origin of the CKM matrix

Let $U' \equiv (u \; c \; t)^T$ repesent the column vector consisting of three up-type quarks, and $D' \equiv (d \; s \; b)^T$, the column vector consisting of the three down-type quarks. The charged current part of the SM Lagrangian, in the basis of flavor eigenstates, is

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \overline{U'_L} \gamma^\mu D'_L W^+_\mu + h.c. \; , \tag{32}$$

where the subscript L represents left chiral component of the quark spinors. Note that the charged current interactions are diagonal in the flavor basis by definition.

The part of the Lagrangian giving mass to the quarks is, in the flavor basis,

$$\mathcal{L}_{\rm mass} = \overline{U'_L} M'_U U'_R + \overline{D'_L} M'_D D'_R + H.c. \ , \tag{33}$$

where the mass matrices M'_U and M'_D are 3×3 matrices that need not be diagonal (and indeed, are not) in this flavor basis. Let the mass eigenstate basis, where the mass matrices become diagonal, be given by U_L, D_L, U_R, D_R such that

$$\begin{aligned} U'_L &= V_{UL} U_L \ , & D'_L &= V_{DL} D_L \ , \\ U'_R &= V_{UR} U_R \ , & D'_R &= V_{DR} D_R \ . \end{aligned} \tag{34}$$

Here, $V_{UL}, V_{DL}, V_{UR}, V_{DR}$ are unitary matrices. In the mass eigenstate basis, the mass part of the Lagrangian becomes

$$\begin{aligned} \mathcal{L}_{\rm mass} &= \overline{U_L} V^\dagger_{UL} M'_U V_{UR} U_R + \overline{D_L} V^\dagger_{DL} M'_D V_{DR} D_R + H.c. \ , \\ &= \overline{U_L} M_U^{diag} U_R + \overline{D_L} M_D^{diag} D_R + H.c. \ , \end{aligned} \tag{35}$$

where

$$\begin{aligned} M_U^{diag} &\equiv V^\dagger_{UL} M'_U V_{UR} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \ , \\ M_D^{diag} &\equiv V^\dagger_{DL} M'_D V_{DR} = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} \ . \end{aligned} \tag{36}$$

The elements of M_U^{diag} and M_D^{diag}, which are the quark masses m_q, are in general complex numbers.

The charged current Lagrangian in the mass basis is

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \overline{U_L} V^\dagger_{UL} \gamma^\mu V_{DL} D_L W^+_\mu + H.c. = \frac{g}{\sqrt{2}} \overline{U_L} \gamma^\mu (V^\dagger_{UL} V_{DL}) D_L W^+_\mu + H.c. \ . \tag{37}$$

Thus in the mass basis, the up-type quarks U_L couple to $(V_{UL}^\dagger V_{DL})D_L$ with the standard weak coupling strength $g/\sqrt{2}$. Therefore, the coupling between the mass eigenstates U_L and D_L is given by $(g/\sqrt{2})V_{CKM}$, where

$$V_{CKM} \equiv V_{UL}^\dagger V_{DL} \; . \tag{38}$$

V_{CKM} is the Cabibbo-Kobayashi-Maskawa (CKM) matrix which characterizes mixing in the quark sector. This is a unitary matrix, since it is formed by a multiplication of two unitary matrices. Note that V_{UR} and V_{DR} do not play any role here.

The elements of the CKM matrix are named in terms of the quarks which they connect:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \; . \tag{39}$$

3.1.1 Parameter counting

The CKM matrix is a complex matrix, so in general it can be paramerized by 9 real and 9 imaginary quantities. The unitarity of the matrix ($V_{CKM}^\dagger V_{CKM} = 1$) provides constraints on 6 real and 3 imaginary quantities, leaving us with 3 real and 6 imaginary ones. We still have the freedom of changing the phase of each of the six quarks individually, however a common phase change for all quarks will not affect V_{CKM}. The remaining 5 phases that affect V_{CKM} should be unphysical, so that only 1 imaginary and 3 real quantities are required to describe the complete physics incorporated in V_{CKM}.

Recall that any rotation in 3 dimesions may be described in terms of three real parameters, the three Euler angles. The rotation required in the 3 dimensional flavor space here thus involves an additional complex component which is parametrized by the imaginary "phase". This is the single complex phase that is responsible for all the CP violation, according to the CKM mechanism.

Problem 6. *For n generations of quarks, calculate the number of real and imaginary quantities required to determine the physics of the quark mixing matrix. Hence, show that 3 is the minimum number of generations for which a complex mixing matrix may be obtained.*

3.2 Parametrization of CKM elements

The general expression for the CKM matrix, showing the complete dependence on the three angles $\theta_{12}, \theta_{23}, \theta_{13}$ and one phase δ_{13} may be written as

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta 13} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta 13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta 13} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta 13} & c_{23}c_{13} \end{pmatrix} , \tag{40}$$

where $s_{ij} \equiv \sin\theta_{ij}$ and $c_{ij} \equiv \cos\theta_{ij}$. Although this form in principle would suffice for all the analysis, a more convenient parametrization is obtained when we use the experimental observation that the angle θ_{12}, which is approximately the same as the Cabibbo angle, is small:

$$\lambda \equiv \sin\theta_{12} \approx 0.2 \; . \tag{41}$$

All the CKM elements may then be written as an expansion in powers of λ.

3.2.1 The parameters λ, A, ρ, η

The magnitudes of some of the CKM elements may be determined through simple tree-level decay modes.

- $|V_{ud}|$: This may be obtained through neutron decay, which is essentially $d \to u\ell^-\bar{\nu}$.
- $|V_{us}|$: This is determined through semileptonic kaon decay, e.g. $K \to \pi\ell\nu$. Cabibbo angle is defined through $\tan\theta_{12} = |V_{us}/V_{ud}|$. The parameter λ is then defined as $\sin\theta_{12}$.
- $|V_{cs}|$: The "strange" decays of "charmed" mesons, $D \to K\ell\nu$, lead to the measurement of $|V_{cs}|$, which is found to be very close to $|V_{ud}|$. From (40), this leads us to conclude that $s_{23}s_{13}$ is extremely small.
- $|V_{cd}|$: The decays $D \to \pi\ell\nu$ yield this quantity, which is found to be extremely close to $|V_{us}|$. Since $s_{23}s_{13}$ is small, we get $V_{cd} = -V_{us}$.

Since $s_{23}s_{13}$ is extremely small, the 2×2 submatrix consisting of the above four elements is almost unitary. In terms of λ, this submatrix is

$$V_{CKM(2\times2)} = \begin{pmatrix} 1-\lambda^2/2 & \lambda \\ -\lambda & 1-\lambda^2/2 \end{pmatrix} + \mathcal{O}(\lambda^4) \; . \tag{42}$$

The phase convention is chosen so as to make V_{ud}, V_{us} and V_{cs} real and positive. The realness of V_{cs} is the consequence of "almost unitarity" of this submatrix. The matrix in (42) is identical to the mixing matrix proposed by Glashow-Iliopoulos-Maiani that had only Cabibbo angle.

The semileptonic decays of B mesons lead us to the measurements of the next two magnitudes:

- $|V_{cb}|$: The decay $B \to D\ell\nu$ determines $|V_{cb}|$. The phase convention is chosen to make V_{cb} real and positive, and its magnitude is defined to be $A\lambda^2$. Experiments imply $A \approx 0.8$–$1.0 \sim \mathcal{O}(1)$, justifying the use of quadratic power of λ. The definition of the parameter A is thus

$$A \equiv |V_{cb}|/\lambda^2 \ . \tag{43}$$

- $|V_{ub}|$: This is obtained through the decay $B \to (\pi/\rho)\ell\nu$. This is a difficult mode to measure experimentally, since removing the background coming from $B \to D\ell\nu$ is a daunting task. We shall not go into the details of the measurement. From (40), V_{ub} has to be a complex quantity, and needs two more parameters for its complete description. We define the parameters ρ and η through

$$V_{ub} \equiv A\lambda^3(\rho - i\eta) \ . \tag{44}$$

  The observed value of $|V_{ub}| = A\lambda^3\sqrt{\rho^2+\eta^2}$ is consistent with the third power of λ used in the definition, so that $\rho, \eta \sim \mathcal{O}(1)$.

Problem 7. *Calculate the maximum energy that an electron can have, when it is a product of (i)* $b \to ce^-\nu$ *, and (ii)* $b \to ue^-\nu$*. Argue how the difference in these energies can be used to identify a pure sample of* $b \to u$ *decays.*

3.2.2 The Wolfenstein parametrization and beyond

The definitions of λ, A, ρ, η through equations (41,43,44), and the unitarity condition, are enough to parametrize the CKM matrix completely as

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4) \ . \tag{45}$$

This is called the Wolfenstein parametrization, and is the common one in use.

Some of the earlier results can now be cast in terms of this parametrization. Comparison of (40) and (45) gives

$$s_{12} = \lambda \ , \quad s_{23} = A\lambda^2 \ , \quad s_{13} = A\lambda^3 \sqrt{\rho^2 + \eta^2} \ , \tag{46}$$

and the complex-ness of the matrix is represented by the nonvanishing value of η.

The original Wolfenstein parametrization is accurate to $\mathcal{O}(\lambda^4)$. A more precise version is sometimes needed, and is given by

$$\begin{aligned} V \ = & \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3[(1 - (1 - \frac{1}{2}\lambda^2)(\rho + i\eta)] & -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix} \\ & + \mathcal{O}(\lambda^6) \ . \end{aligned} \tag{47}$$

Note that the freedom to choose the 5 relative phases has been used to make the elements $V_{ud}, V_{us}, V_{cs}, V_{cb}$ and V_{tb} real and positive. The constraints quoted on the elements of the CKM matix are often in terms of $\overline{\rho}$ and $\overline{\eta}$, where

$$\overline{\rho} \equiv \rho(1 - \frac{1}{2}\lambda^2) \ , \quad \overline{\eta} \equiv \eta(1 - \frac{1}{2}\lambda^2) \ . \tag{48}$$

3.3 Unitarity triangles

The unitarity of the CKM matrix implies the relation $V^\dagger V = 1$. This can be viewed as conditions on combinations of CKM elements in a complex plane. For example,

$$[V^\dagger V]_{32} = 0 \quad \Rightarrow \qquad V^*_{ub}V_{ud} + V^*_{cb}V_{cd} + V^*_{tb}V_{td} = 0 \ . \tag{49}$$

This relation may be represented as a triangle in the complex plane, whose sides are the three complex quantities $V^*_{ub}V_{ud}$, $V^*_{cb}V_{cd}$ and $V^*_{tb}V_{td}$. This triangle is shown in Fig. 2.

The CKM matrix satisfies three distinct relations of the form $[V^\dagger V]_{ij} = 0$ $(i \neq j)$. These give rise to three different unitarity triangles. The Wolfenstein parametrization (45) can be used to determine some of the features of these triangles.

- The $[V^\dagger V]_{31} = 0$ triangle $V^*_{ub}V_{us} + V^*_{cb}V_{cs} + V^*_{tb}V_{ts} = 0$:
  All sides of this triangle are $\mathcal{O}(\lambda^3)$, hence of comparable lengths, and

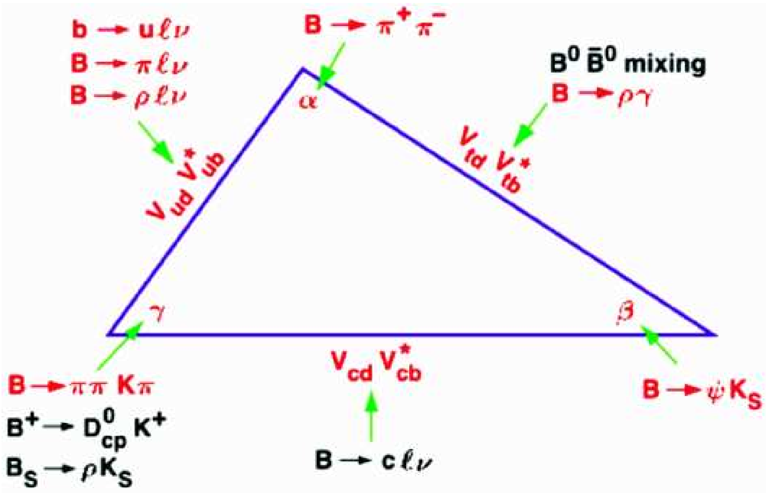

Figure 2: The unitarity triangle obtained from $[V^\dagger V]_{32} = 0$. The B decay modes useful for determining its sides and angles have also been indicated. The figure has been taken from [10].

all the angles are $\mathcal{O}(1)$. This is the "standard" unitarity triangle, whose angles are defined as

$$\alpha \equiv \text{Arg}\left(-\frac{V_{td}V^*_{tb}}{V_{ud}V^*_{ub}}\right) \ , \quad \beta \equiv \text{Arg}\left(-\frac{V_{cd}V^*_{cb}}{V_{td}V^*_{tb}}\right) \ , \quad \gamma \equiv \text{Arg}\left(-\frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}}\right) \ . \tag{50}$$

These angles, shown in Fig. 2, are probed mainly via B_d–$\overline{B}_d$ mixing.

Note that $\tan\alpha \cdot \tan\beta \cdot \tan\gamma = -1$, implying that $\alpha + \beta + \gamma = \pi$ (in radians). This is just an identity. In literature, the angles α, β, γ have also been referred to as ϕ_2, ϕ_1, ϕ_3 respectively.

- The $[V^\dagger V]_{32} = 0$ triangle $V^*_{ub}V_{us} + V^*_{cb}V_{cs} + V^*_{tb}V_{ts} = 0$:
  Two of the sides of this triangle are $\mathcal{O}(\lambda^2)$, and one is $\mathcal{O}(\lambda^4)$. This triangle is thus much flatter than the previous one. The smallest angle of this triangle,

$$\beta_s \equiv \text{Arg}\left(-\frac{V_{cs}V^*_{cb}}{V_{ts}V^*_{tb}}\right) \ , \tag{51}$$

  is relevant in B_s–$\overline{B}_s$ mixing.

- The $[V^\dagger V]_{12} = 0$ triangle $V_{ud}^* V_{us} + V_{cd}^* V_{cs} + V_{td}^* V_{ts} = 0$:
  This triangle is the flattest of them all, two of its sides being $\mathcal{O}(\lambda)$ and the third one $\mathcal{O}(\lambda^5)$. The smallest angle is relevant for K–$\overline{K}$ mixing, and is defined as
  $$\beta_K \equiv \mathrm{Arg}\left(-\frac{V_{cs} V_{cd}^*}{V_{ts} V_{td}^*} \right) . \qquad (52)$$
  The smallness of CP violation observed in K system $[\mathcal{O}(10^{-3}]$ as compared to that in the B_d system ($\mathcal{O}(1)$) can be traced to $\beta_K \ll \beta$.

There are also three unitarity triangles corresponding to the relations $[VV^\dagger]_{31} = 0, [VV^\dagger]_{32} = 0$ and $[VV^\dagger]_{21} = 0$. However, they can be derived from the thee $[V^\dagger V]_{ij} = 0$ relations, and do not offer any extra insight.

Problem 8. *Prove the "unitarity relation" between the angles of two of the unitarity triangles:*

$$\sin \beta_s = \left| \frac{V_{us}}{V_{ud}} \right|^2 \frac{\sin \beta \ \sin(\gamma + \beta_s)}{\sin(\beta + \gamma)} \left[1 + \mathcal{O}(\lambda^4) \right] . \qquad (53)$$

Problem 9. *Determine the angles β_s and β_K to leading power in λ.*

3.3.1 Areas of unitarity triangles

The area of a triangle in the complex plane, two of whose sides are represented by the comples numbers x and y, is given by

$$\mathrm{Area} = \frac{1}{2} |\vec{x} \times \vec{y}| = \frac{1}{2} \mathrm{Im}(x^* y) \qquad (54)$$

Using this result, it is clear, for example, that the area of the triangle in Fig. 2 is $|\mathrm{Im}(V_{cb}^* V_{cs} V_{tb} V_{ts}^*)|/2$.

Using only the unitarity of CKM matrix, it can be shown that the quantity

$$J \equiv |\mathrm{Im}(V_{\alpha i}^* V_{\alpha j} V_{\beta i} V_{\beta j}^*)| \qquad (55)$$

is the same for all sets of $\alpha \neq \beta$ and $i \neq j$. This quantity, called the Jarlskog invariant [11], is twice the area of the corresponding unitarity triangle. Thus, we have the result that all the unitarity triangles have the same area. In terms of the CKM parametrizations we have discussed, the Jarlskog invariant is

$$J = s_{12} s_{23} s_{13} c_{12} c_{23} c_{13}^2 \sin \delta \approx A^2 \lambda^6 \eta . \qquad (56)$$

The Jarlskog invariant is also a "rephase invariant", i.e. it does not depend on the 5 relative phases chosen for the quarks [12].

The area of the unitarity triangles can vanish only if all the CKM matrix elements are real, which corresponds to CP conservation. This area, as will be seen later, is in fact proportional to the CP violation in the corresponding process. The area of the triangles, or the Jarlskog invariant, is thus the single quantity that is responsible for all the CP violation that can be described by the CKM paradigm.

Problem 10. *Prove that J is indeed independent of i, j, α, β, as long as $i \neq j$ and $\alpha \neq \beta$, using only the unitarity relations $[V^\dagger V]_{ij} = 0$ and $[VV^\dagger]_{ij} = 0$.*

4 CP violation and the CKM matrix

In this section, we shall employ the CKM formalism developed so far to explicitly calculate CP violation in various processes, leading to an overdetermination of the CKM matrix elements. This will help us measure them to a good accuracy, at the same time allowing us to test the mechanism.

Let us start with some explicit examples of CP violation through decay, mixing, and their interference.

4.1 CP violation through decay only

Consider the CP-conjugate decays $B^\pm \to K^\pm \pi^0$ with amplitudes $A_\pm$. If the only channel through which this decay occurs were the one shown in Fig. 3, then we would have

$$A_+ = V_{ub}^* V_{us} A_1 \ , \quad A_- = V_{ub} V_{us}^* A_1 \ , \tag{57}$$

where A_1 includes all the hadronic factors as well as phase space factors, which are identical for these two processes. Then, even though the amplitudes A_+ and A_- are different, the net decay rates $\Gamma_\pm$ are identical, and hence no observable CP violation will be present. Indeed, this is always the case for the "decay only" CP violation when there is only one CKM combination involved.

The actual situation for $B^\pm \to K^\pm \pi^0$ is different, since this decay can also proceed through the "penguin" diagram shown in Fig. 4. The amplitudes

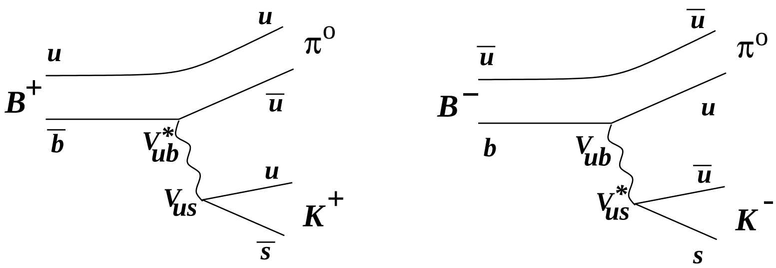

Figure 3: Tree diagrams for $B^{\pm} \to K^{\pm}\pi^0$. The relevant CKM elements are also shown.

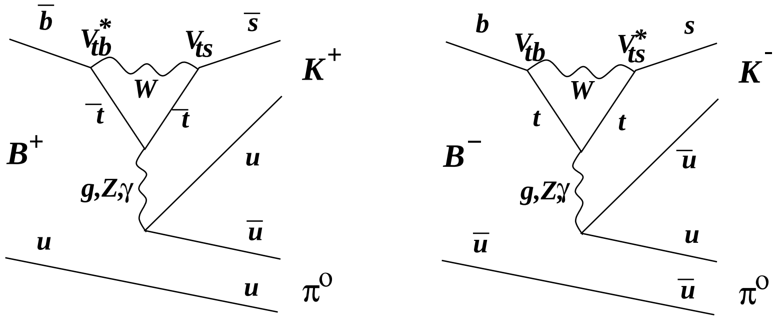

Figure 4: Penguin diagrams for $B^{\pm} \to K^{\pm}\pi^0$, mediated by the top quark. The relevant CKM elements are also shown. The $u\overline{u}$ pair may be produced by a gluon, Z boson or a photon.

$A_{\pm}$ are then a sum of two contributions each:

$$A_+ = V_{ub}^* V_{us} A_1 + V_{tb}^* V_{ts} A_2 \; , \tag{58}$$

$$A_+ = V_{ub} V_{us}^* A_1 + V_{tb} V_{ts}^* A_2 \; , \tag{59}$$

where A_2 includs the hadronic as well as phase space factors for the penguin diagram. Note that we have only taken the penguin diagram mediated by the top quark, since this happens to dominate over the ones with intermediate charmed or up quark. The decay rates for these processes will now be different:

$$\Gamma_+ - \Gamma_- = 4\,\mathrm{Im}(V_{ub}^* V_{us} V_{tb} V_{ts}^*)\,\mathrm{Im}(A_1 A_2^*) \; . \tag{60}$$

Thus, observable CP violation requires that the terms $(V_{ub}^* V_{us} V_{tb} V_{ts}^*)$ as well as $(A_1 A_2^*)$ are not completely real. This result is sometimes also stated as "there must be a weak (CKM) phase difference as well as a strong phase difference". Note that the CKM contribution to the CP violation is indeed proportional to the Jarlskog invariant J.

Problem 11. *Find the leading power of λ present in the "direct" CP asymmetry $\mathcal{A}_{dir} = (\Gamma_+ - \Gamma_-)/(\Gamma_+ - \Gamma_-)$, for (i) $K \to \pi\pi$, (ii) $D \to K\pi$, (iii) $B \to D\pi$. Argue why B decays should typically show more asymmetry than D or K.*

4.2 CP violation through mixing only

For this type of CP violation, we should look for $|q/p| \neq 1$. The experimental measurements give [13]

$$|q/p|_d = 1.0002 \pm 0.0028 \; , \quad |q/p|_s = 1.0015 \pm 0.0051 \qquad (61)$$

for the B_d and B_s system respectively. We are thus far from a nonzero measurement of "CP violation through mixing only" in the B meson systems.

This may be understood theoretically from the expression

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}} \; . \qquad (62)$$

Calculation of the dispersive and absorptive parts of the box diagram (see Fig. 5) yields $\Gamma_{12} \ll M_{12}$ in both, B_d as well as B_s systems. As a result, $|q/p| \approx 1$, and CP violation through mixing only is not observed in B decays. Indeed, the semileptonic asymmetry defined in (27) is found to be

$$\mathcal{A}_{SL}^d = -0.0005 \pm 0.0056 \quad \text{and} \quad \mathcal{A}_{SL}^s = -0.0030 \pm 0.0101 \qquad (63)$$

in B_d and B_s systems respectively

Problem 12. *In the kaon system, where $\Delta m = 3.5 \times 10^{-12}$ MeV and $\tau_{K_S} = 0.9 \times 10^{-10}$ sec, estimate $|q/p|$ and $\mathcal{A}_{SL}$. Compare with the measured value. Is any more information needed ?*

4.3 CP violation through decay-mixing interference

Let us consider the "golden channel" $B_d/\overline{B}_d \to J/\psi K_S$, which has given us a rather clean measurement of β, one of the angles of the unitarity triangle.

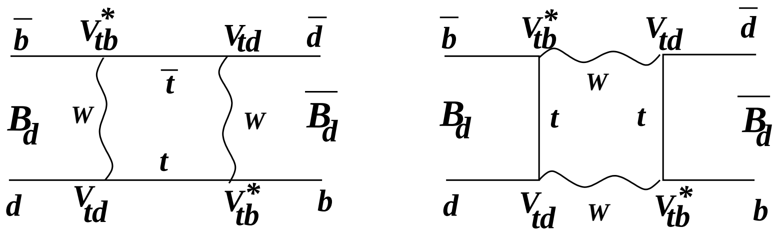

Figure 5: The B_d–$\overline{B}_d$ mixing box diagram intermediated by the top quark. The relevant CKM elements are also shown.

As mentioned in the previous section, $\Gamma_{12} \ll M_{12}$ in the B_d system, so from (62), we get

$$q/p = \exp[-i\,\mathrm{Arg}(M_{12})] \ . \tag{64}$$

Let us use the phase convention where $CP|B_d\rangle = |\overline{B}_d\rangle$. In this convention, from the box diagram in Fig. 5,

$$\mathrm{Arg}(M_{12}) = \mathrm{Arg}(V_{tb}^* V_{td} V_{tb}^* V_{td}) \approx -2\beta \ . \tag{65}$$

As a result, $q/p \approx e^{2i\beta}$.

The dominant contribution to A_f and $\overline{A}_f$ is from the tree diagram shown in Fig. 6. [1] This tree amplitude is proportional to the CKM combination $V_{cb}V_{cs}^*$ ($V_{cb}^*V_{cs}$) for B_d ($\overline{B}_d$) decay.

Thus we get

$$\frac{\overline{A}_f}{A_f} \approx \frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs})} \approx 1 \ . \tag{66}$$

Consequently,

$$\lambda_f = \frac{q}{p}\frac{\overline{A}_f}{A_f} = e^{2i\beta} \ . \tag{67}$$

In the B_d system, the lifetime difference $\Delta\Gamma$ is expected to be extremely small: in the SM, $\Delta\Gamma/\Gamma \approx 0.5\%$. Then we can use (19) and (20) to get the

[1] The penguin contribution is suppressed since it involves the production of a $c\bar{c}$ pair. Moreover, even among the penguin processes, the one intermediated by the top quark dominates, and the CKM phase it provides is almost identical to the tree diagram CKM phase, since from the Wolfenstein parametrization, it can be seen that $V_{tb}V_{ts}^* = V_{cb}V_{cs}^* + \mathcal{O}(\lambda^4)$.

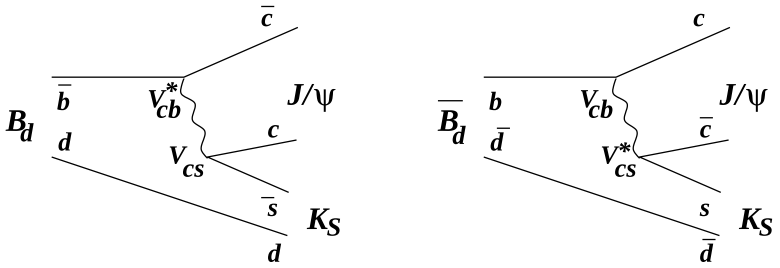

Figure 6: Feynman diagrams for $A_f \equiv A(B_d \to J/\psi K_S)$ and $\overline{A}_f \equiv A(\overline{B}_d \to J/\psi K_S)$

asymmetry

$$\mathcal{A}_{J/\psi K_S} \equiv \frac{\frac{d\Gamma}{dt}(\overline{B}_d(t) \to J/\psi K_S) - \frac{d\Gamma}{dt}(B_d(t) \to J/\psi K_S)}{\frac{d\Gamma}{dt}(\overline{B}_d(t) \to J/\psi K_S) + \frac{d\Gamma}{dt}(B_d(t) \to J/\psi K_S)} \approx \sin 2\beta \sin(\Delta m t) \ . \tag{68}$$

The observation of this asymmetry thus gives a direct measurement of the phase β, which testifies to the presence of CP violation through the interference of decay and mixing.

Problem 13. *Determine the time dependent CP asymmetry in $B_s/\overline{B}_s \to J/\psi\phi$.*

4.4 Constraining the unitarity triangle

The bulk of the tests carried out so far have can be expressed in terms of the standard unitarity triangle. When all sides of this triangle are divided by $A\lambda^3$, it becomes a triangle all of whose sides are $\mathcal{O}(1)$. Its vertices are at the points $(0,0), (1,0), (\overline{\rho}, \overline{\eta})$.

Various experiments constrain different combinations of ρ and η:

- $|V_{ub}/V_{cb}|^2$ gives $\overline{\rho}^2 + \overline{\eta}^2$, as shown in Sec. 3.2.
- In the ratio of mass differences in B_d and B_s systems, many common factors (like QCD corrections, dependence on top quark mass, etc.) cancel out, and one obtains

$$\frac{\Delta m_d}{\Delta m_s} = \frac{M_{B_d}}{M_{B_s}} \frac{\hat{B}_{B_d} f_{B_d}^2}{\hat{B}_{B_s} f_{B_s}^2} \left| \frac{V_{td}}{V_{ts}} \right|^2 , \tag{69}$$

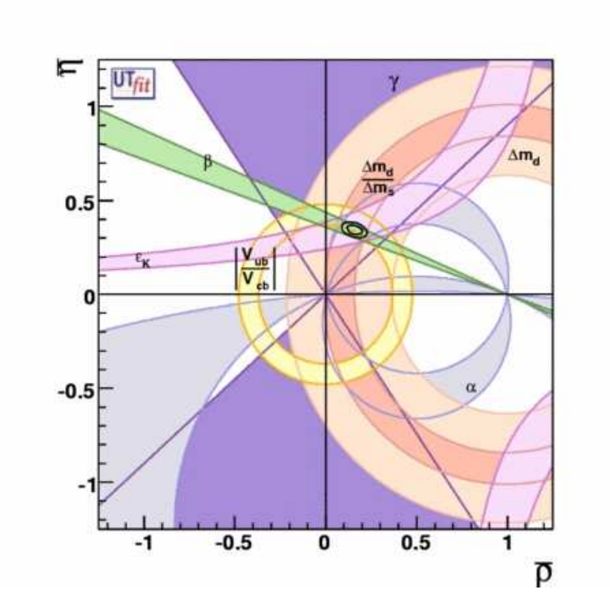

Figure 7: Constraints on the unitarity triangle in the $\rho - \eta$ plane [14]

where f_{B_q} are the decay constants and $\hat{B}_{B_q}$ take care of nonperturbative corrections. These quantities can be reasonably well calculated using lattice methods, and hence the measurement of the ratio (69) gives information on $|V_{td}/V_{ts}|^2$, or equivalently, on $(1-\overline{\rho})^2 + \overline{\eta}^2$.

- As seen in Sec. 4.3, the time dependent asymmetry observed in $B_d/\overline{B}_d \to J/\psi K_S$ yields the angle β, equivalently the combination $\tan\beta = \overline{\eta}/(1-\overline{\rho})$.

- In addition, the measurements of ϵ parameter in the kaon system, various measurements of the unitarity angles α, γ, all conspire to overconstrain the values of $\overline{\rho}$ and $\overline{\eta}$. (For details, see [14]).

It is remarkable that, even with rather accurate measurements of some of the above quantities, all the current constraints overlap in a small region in the $\overline{\rho} - \overline{\eta}$ plane (see Fig. 7). This is a strong evidence that the CKM paradigm is working well and perhaps its strong claim to a single source of CP violation in the quark sector is valid.

5 Concluding remarks

Since the CKM mechanism makes such strong predictions about CP violation observed in the quark sector, it lends itself amenable to testing from various angles. The fact that it has passed all the tests so far indicates that the sources of CP violation apart from the CKM matrix are likely to be small. However, with more and more accurate data expected from the B factories and the LHC, perhaps the limits of the CKM mechanism will be reached and we shall obtain perhaps the first evidence for physics beyond the SM in the quark sector.

Crucial in this context are two kinds of processes.

- Those that can be predicted very accurately within the SM, so that a deviation from this prediction is a robust signature of new physics. These include the consistency checks of the CKM matrix elements, as well as channels like the radiative decay $b \to s\gamma$ which has already constrained new physics to a great degree.

- Processes that lead to quantities that vanish or are extremely small in the SM, but can be enhanced by orders of magnitude by new physics. These mainly involve loops in which new particles propagate, but the enhancement can also be obtained from new couplings. Some examples of such processes are (i) CP violation in the B_s–$\overline{B}_s$ system, (ii) lifetime difference in the B_d–$\overline{B}_d$ system, (iii) branching ratio and polarization asymmetry in $B_d/B_s \to \mu^+\mu^-$, (iv) forward-backward asymmetry in $B_d \to K\mu^+\mu^-$.

These lectures have not dwelt much on the calculation of actual decay rates of processes, which is often a daunting task involving subleading QCD corrections and estimations of hadronic matrix elements that are often non-perturbative quantities. One tries to get around this by using symmetry arguments like flavor SU(3) to relate the amplitudes of different decays. Techniques like the QCD-improved factorization or Soft Collinear Effective Theory are also being developed.

This short course was aimed towards those who were being exposed to B physics, as well as to ideas about CP violation, for the first time. It is hoped that these lectures will give them a basic understanding of these topics and motivation to pursue them.

Acknowledgements

I would like to thank the SERC school organisers for inviting me to give these lectures, and later putting enough pressure on me to write these notes down. I would also like to thank S. Uma Sankar, who was the Guest Lecturer for this course, for his invaluable help right from the planning stage, and Ashutosh Kumar Alok for a careful reading of these lecture notes.

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