Neutrino oscillations and supernovae

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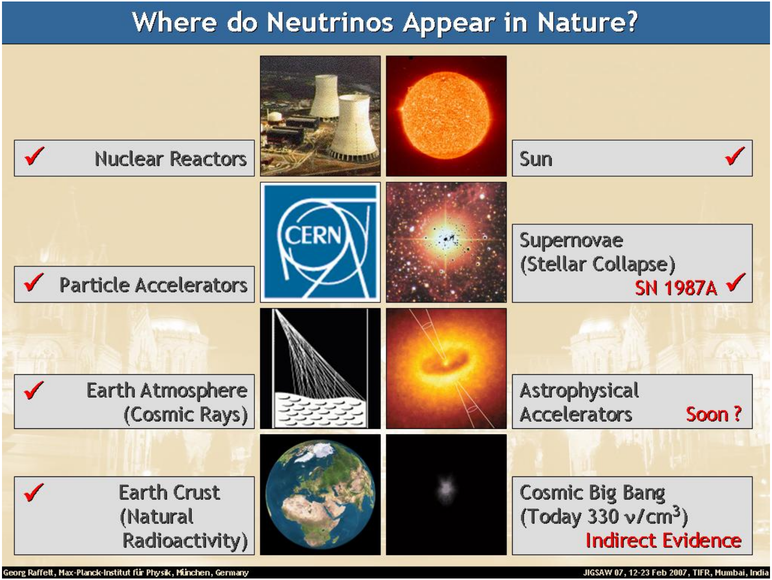
Outline

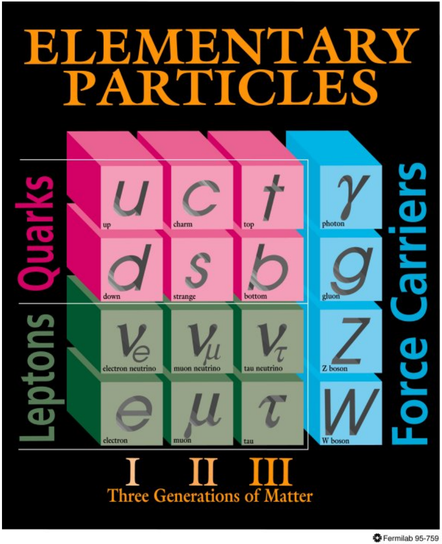
1. Current understanding of neutrinos
   - Various facets of neutrinos
   - Atmospheric neutrino puzzle
   - Solar neutrino puzzle
   - What we know, what we do not

2. Core collapse supernova
   - Collapse, explosion and neutrino emission
   - Neutrino propagation and flavor conversions
   - Observable signals at the detectors

The most abundant particles in the universe



The Standard Model of Particle Physics

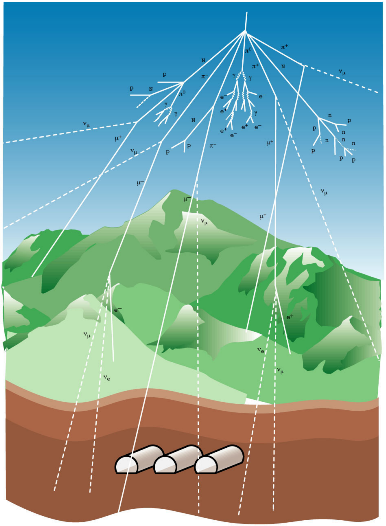


- 3 neutrinos:

  ν_e, ν_μ, ν_τ
- chargeless
- spin 1/2
- almost massless
- Only weak interactions

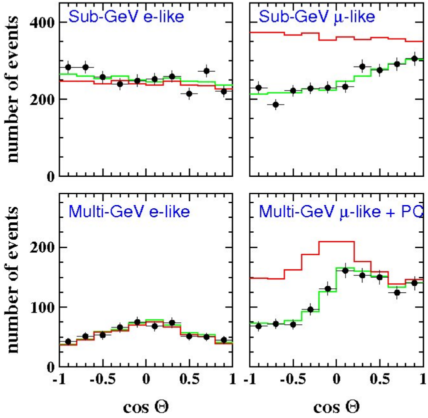
Outline

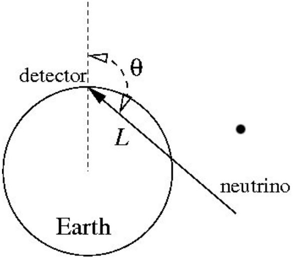
Neutrinos from cosmic rays



- $\pi^+ \rightarrow \mu^+ + \nu_\mu$
- $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$
- "ν_μ" flux $= 2\times$ "ν_e" flux
- "Down" flux = "Up" flux

Zenith angle dependence





Super-Kamiokande

Missing muon neutrinos !

Solution through "vacuum oscillations"

- Effective Hamiltonian:

$$H = \frac{1}{2E}\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

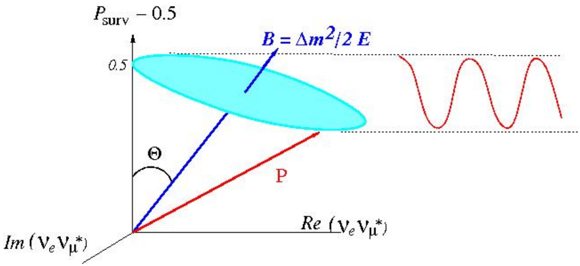
$$= \frac{m_2^2+m_1^2}{2E} + \frac{1}{4E}\begin{pmatrix} -\Delta m^2\cos 2\theta & \Delta m^2 \sin 2\theta \\ \Delta m^2 \sin 2\theta & \Delta m^2 \cos 2\theta \end{pmatrix}$$

- Eigenvalues: $\frac{m_1^2}{2E}, \frac{m_2^2}{2E}$
- Survival probability

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

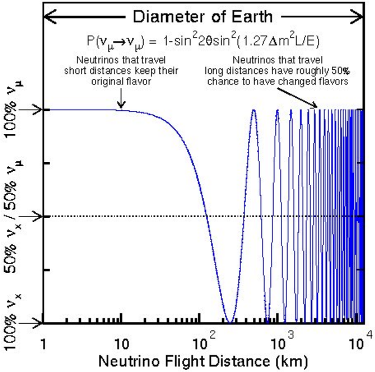
$$\Delta m^2 \equiv m_2^2 - m_1^2$$

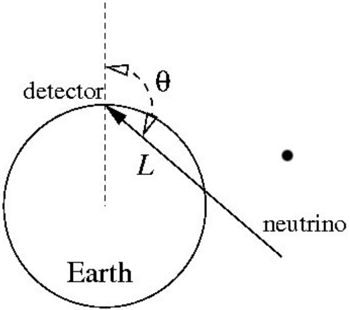
Precession of the polarization vector



- Density matrix $\rho = P_0/2 + \vec{P} \cdot \vec{\sigma}$
- Half-angle of precession $= \theta =$ mixing angle
- Different energies: same cone, different precession speeds

Solution of the atmospheric neutrino puzzle





- ν_μ oscillate into ν_τ
- ν_e do not participate

$$\Delta m^2_{\rm atm} \approx (1.3\text{–}3.4) \times 10^{-3}\ \text{eV}^2$$

Mixing angle $\theta_{\rm atm} \approx 36^\circ\text{–}54^\circ$

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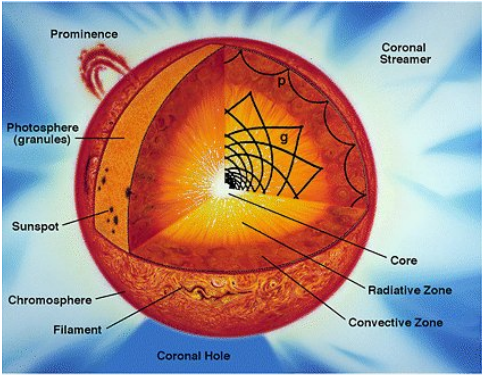
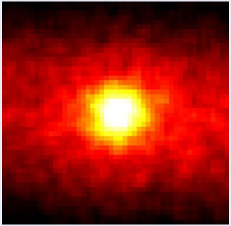
How the Sun shines



- Nuclear fusion reactions: mainly $4\,{}^1_1\mathrm{H} \rightarrow {}^4_2\,\mathrm{He} + 2e^+ + 2\nu_e$
- Light cannot be produced unless neutrinos are produced !!
- Davis-Koshiba Nobel prize 2002

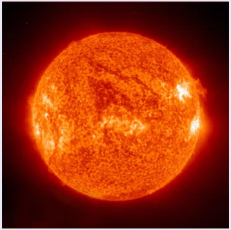
Sun: now and then

Sun in neutrinos: 8 minutes ago



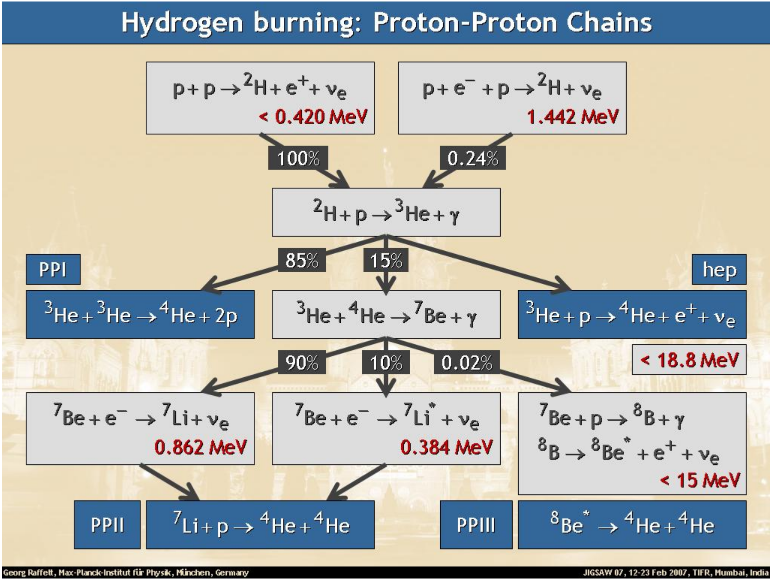
Angular size $\sim 20^\circ$

Sun in photons: a few million years ago

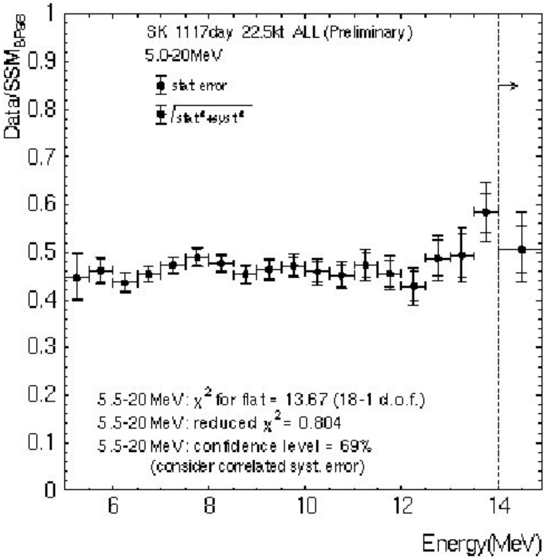


Angular size $\sim 1^\circ$

Nuclear reactions inside the Sun



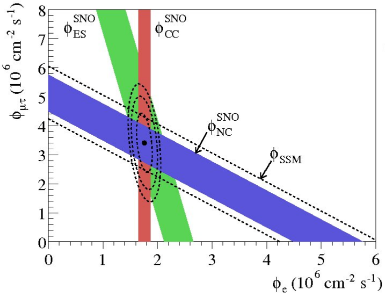
Mystery of missing solar neutrinos



Where did the missing neutrinos go ?

Super-Kamiokande

Solar ν_e convert to ν_μ and ν_τ



- $\nu_e\, D \to p\, p\, e^-$
- $\nu_{e,\mu,\tau}\, e^- \to \nu_{e,\mu,\tau}\, e^-$
- $\nu_{e,\mu,\tau}\, D \to n\, p\, \nu_{e,\mu,\tau}$

SNO

- ν_e oscillate into ν_μ and ν_τ

2-ν level crossing: MSW resonance

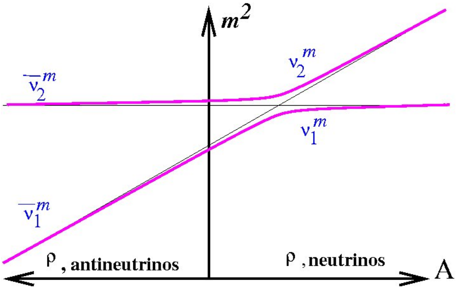
- Effective Hamiltonian:

$$H = \frac{1}{4E}\begin{pmatrix} -\Delta m^2\cos 2\theta + 2A & \Delta m^2 \sin 2\theta \\ \Delta m^2 \sin 2\theta & \Delta m^2 \cos 2\theta \end{pmatrix}$$

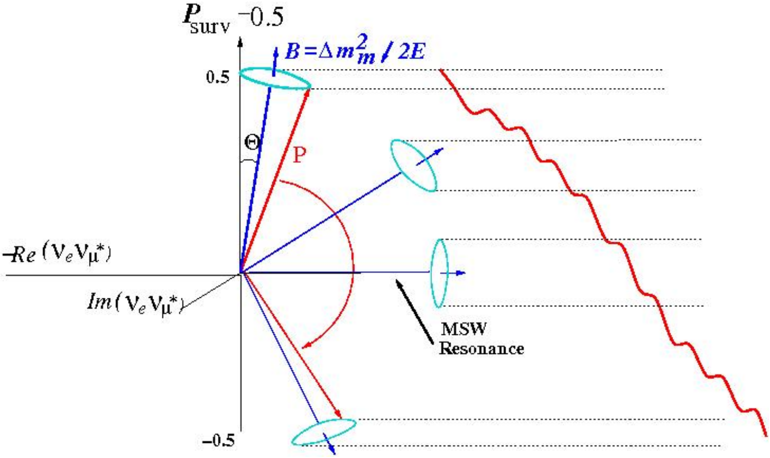
$$(A = 2\sqrt{2}G_F N_e E)$$

- Eigenvalues:

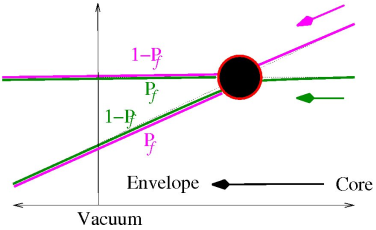
$$\frac{m_i^2}{2E} = \frac{1}{2E}\left[\frac{A}{2} \mp \sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}\right]$$



Precession picture of MSW resonance



Solution of solar neutrino puzzle



P_f depends on: Δm^2, mixing angle $\theta_\odot$, density profile

- Survival probability:

$$P(\nu_e \to \nu_e) \approx P_f \cos^2\theta_\odot + (1 - P_f)\sin^2\theta_\odot$$

- No oscillations ! (Mass eigenstates have decohered)

$\Delta m^2_\odot \approx (7.2\text{–}9.5) \times 10^{-5}\ \text{eV}^2$
Mixing angle $\theta_\odot \approx 28^\circ\text{–}36^\circ$

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Our current knowledge about neutrinos

- ν_e, ν_μ, ν_τ mix among each other
- Atmospheric neutrinos:
  $\Delta m^2_{\rm atm} \approx 2 \times 10^{-3}$ eV^2, $\theta_{\rm atm} \approx 45°$
- Solar neutrinos:
  $\Delta m^2_{\odot} \approx 8 \times 10^{-5}$ eV^2, $\theta_{\odot} \approx 32°$
- Reactor neutrinos:
  the "third" angle: very small ($\theta_{13} < 12°$, may even be zero).

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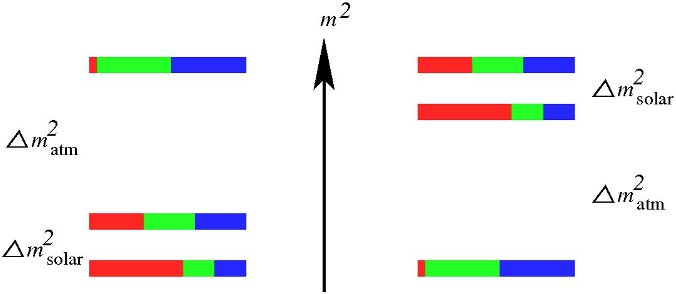
Open questions in neutrino physics

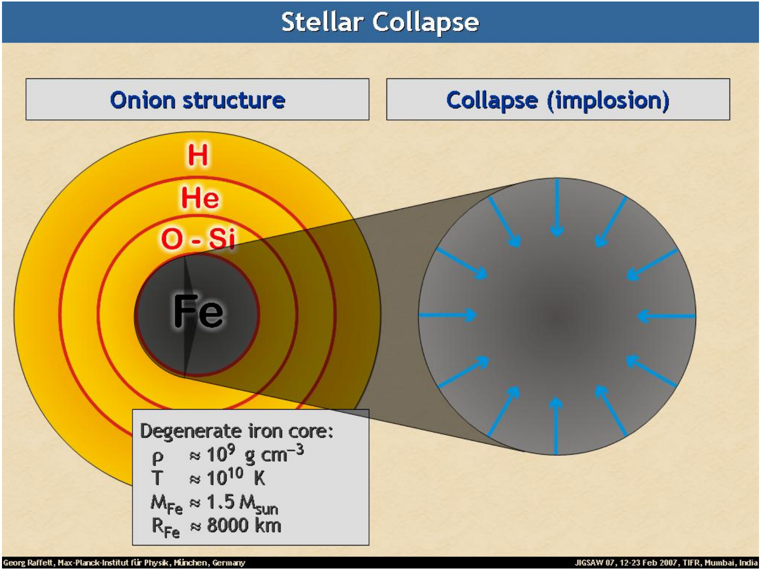
- Mass hierarchy: Normal or Inverted ?
  (red ν_e, green ν_μ, blue ν_τ)



- Absolute neutrino masses
- Are there more than 3 neutrinos ?
- CP violation ? own antiparticles ? ...

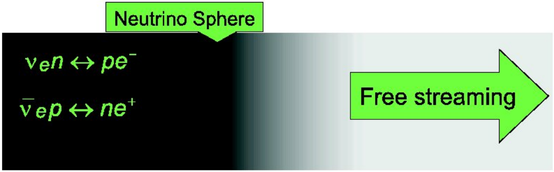
Outline

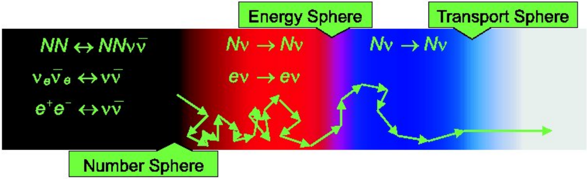
The onion ring structure



Trapped neutrinos before the collapse

- Neutrinos trapped inside "neutrinospheres" around $\rho \sim 10^{10}$g/cc.

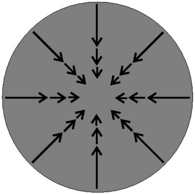


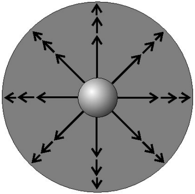


- Escaping neutrinos: $\langle E_{\nu_e} \rangle < \langle E_{\bar{\nu}_e} \rangle < \langle E_{\nu_x} \rangle$

Core collapse, shock wave, and explosion

Gravitational core collapse $\Rightarrow$ Shock Wave





Neutronization burst:

ν_e emitted for $\sim$ 10 ms

Cooling through neutrino emission: $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$

Duration: About 10 sec

Emission of 99% of the SN energy in neutrinos

¿¿¿ *Explosion* ???

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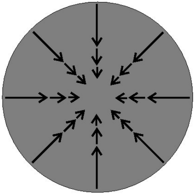

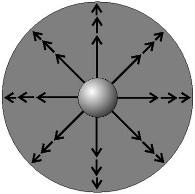

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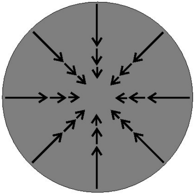

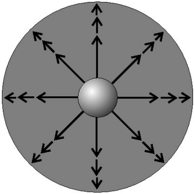

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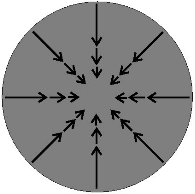

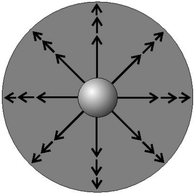

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Cooling through neutrino emission: $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$

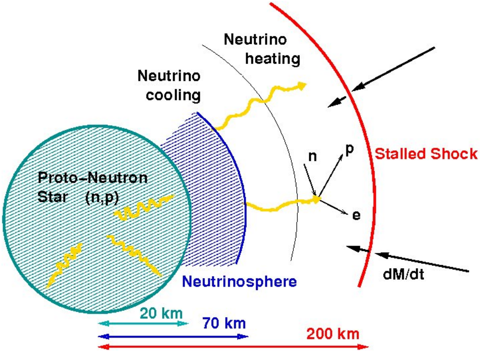
Duration: About 10 sec

Emission of 99% of the SN energy in neutrinos

¿¿¿ *Explosion* ???

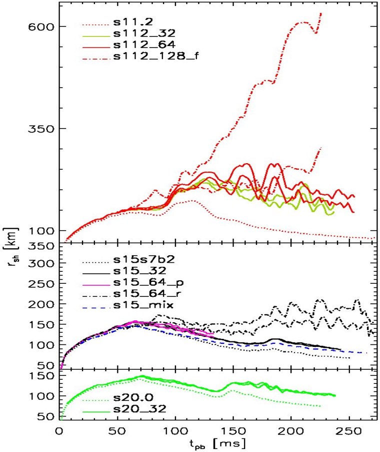
Role of neutrinos in explosion

Neutrino heating needed for pushing the shock wave



- Neutrino heating essential, but not enough
- No spherically symmetric (1-D) simulations show robust explosions

Ingredients required for explosion



- Higher ν opacity
- Stiffer equation of state for the core
- Rotation of the star
- Large scale convection modes

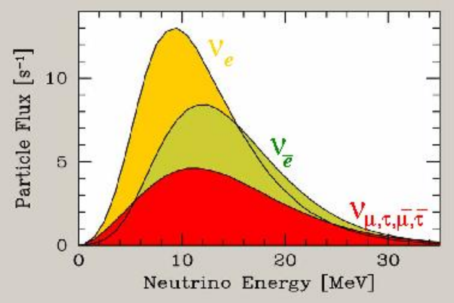
R. Buras, H.-T. Janka, M. Rampp
K. Kifonidis, astro-ph/0512189

An explosion

The explosion movie

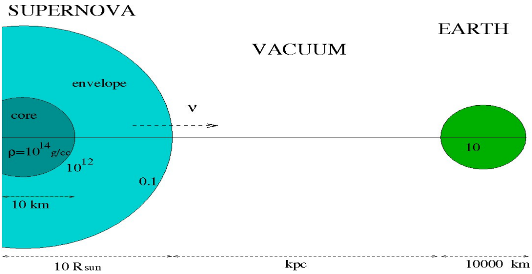
Outline

Primary fluxes and spectra



- Almost blackbody spectra, slightly "pinched"
- Energy hierarchy: $E_0(\nu_e) < E_0(\bar{\nu}_e) < E_0(\nu_x)$
- $E_0(\nu_e) \approx$ 10–12 MeV
  $E_0(\bar{\nu}_e) \approx$ 13–16 MeV
  $E_0(\nu_x) \approx$ 15–25 MeV

Propagation through matter of varying density



Inside the SN: *flavour conversion*

Collective effects and MSW matter effects

Between the SN and Earth: *no flavour conversion*

Mass eigenstates travel independently

Inside the Earth: *flavour conversion*

MSW matter effects (*if detector is on the other side*)

Nonlinear effects due to $\nu - \nu$ coherent interactions

- Large neutrino density $\Rightarrow$ substantial ν–ν potential

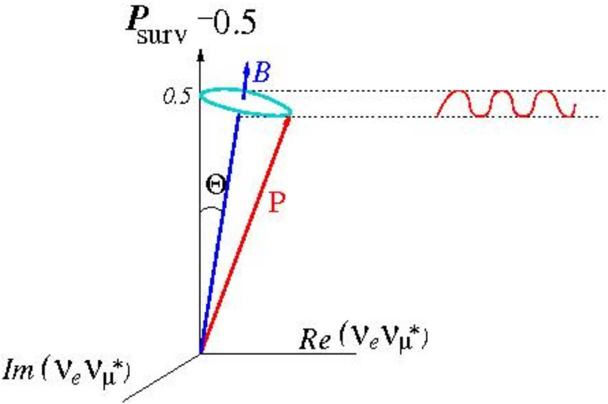
$$H = H_{vac} + H_{MSW} + H_{\nu\nu}$$

$$\begin{aligned} H_{vac}(\vec{p}) &= M^2/(2p) \\ H_{MSW} &= \sqrt{2} G_F n_{e^-} diag(1,0,0) \\ H_{\nu\nu}(\vec{p}) &= \sqrt{2} G_F \int \frac{d^3 q}{(2\pi)^3} (1 - \cos\theta_{pq}) \left(\rho(\vec{q}) - \bar{\rho}(\vec{q})\right) \end{aligned}$$

- $$\frac{d\rho}{dt} = i[H(\rho), \rho] \quad \Rightarrow \qquad \text{Nonlinear effects !}$$

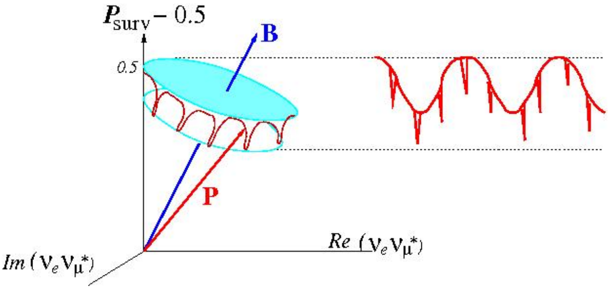
Synchronized osc. $\rightarrow$ Bipolar osc. $\rightarrow$ Spectral split

Synchronized oscillations



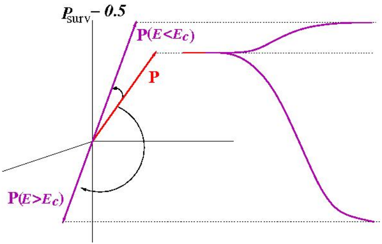
- ν and $\bar{\nu}$ of all energies oscillate with the same frequency
- No significant flavour change since mixing angle is small

Bipolar oscillations

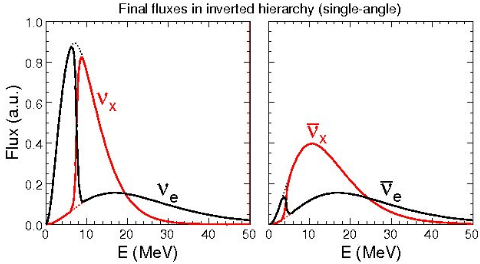


- Coherent $\nu_e \bar{\nu}_e \leftrightarrow \nu_x \bar{\nu}_x$ oscillations
- A nutating top ??
- Take place in inverted hierarchy
- Even $\theta_{13} \lesssim 10^{-10}$ OK !
- Prepare neutrinos for the "spectral split"

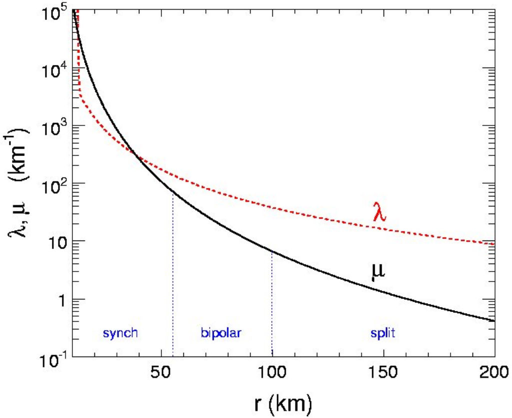
Spectral split



- $\bar{\nu}_e$ and $\bar{\nu}_x$ spectra interchange completely
- ν_e and ν_x spectra interchange for $E > E_c$
- Occurs in inverted hierarchy

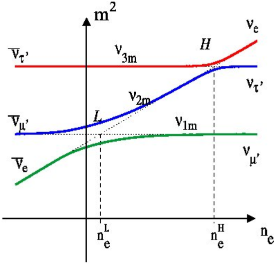


Sequential dominance of processes

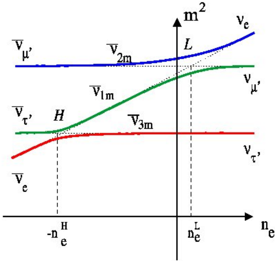


MSW Resonances inside a SN

Normal mass ordering



Inverted mass ordering



H resonance: $(\Delta m^2_{\rm atm}, \theta_{13})$, $\rho \sim 10^3$–10^4 g/cc

- In $\nu(\bar{\nu})$ for normal (inverted) hierarchy
- Adiabatic (non-adiabatic) for $\sin^2\theta_{13} \gtrsim 10^{-3} (\lesssim 10^{-5})$

L resonance: $(\Delta m^2_\odot, \theta_\odot)$, $\rho \sim 10$–100 g/cc

- Always adiabatic, always in ν

Fluxes arriving at the Earth

Mixture of initial fluxes:

$$
\begin{aligned}
F_{\nu_e} &= p\, F^0_{\nu_e} + (1-p)\, F^0_{\nu_x} \,, \\
F_{\bar\nu_e} &= \bar p\, F^0_{\bar\nu_e} + (1-\bar p)\, F^0_{\nu_x} \,, \\
4F_{\nu_x} &= (1-p)\, F^0_{\nu_e} + (1-\bar p)\, F^0_{\bar\nu_e} + (2+p+\bar p)\, F^0_{\nu_x} \,.
\end{aligned}
$$

Survival probabilities in different scenarios:

	Hierarchy	$\sin^2\theta_{13}$	p	$\bar p$
A	Normal	Large	0	$\sin^2\theta_\odot$
B	Inverted	Large	$\cos^2\theta_\odot \mid 0$	$\cos^2\theta_\odot$
C	Normal	Small	$\sin^2\theta_\odot$	$\cos^2\theta_\odot$
D	Inverted	Small	$\cos^2\theta_\odot \mid 0$	0

- "Small": $\sin^2\theta_{13} \lesssim 10^{-5}$, "Large": $\sin^2\theta_{13} \gtrsim 10^{-3}$.
- All four scenarios separable in principle !!

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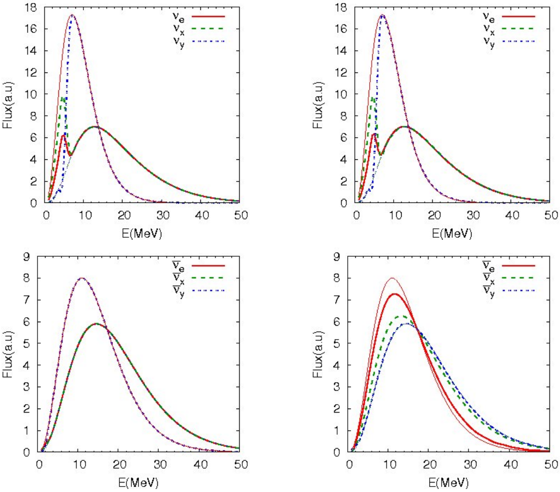
\begin{aligned}
F_{\nu_e} &= p\, F^0_{\nu_e} + (1-p)\, F^0_{\nu_x} \,, \\
F_{\bar{\nu}_e} &= \bar{p}\, F^0_{\bar{\nu}_e} + (1-\bar{p})\, F^0_{\nu_x} \,, \\
4F_{\nu_x} &= (1-p)\, F^0_{\nu_e} + (1-\bar{p})\, F^0_{\bar{\nu}_e} + (2+p+\bar{p})\, F^0_{\nu_x} \,.
\end{aligned}
$$

Survival probabilities in different scenarios:

	Hierarchy	$\sin^2\theta_{13}$	p	$\bar{p}$
A	Normal	Large	0	$\sin^2\theta_\odot$
B	Inverted	Large	$\cos^2\theta_\odot \mid 0$	$\cos^2\theta_\odot$
C	Normal	Small	$\sin^2\theta_\odot$	$\cos^2\theta_\odot$
D	Inverted	Small	$\cos^2\theta_\odot \mid 0$	0

- "Small": $\sin^2\theta_{13} \lesssim 10^{-5}$, "Large": $\sin^2\theta_{13} \gtrsim 10^{-3}$.
- All four scenarios separable in principle !!

Final spectra for inverted hierarchy



Neutrinos

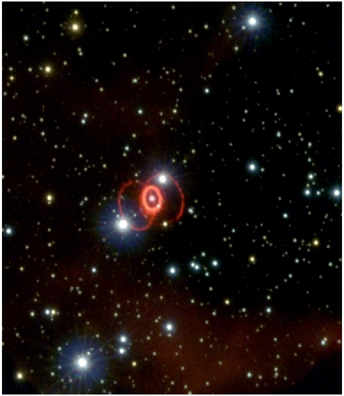
Antineutrinos

Small θ_{13}

Large θ_{13}

Outline

SN1987A



(Hubble image)

- Confirmed the SN cooling mechanism through neutrinos
- Number of events too small to say anything concrete about neutrino mixing
- Some constraints on SN parameters obtained

Signal expected from a galactic SN (10 kpc)

Water Cherenkov detector:

- $\bar{\nu}_e p \to n e^+$: $\approx$ 7000 – 12000*
- $\nu e^- \to \nu e^-$: $\approx$ 200 – 300*
- $\nu_e + ^{16}O \to X + e^-$: $\approx$ 150–800*

* Events expected at Super-Kamiokande with a galactic SN at 10 kpc

Carbon-based scintillation detector:

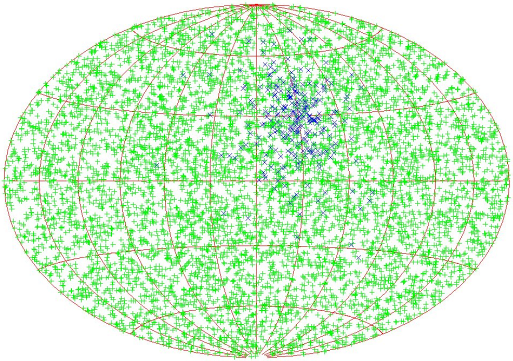
- $\bar{\nu}_e p \to n e^+$
- $\nu + {}^{12}C \to \nu + X + \gamma$ (15.11 MeV)

Liquid Argon detector:

- $\nu_e + {}^{40}Ar \to {}^{40}K^* + e^-$

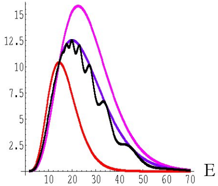
Pointing to the SN in advance

- Neutrinos reach 6-24 hours before the light from SN explosion (SNEWS network)
- $\bar{\nu}_e p \to n e^+$: nearly isotropic background
- $\nu e^- \to \nu e^-$: forward-peaked "signal"
- Background-to-signal ratio: $N_B/N_S \approx$ 30–50
- SN at 10 kpc may be detected within a cone of $\sim 5^\circ$ at SK

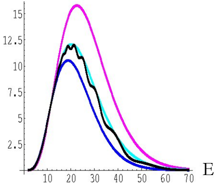


Earth matter effects

Neutrinos



Antineutrinos



(ν_e, ν_x, mixed ν)

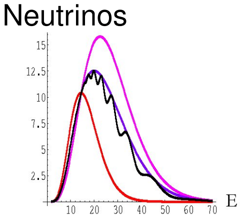
($\bar{\nu}_e$, $\bar{\nu}_x$, mixed $\bar{\nu}$)

- "Earth effect" oscillations

Presence or absence of Earth matter effects:

	Hierarchy	$\sin^2\theta_{13}$	ν_e	$\bar{\nu}_e$
A	Normal	Large	X	$\surd$
B	Inverted	Large	X	$\surd$
C	Normal	Small	$\surd$	$\surd$
D	Inverted	Small	X	X

Earth matter effects



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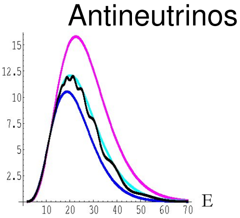

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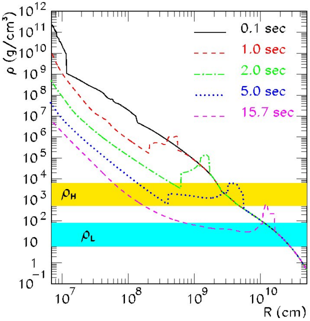
Shock wave and adiabaticity breaking

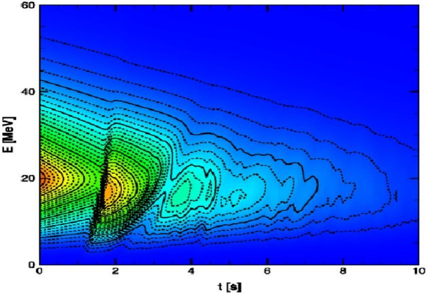
When shock wave passes through a resonance region (density ρ_H or ρ_L):



- adiabatic resonances may become momentarily non-adiabatic
  scenario A $\rightarrow$ scenario C
  scenario B $\rightarrow$ scenario D
- Sharp changes in the final spectra even if the primary spectra change smoothly

Shock wave effects

- Time dependent spectral evolution



Kneller, Mclaughlin, Brockman, PRD77, 045023 (2008)

Presence or absence of shock effects

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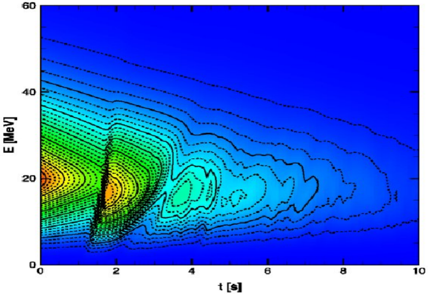

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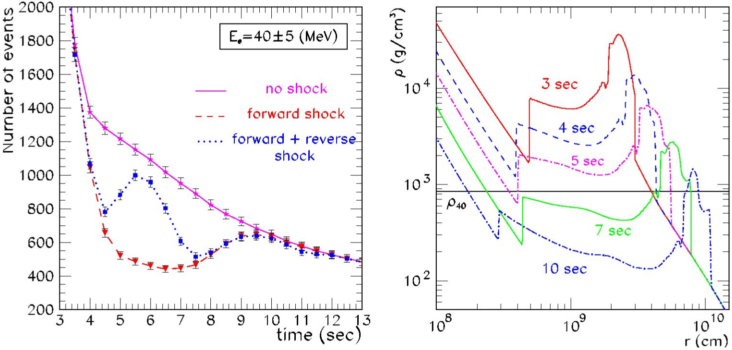
Tracking the shock fronts



- At $t \approx 4.5$ sec, (reverse) shock at ρ_{40}
- At $t \approx 7.5$ sec, (forward) shock at ρ_{40}
- Multiple energy bins $\Rightarrow$ the times the shock fronts reach different densities of $\rho \sim 10^2$–10^4 g/cc

Neutrinocracy in Nova-land

- Explosion of the neutrinos
  - $\mathcal{O}(10^{65})$ neutrinos emitted, $\mathcal{O}(10^{53})$ erg carried away
  - Helps locating the SN before/without the optical signal
- Explosion by the neutrinos
  - Neutrino heating essential for shock propagation
  - Neutrino cooling essential for transporting energy
- Explosion for the neutrinos
  - Identifying the neutrino mixing scenario (A/B/C/D)
  - Tracking the shock wave while still inside the mantle

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Future perspective

Theoretical challenges

- Neutrino transport inside the SN
- Primary neutrino spectra
- Many aspects of the nonlinear effects

Experimental challenges

- Reconstruction of ν_e spectrum (liq Ar detector ?)
- Multiple megaton-class water Cherenkov detectors

A rare event is a lifetime opportunity

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Polarization analogy in two flavours

2-ν flavors : Formalism

- Expand all matrices in terms of Pauli matrices as

$$X = \frac{I}{2} + \frac{1}{2} \sum_{i=1,2,3} X_i \, \sigma_i$$

- The following vectors result from the matrices

$$\rho_{\rm p} \Leftrightarrow \mathbf{P}_\omega$$

$$H_{\rm p}^0 \Leftrightarrow \omega \, \mathbf{B}$$

$$V \Leftrightarrow \sqrt{2} G_F N_e \, \mathbf{L} \equiv \lambda \, \mathbf{L}$$

$$H_{\rm p}^{\nu\nu} \Leftrightarrow \sqrt{2} G_F (n + \bar{n}) \int d\omega \, f(\omega) \, \mathbf{P}_\omega \, \mathrm{sgn}(\omega) \equiv \mu \, \mathbf{D}$$

- EOM resembles spin precession

$$\frac{d}{dr} \mathbf{P}_\omega = (h\omega \, \mathbf{B} + \lambda \, \mathbf{L} + \mu \, \mathbf{D}) \times \mathbf{P}_\omega \equiv \mathbf{H}_\omega \times \mathbf{P}_\omega$$

Polarization analogy in three flavours

3-ν flavors : Formalism

- Expand all matrices in terms of Gell-Mann matrices as

$$X = \frac{I}{3} + \frac{1}{2}\sum_{i=1-8} X_i \, \Lambda_i$$

- The following vectors result from the matrices

$$\rho_{\mathrm{p}} \Leftrightarrow \mathbf{P}_\omega$$

$$H_{\mathrm{p}}^0 \Leftrightarrow \omega\,\mathbf{B}$$

$$V \Leftrightarrow \sqrt{2}G_F N_e \,\mathbf{L} \equiv \lambda\,\mathbf{L}$$

$$H_{\mathrm{p}}^{\nu\nu} \Leftrightarrow \sqrt{2}G_F (n+\bar{n}) \int d\omega\, f(\omega)\,\mathbf{P}_\omega \,\mathrm{sgn}(\omega) \equiv \mu\,\mathbf{D}$$

- EOM formally resembles spin precession

$$\frac{d}{dr}\mathbf{P}_\omega = (\omega\,\mathbf{B} + \lambda\,\mathbf{L} + \mu\,\mathbf{D}) \times \mathbf{P}_\omega \equiv \mathbf{H}_\omega \times \mathbf{P}_\omega$$

Analogy to a spinning top

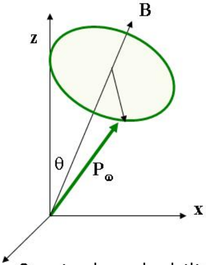
The spinning top analogy

- Motion of the average $\mathbf{P}_\omega$ defined by $\mathbf{S} = \int d\omega \, f(\omega) \, \mathbf{P}_\omega$
- Construct the "Pendulum" vector $\mathbf{Q} = \mathbf{S} - \frac{\omega_{avg}}{\mu} \mathbf{B}$
- EOMs are given by $\dot{\mathbf{Q}} = \mu \, \mathbf{D} \times \mathbf{Q}$, $\dot{\mathbf{D}} = \omega_{avg} \mathbf{B} \times \mathbf{Q}$
- Mapping to Top : $\mathbf{Q}/\mathrm{Q} \equiv \mathbf{r}$, $\mathbf{D} \equiv \mathbf{j}$, $\omega_{avg} \mu \mathrm{Q} \mathbf{B} \equiv \mathbf{g}$

  $\mu^{-1} \equiv m$, $\mathbf{D}.\mathbf{Q}/\mathrm{Q} \equiv \sigma$
- EOMs now become $\mathbf{j} = m\mathbf{r} \times \dot{\mathbf{r}} + \sigma \mathbf{r}$, $\dot{\mathbf{j}} = m\mathbf{r} \times \mathbf{g}$
- Note that these are equations of a spinning top!!! (Hannestad, Raffelt, Sigl, Wong: astro-ph/0608695; Fogli, Lisi, Mirizzi, Marrone: hep-ph/0707.1998)

Synchronized oscillation

- Spin is very large : Top precesses about direction of gravity
- At large $\mu \gg \omega_{avg}$: $\mathbf{Q}$ precesses about $\mathbf{B}$ with frequency ω_{avg}
- Therefore $\mathbf{S}$ precesses about $\mathbf{B}$ with frequency ω_{avg}
- Large μ: all $\mathbf{P}_\omega$ are bound together: same EOM



$$\frac{d}{dr}\mathbf{P}_\omega = (\omega\,\mathbf{B} + \lambda\,\mathbf{L} + \mu\,\mathbf{D}) \times \mathbf{P}_\omega$$

(Pastor, Raffelt, Semikoz: hep-ph/0109035)

Precession = Sinusoidal Oscillation

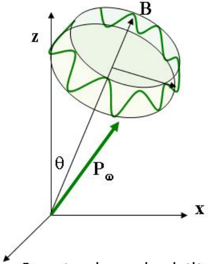
- Survival probability : $\left|\langle \nu_e | \nu_e(r) \rangle\right|^2 = (1 + \mathrm{P}_z)/2$

$$= 1 - \sin^2 2\theta \sin^2 \omega_{avg} r$$

Bipolar oscillations

Bipolar oscillation

- Spin is not very large : Top precesses and nutates
- At large $\mu \geq \omega_{avg}$: $\mathbf{Q}$ precesses + nutates about $\mathbf{B}$
- Therefore $\mathbf{S}$ does the same
- All $\mathbf{P}_\omega$ are still bound together, same EOM:



$$\frac{d}{dr}\mathbf{P}_\omega = (\omega\,\mathbf{B} + \lambda\,\mathbf{L} + \mu\,\mathbf{D}) \times \mathbf{P}_\omega$$

(Hannestad, Raffelt, Sigl, Wong: astro-ph/0608695; Duan, Fuller, Carlson, Qian: astro-ph/0703776)

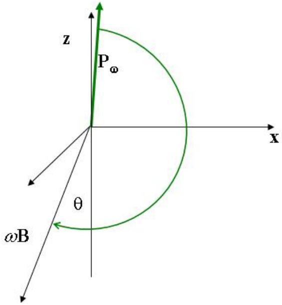
Nutation = Complicated elliptic functions

- Survival probability : $\left|\langle \nu_e | \nu_e(r) \rangle\right|^2 = (1 + \mathrm{P}_z)/2$

Spectral split

Adiabatic spectral split

- Top falls down when it slows down (when mass increases)
- If μ decreases slowly $\mathbf{P}_\omega$ keeps up with $\mathbf{H}_\omega$
- As $\mu \rightarrow 0$ from its large value : $\mathbf{P}_\omega$ aligns with $h\omega\mathbf{B}$
- For inverted hierarchy $\mathbf{P}_\omega$ has to flip, **BUT…**



$$\frac{d}{dr}\mathbf{B.D} \approx \mathbf{B}.\dot{\mathbf{D}} = \mathbf{B}.(\omega_{avg}\mathbf{B}\times\mathbf{Q}) = 0$$

- $\mathbf{B.D}$ is conserved so all P_ω can't flip
- Low energy modes anti-align
- All $\mathbf{P}_\omega$ with $\omega < \omega_c$ flip over
- Spectral Split

(Raffelt, Smirnov:hep-ph/0705.1830)