Neutrino Physics: an Introduction

Lecture 2: Neutrino mixing and oscillations

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Lecture 1: Neutrino detection and basic properties

- Unique properties
- Discovery of neutrino flavours
- Measuring mass, helicity, interactions

Lecture 2: Neutrino mixing and oscillations

- Solar and atmospheric puzzles and solutions
- Neutrino mixing, oscillations, flavour conversions
- The three-neutrino mixing picture

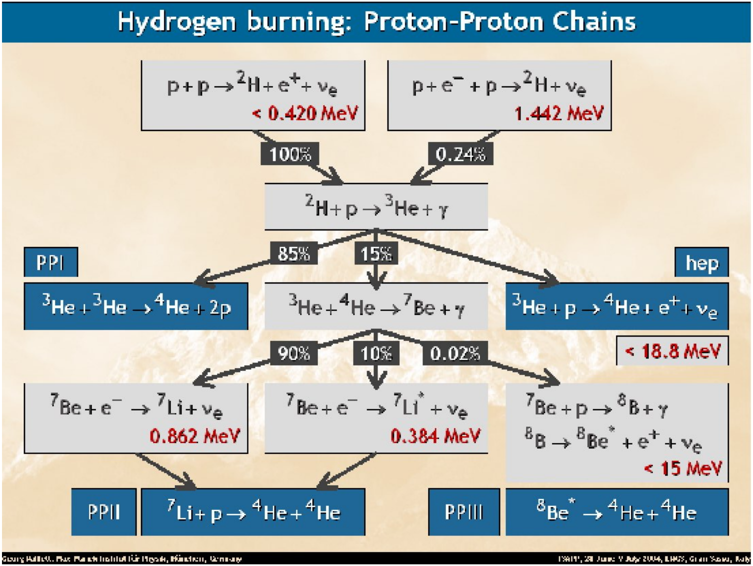
Lecture 3: Neutrinos in astrophysics and cosmology

- Low-energy (meV) cosmological neutrinos
- Medium-energy (MeV) supernova neutrinos
- High-energy ($>$ TeV) astrophysical neutrinos

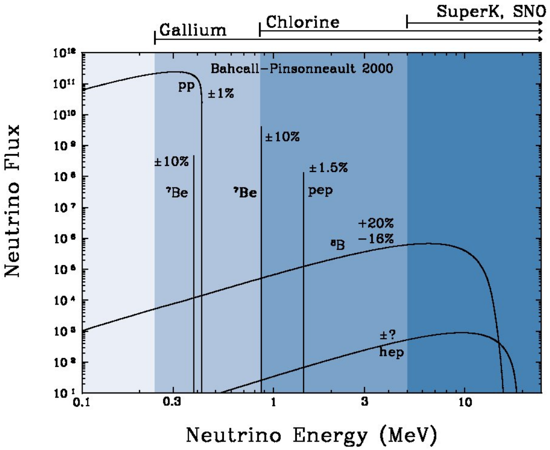
1. Solar and atmospheric neutrino puzzles
2. Atmospheric ν solution: mixing and vacuum oscillations
3. The path to the solution for solar ν puzzle
4. The three-neutrino mixing picture

1. Solar and atmospheric neutrino puzzles
2. Atmospheric ν solution: mixing and vacuum oscillations
3. The path to the solution for solar ν puzzle
4. The three-neutrino mixing picture

Neutrinos from the Sun



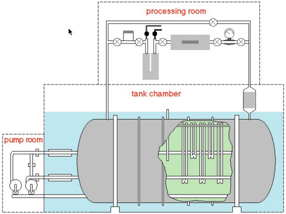
The solar neutrino spectra



- Magnitudes of fluxes depend on details of solar interior
- Spectral shapes robustly known

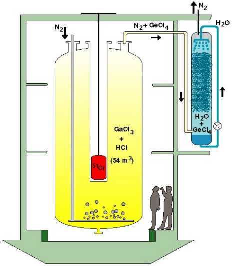
Detecting neutrinos from the Sun

- The Sun produces ν_e
- These ν_e can be detected at Earth: difficult, but possible



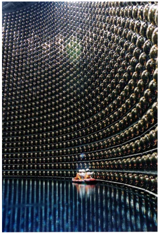
$\nu_e + \mathrm{Cl} \to \mathrm{Ar} + e^-$

Homestake



$\nu_e + \mathrm{Ga} \to \mathrm{Ge} + e^-$

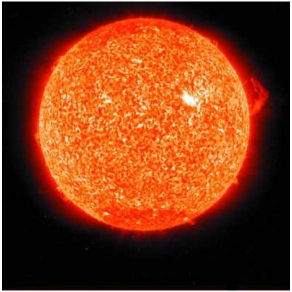
Gallex



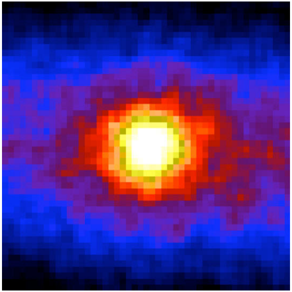
$\nu_e + e^- \to \nu_e + e^-$

SuperKamiokande

Seeing the Sun with neutrinos



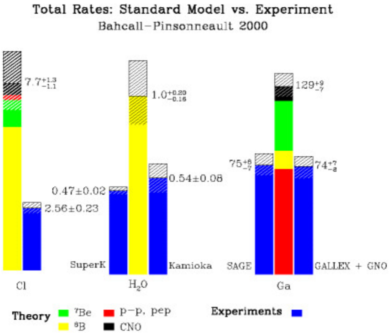
- Light from the Sun's surface:
  due to nuclear reactions
  millions of years ago
- Neutrinos from the Sun's core:
  due to nuclear reactions
  8 minutes ago

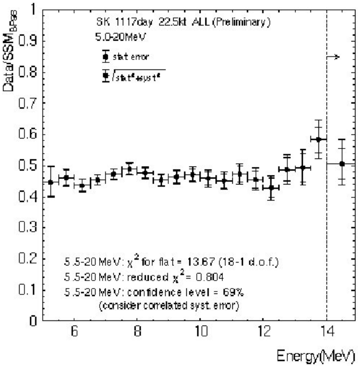


- We know how much light we get from the Sun...
- So we know how many neutrinos should arrive.

BUT...

Do we really understand how the Sun shines ?





The solar neutrino puzzle

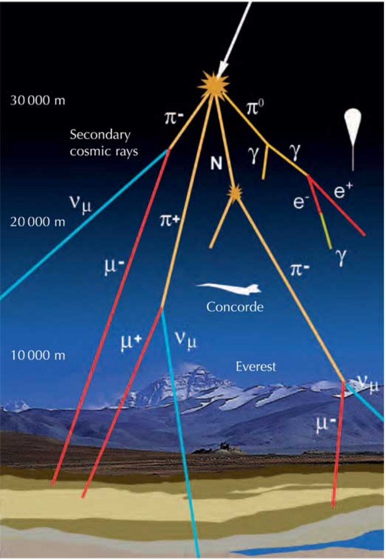
- Only about 30%–50% of neutrinos from the Sun found
- Different experiments give different neutrino loss...
  (They look at different energy ranges, of course..)
- SuperKamiokande shows similar neutrino loss at all energies

Possible resolutions of the puzzle

- The astrophysicists cannot calculate accurately
- The experimentalists cannot measure accurately
- Neutrinos behave differently from what everyone thought !

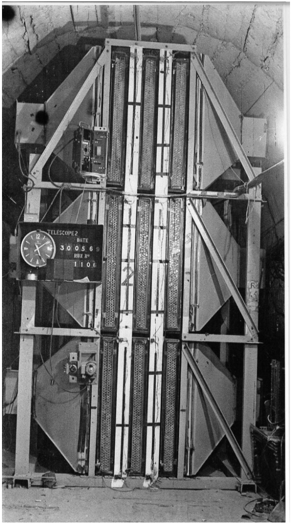
.... remained unresolved for about 40 years !

Neutrino production from cosmic rays



- $\pi^+ \rightarrow \mu^+ + \nu_\mu$
- $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$

The first “atmospheric” neutrinos detected in India



Detector in
Kolar Gold Fields

DETECTION OF MUONS PRODUCED BY COSMIC RAY NEUTRINO DEEP UNDERGROUND

C. V. ACHAR, M. G. K. MENON, V. S. NARASIMHAM, P. V. RAMANA MURTHY and B. V. SREEKANTAN,
Tata Institute of Fundamental Research, Colaba, Bombay

K. HINOTANI and S. MIYAKE,
Osaka City University, Osaka, Japan

D. R. CREED, J. L. OSBORNE, J. B. M. PATTISON and A. W. WOLFENDALE
University of Durham, Durham, U.K.

Received 12 July 1965

Physics Letters 18, (1965) 196

(15th Aug 1965)

EVIDENCE FOR HIGH-ENERGY COSMIC-RAY NEUTRINO INTERACTIONS*

F. Reines, M. F. Crouch, T. L. Jenkins, W. R. Kropp, H. S. Gurr, and G. R. Smith

Case Institute of Technology, Cleveland, Ohio

and

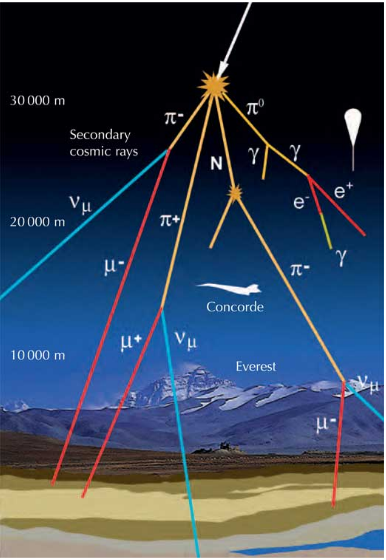
J. P. F. Sellschop and B. Meyer

University of the Witwatersrand, Johannesburg, Republic of South Africa
(Received 26 July 1965)

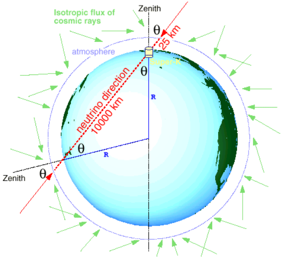
PRL 15, (1965) 429

(30th Aug 1965)

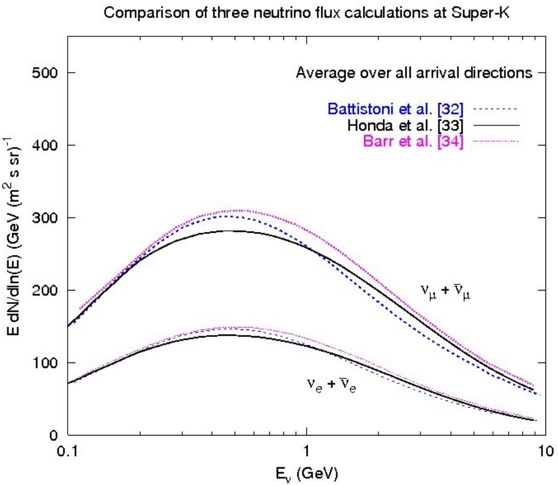
Neutrino production from cosmic rays



- $\pi^+ \to \mu^+ + \nu_\mu$
- $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$
- "ν_μ" flux $= 2\times$ "ν_e" flux
- "Down" flux = "Up" flux

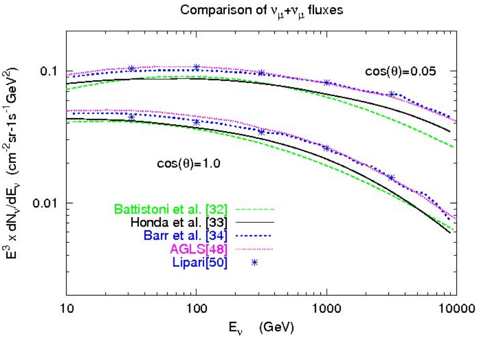


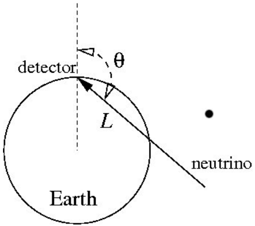
Expected neutrino fluxes at the Earth surface



- ν_μ/ν_e ratio: increases with energy

Expected neutrino fluxes at Earth surface

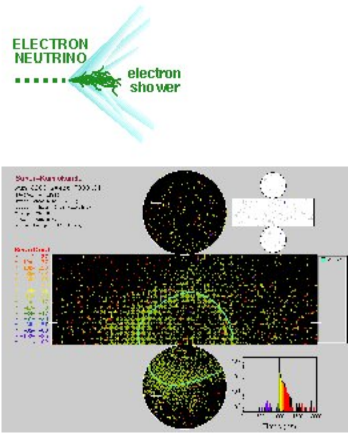




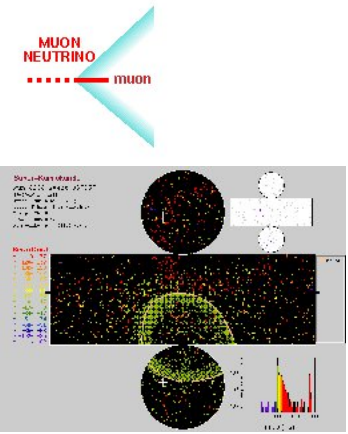
- ν_μ flux: smaller at larger $|\cos\theta|$
- However:

$$\text{Flux}_{\cos\theta=a} = \text{Flux}_{\cos\theta=-a}$$

How to detect ν_e and ν_μ through Cherenkov cones



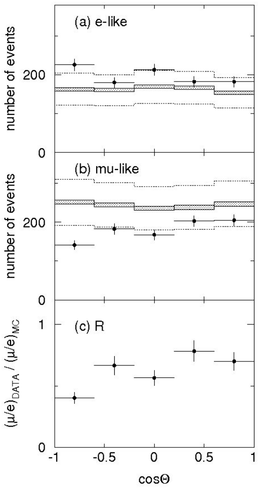
Diffused ring



Sharp ring

Atmospheric neutrino puzzle

Double ratio: $R = \frac{(\mu/e)_{\text{measured}}}{(\mu/e)_{\text{predicted}}}$

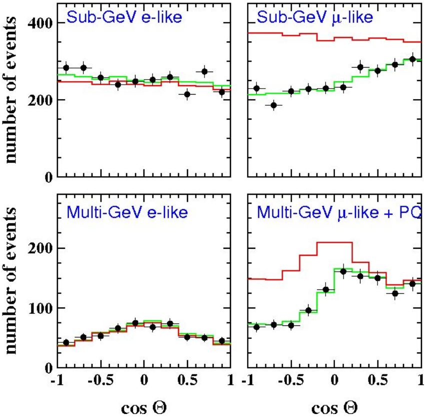


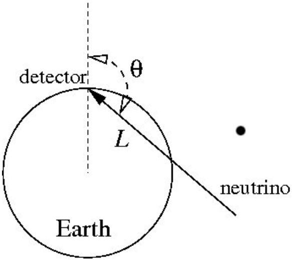
- Expected $R = 1$
- Observed $R < 1$

Super-Kamiokande

Atmospheric neutrino puzzle

Zenith angle dependence:





Super-Kamiokande

Preliminary observations from zenith angle data

- Electron neutrinos match predictions
- High energy ν_μ from above: match predictions
- High energy ν_μ through the earth: partially lost
- Low energy ν_μ: lost even when coming from above, loss while passing through the Earth even greater

Where are we now

- About 20 years ago: in the middle of two long-standing puzzles

The breakthrough idea

Bruno Pontecorvo
(original idea suggested for solar neutrinos, with neutrino-antineutrino mixing.)

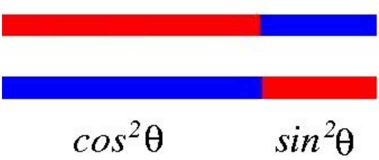
Maybe the neutrino flavours change !

- All the experiments are looking for ν_e and ν_μ
- What if ν_e / ν_μ are getting converted to ν_τ ?
- This is possible, but only if the neutrinos have different masses and they mix !

What is meant by neutrino mixing ?

Neutrino flavours ν_e, ν_μ, ν_τ do not have fixed masses !!

For example, ν_e–ν_μ mixing:



$$\nu_2 = -\nu_e \sin\theta + \nu_\mu \cos\theta$$

$$\nu_1 = \nu_e \cos\theta + \nu_\mu \sin\theta$$

- Only ν_1 and ν_2 have fixed masses
  (*They are eigenstates of energy / eigenstates of evolution*)
- Then, if you produce ν_e, it may be observed as ν_μ !

Effective Hamiltonian for a single neutrino

$$H = \sqrt{p^2 + m^2} \approx p + \frac{m^2}{2p} \approx p + \frac{m^2}{2E}$$

Schrödinger's equation:

$$i\frac{d}{dt}|\nu(t)\rangle = H|\nu(t)\rangle$$

Time evolution:

$$\begin{aligned} |\nu(t)\rangle &= |\nu(0)\rangle e^{-iHt} \\ &= |\nu(0)\rangle e^{-ipt} e^{-i\frac{m^2}{2E}t} \end{aligned}$$

- Simple for a mass eigenstate with fixed momentum !

Time evolution for a flavour eigenstate

- Initial flavour state $|\nu_\alpha\rangle$:

$$|\nu_\alpha\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$

- State after time t:

$$|\nu_\alpha(t)\rangle = \cos\theta|\nu_1\rangle e^{-ipt} e^{-i\frac{m_1^2}{2E}t} + \sin\theta|\nu_2\rangle e^{-ipt} e^{-i\frac{m_2^2}{2E}t}$$

- "Survival" probability of finding the flavour $|\nu_\alpha\rangle$ at time t:

$$P(\nu_\alpha \to \nu_\alpha) = |\langle\nu_\alpha|\nu_\alpha(t)\rangle|^2$$

Vacuum oscillations

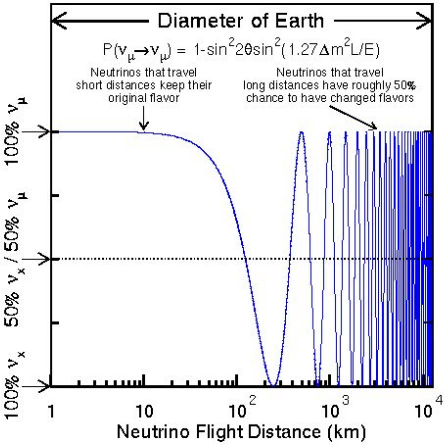
$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$\Delta m^2 \equiv m_2^2 - m_1^2$

(In Natural units, where $c = 1 = \hbar$)

Amplitude, wavelength:

Neutrino oscillations as a function of distance travelled



- More neutrinos 'lost" when $\cos(\Theta) < 0$

(Θ : angle made with the zenith)

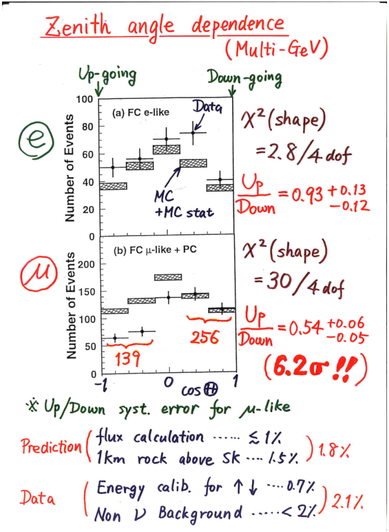
$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\theta \sin^2\left(1.27\frac{\Delta m^2\,(\mathrm{eV}^2)\,L\,(\mathrm{km})}{E\,(\mathrm{GeV})}\right)$$

Broad features of atmospheric ν data explained

- Electron neutrinos match predictions
- High energy ν_μ from above: match predictions
- High energy ν_μ through the earth: partially lost
- Low energy ν_μ: lost even when coming from above, loss while passing through the Earth even greater

The zenith angle dependence (1998) !

- Indeed more ν_μ travelling through the Earth are lost
- The zenith angle dependence fits the form of the probability expressions exactly
- Neutrino oscillation hypothesis proved !

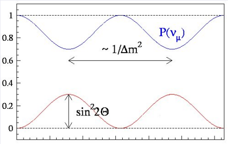


Atmospheric ν solution through "vacuum oscillations"

Prerequisites

- Neutrino flavours mix with each other
- Neutrinos have different masses
- ν_e do not participate in the oscillations

Neutrino oscillations: ν_μ oscillate into ν_τ



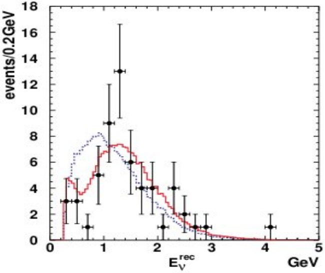
$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$\Delta m^2 \equiv m_2^2 - m_1^2$$

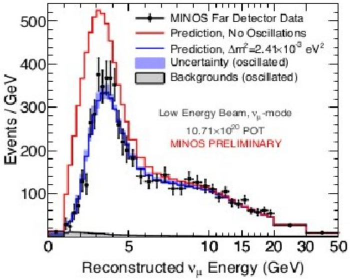
- Measurements can determine $\sin 2\theta_{\rm atm}$ and $\Delta m^2_{\rm atm}$.

Confirming from terrestrial experiments

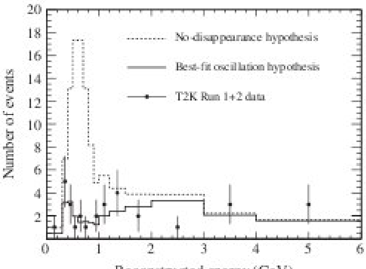
K2K: 285 km



MINOS: 735 km

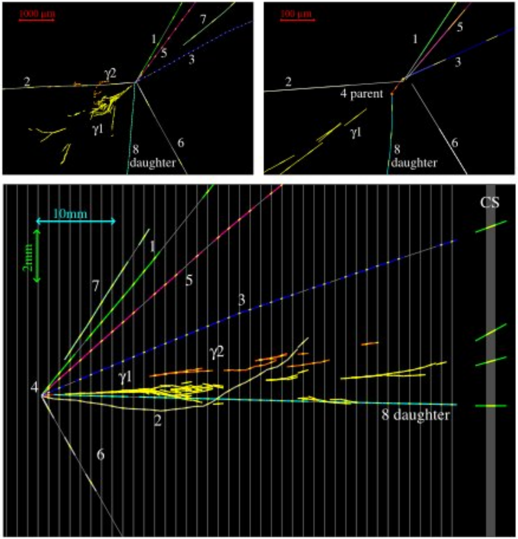


T2K: 295 km



OPERA experiment for ν_τ appearance

Oscillation Project with Emulsion tRacking Apparatus



The breakthrough idea

Bruno Pontecorvo

Original idea with $\nu - \bar{\nu}$ mixing

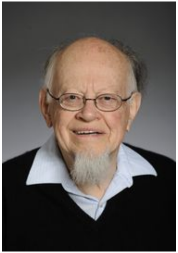
Maybe the neutrino flavours change !

- All the experiments are looking for ν_e
- What if ν_e are getting converted to other flavours of neutrinos (ν_μ or ν_τ) ?
- This is possible, but only if the neutrinos have different masses and they mix !

Neutrino flavour changes inside the Sun

John Bahcall

Lincoln Wolfenstein

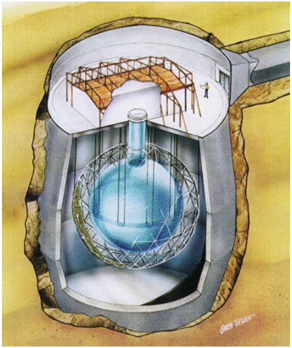


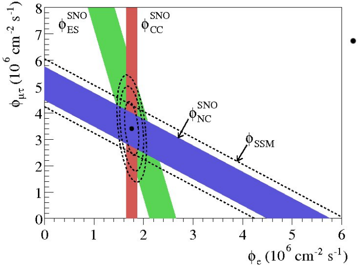
Stanislav Mikheyev

Alexei Smirnov

- Bahcall: Calculated the neutrino production inside the Sun in detail
- Wolfenstein: Showed that the neutrino mixing gets affected by the matter inside the Sun
- Mikheyev – Smirnov: Showed how these matter effects affect the neutrino flavour changes

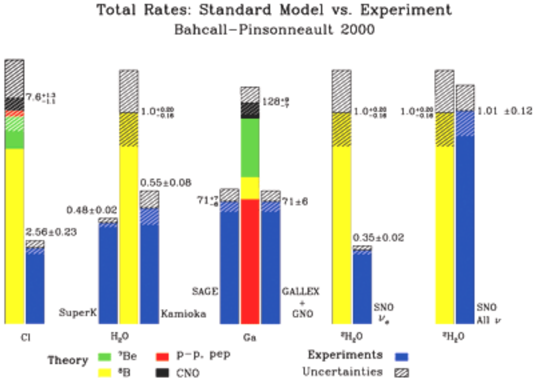
Heavy water Cherenkov experiment: SNO





- Heavy water Cherenkov
- $\nu_e\, D \to p\, p\, e^-$
  sensitive to Φ_e
- $\nu_{e,\mu,\tau}\ e^- \to \nu_{e,\mu,\tau}\ e^-$
  Sensitive to $\Phi_e + \Phi_{\mu\tau}/6$
- $\nu_{e,\mu,\tau}\ D \to n\, p\, \nu_{e,\mu,\tau}$
  sensitive to $\Phi_e + \Phi_{\mu\tau}$
- Neutral current: no effect of oscillations

Solar neutrino problem settled (2002)



- All neutrinos from the Sun are now accounted for !
- Our understanding of the Sun is vindicated...

Solution of solar neutrino problem

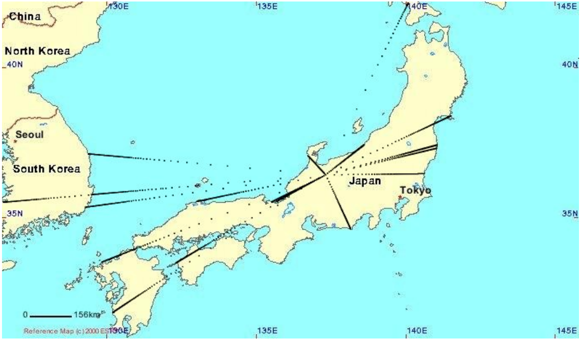
- ν_e mixes with ν_μ/ν_τ
  There is a "MSW resonance" inside the Sun

- Survival probability is almost flat for $E \gtrsim 1$ MeV:
  $$P_{ee} = \sin^2\theta_\odot$$
  No oscillations observable but "flavour conversions"

- The measurements can determine $\sin^2\theta_\odot$

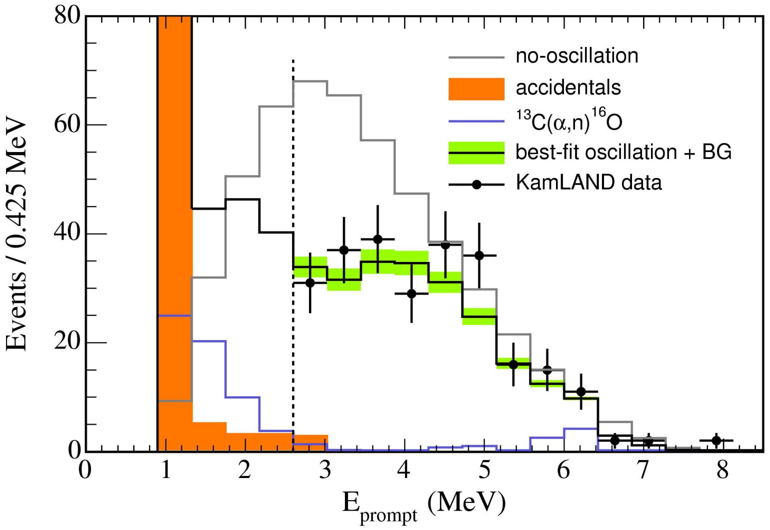
- To determine $\Delta m^2_\odot$ accurately, have to conduct terrestrial experiments (using reactors)

Reactor $\bar{\nu}_e$ from KamLAND

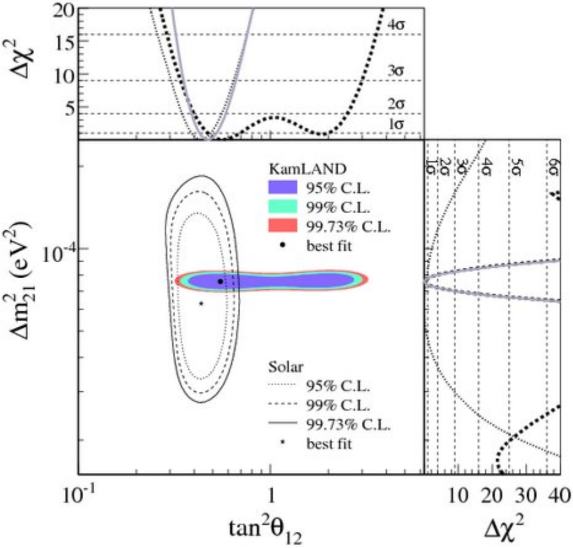


- The oscillations are consistent with vacuum oscillations
- ... with the same parameters Δm^2 and θ

$\bar{\nu}_e$ spectrum at KamLAND



Overlap of solar and KamLAND allowed regions



Assigning Δm^2 values from available data

ν_1, ν_2, ν_3: in decreasing order of ν_e content

Solar data

$$\Delta m^2_{\odot} = m_2^2 - m_1^2 \equiv \Delta m^2_{21}$$

- MSW resonance $\Rightarrow \Delta m^2_{21} > 0$

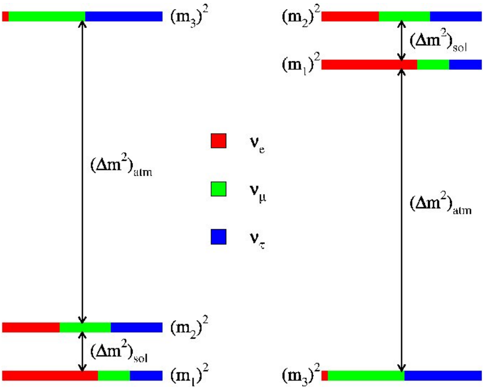
Atmospheric data

$$\Delta m^2_{atm} \equiv m_3^2 - (m_2^2 + m_1^2)/2$$

- $\Delta m^2_{31} \equiv m_3^2 - m_1^2$
- $\Delta m^2_{32} \equiv m_3^2 - m_2^2$
- $\Delta m^2_{31} \approx \Delta m^2_{32} \approx \Delta m^2_{atm}$
- Sign of Δm^2_{atm} unknown:
  Measured quantity $P_{\mu\mu} = 1 - \sin^2 2\theta \sin^2(\Delta m^2_{atm} L/(4E))$

The two mass orderings / mass hierarchies

Mixing of $\nu_e, \nu_\mu, \nu_\tau \Rightarrow \nu_1, \nu_2, \nu_3$ (mass eigenstates)



- $\Delta m^2_{\rm atm} \approx 2.4 \times 10^{-3}$ eV^2
- $\Delta m^2_{\odot} \approx 7.5 \times 10^{-5}$ eV^2
- $\theta_{\rm atm} \approx 45°$
- $\theta_{\odot} \approx 32°$
- $\theta_{\rm reactor} \approx 9°$

The mixing matrix

$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

PMNS matrix: elements in general complex

$$U^\dagger U = 1$$

A general parameterization

$$U_{PMNS} = \Phi(\chi_1,\chi_2,\chi_3) R_{23}(\theta_{23}) U_{13}(\theta_{13},\delta) R_{12}(\theta_{12}) \Phi(\phi_1,\phi_2,0)$$

$$\Phi(\chi_1,\chi_2,\chi_3) = \begin{pmatrix} e^{i\chi_1} & 0 & 0 \\ 0 & e^{i\chi_2} & 0 \\ 0 & 0 & e^{i\chi_3} \end{pmatrix}, \Phi(\phi_1,\phi_2,0) = \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\widetilde{U}_{PMNS} \equiv R_{23}(\theta_{23}) U_{13}(\theta_{13},\delta) R_{12}(\theta_{12}) =$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$c_i \equiv \cos\theta_i$, $s_i \equiv \sin\theta_i$

Oscillation probability and phases

$$P_{\alpha\beta} = \delta_{\alpha\beta} - \sum_{i<j} 4\mathrm{Re}(\square_{\alpha\beta ij})\sin^2(\Delta_{ji}) - 2\sum_{i<j}\mathrm{Im}(\square_{\alpha\beta ij})\sin(2\Delta_{ji})$$

$$\square_{\alpha\beta ij} \equiv U_{\alpha_i} U_{\beta j} U^*_{\alpha j} U^*_{\beta i}$$

- No-oscillation term
- CP-conserving oscillation term
- CP-violating oscillation term
- Among the phases, only δ appears in oscillation probabilities
- Matter effects may change values of mixing elements

Open questions in neutrino oscillation physics

- Mass ordering: Normal or Inverted ?
- What are the absolute neutrino masses ?
- Are there more than 3 neutrinos ?
- Is there leptonic CP violation ?
- Can neutrinos be their own antiparticles ?

And how do neutrinos get their mass at all ?

- In Standard Model of particle physics, the mass arises from the interaction between a left-handed particle, a right-handed particle, and Higgs.
  For example, e_L, e_R and h come together to give mass to the electron, which contains both e_L and e_R.

- But there is no right-handed neutrino !
  $\Rightarrow$ Higgs mechanism is not enough

- There *has to be* something beyond the Standard Model, perhaps even beyond our current imagination.

Where are we now (end of Lecture 2)

- Atmospheric neutrino problem solved through neutrino mixing and vacuum oscillations
- Solar neutrino problem solved through neutrino mixing, and modification of vacuum mixing due to matter
- Date from neutrino flavour conversion experiments can fitted into a three-neutrino mixing framework, with the mising matrix U_{PMNS}.