# Flavour physics: Lecture 1 History, CP violation, Neutral meson mixing

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SUSY 2017 pre-school, TIFR Mumbai, Dec 4-8, 2017

- $\tau \theta$  puzzle  $\Rightarrow$  Parity violation
- Cabibbo angle ⇒
   weak coupling universality ⊕ quark mixing
- GIM mechanism ⇒ no FCNC at tree level, charm
- CKM paradigm ⇒ (at least) three quark families
- Large  $B-\overline{B}$  mixing  $\Rightarrow$  heavy top quark
- Rate of radiative B decay ⇒ top quark mass
- The standard model looks complete now, with no confirmed signals of BSM physics at the LHC yet!
- Flavour physics may give some indirect hints....



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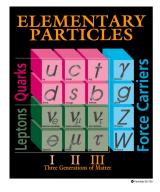
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#### Flavour structure of the Standard Model



- Three families of quarks and leptons
- Quark masses (GeV):

```
up charmed top 0.005 1.5 175
```

down strange bottom 0.01 0.1 5

Mixing between families:
 Strong within the same family,
 Weak across families

Pions:

$$\pi^+ \equiv u \bar d \;, \pi^- \equiv d \bar u \;, \pi^0 \equiv (u \bar u - d \bar d)/\sqrt{2} \;\; (\sim 0.15 \; {\rm GeV})$$

• Strange mesons (Kaons):  $K^+ \equiv u\bar{s}$ ,  $K^- \equiv s\bar{u}$ ,  $K^0 \equiv d\bar{s}$ ,  $\overline{K^0} \equiv s\bar{d}$  ( $\sim 0.5 \text{ GeV}$ )

Charmed mesons:

$$D^+ \equiv c \bar{d} \; , D_s^+ \equiv c \bar{s} \; , D^0 \equiv c \bar{u}$$
 (And antiparticles:  $\sim 1.5 - 1.8 \; {\rm GeV}$ )

B mesons:

$$B^+\equiv uar{b}$$
 ,  $B^0\equiv dar{b}$  ,  $B_s\equiv sar{b}$  ,  $B_c\equiv car{b}$  (And antiparticles:  $\sim 5-6.5$  GeV)

$$\phi \equiv s\bar{s} \quad (\sim 0.7 \text{ GeV})$$
 $J/\psi \equiv c\bar{c} \quad (\sim 3.1 \text{ GeV})$ 
 $\Upsilon \equiv b\bar{b} \quad (\sim 10 \text{ GeV})$ 



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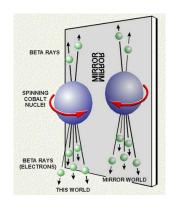
#### Contents

- Some historical perspective
- 2 CP violation: discovery and interpretation
- The Kobayashi-Maskawa mechanism
- Meutral meson mixing and oscillations
- 5 Flavour tests of the CKM paradigm

#### **Outline**

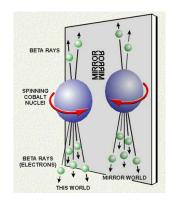
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### Discovery of parity violation: 1956-57



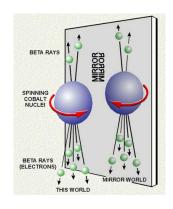
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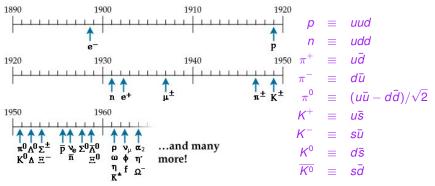
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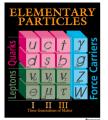
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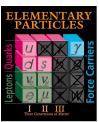


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### The particle zoo (1960's)







### Universality of weak interactions: Cabibbo angle



#### Interrelated coupling constants:

- (i) muon decay:  $g_{e\mu}$  $\mu^- \rightarrow \nu_{\mu} e^- \bar{\nu}_e$
- (ii) neutron decay :  $g_{ud}$  $n \rightarrow pe^-\bar{\nu}_e$  ( $d \rightarrow ue^-\bar{\nu}_e$ )
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  u}_e)$   $|g_{e\mu}|^2 = |g_{ud}|^2 + |g_{us}|^2$

#### Universality:

- ullet There is only one coupling constant,  $g=g_{e\mu}$
- u quark couples to only one combination of d and s:  $d' \equiv \cos \theta_c \cdot d + \sin \theta_c \cdot s$
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# Suppression of flavor-changing neutral currents

Cabibbo angle unable to explain why

$$\Gamma(K_L o \mu^+ \mu^-) << \Gamma(K^+ o \mu^+ \nu_\mu)$$

 Possible explanation via another "c" quark: charge +2/3, couples to

$$s' \equiv -\sin\theta_c \cdot d + \cos\theta_c \cdot s$$

- The  $s \rightarrow u \rightarrow d$  and  $s \rightarrow c \rightarrow d$  contribution cancel, leading to the suppression of FCNC  $s \rightarrow d$
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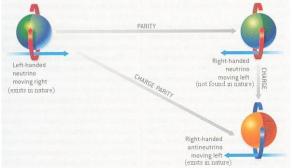
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### Can Charge ⊕ Parity may be conserved?





- Neutrinos violate parity: they are only left-handed
- But antineutrinos are right-handed!
- Does that mean C and P violations cancel each other to give CP conservation?

# Prediction of CP violation in K decay

$$K^0 \equiv d\bar{s}$$
  $\overline{K^0} \equiv s\bar{d}$ 

• CP eigenstates:

$$K_1 \equiv (K^0 + \overline{K})/\sqrt{2}$$
 (CP even)  
 $K_2 \equiv (K^0 - \overline{K})/\sqrt{2}$  (CP odd)

- CP even decay channel:  $\pi\pi$
- CP odd decay channel:  $\pi\pi\pi$
- CP conservation ⇒

$$K_1 
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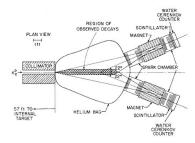
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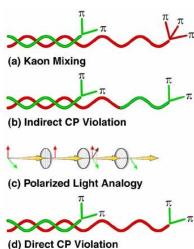


### Discovery of CP violation: 1964

#### Cronin-Fitch experiment



Nobel prize 1980



# Charge-parity violated slightly



"Day and Night", M.C.Escher

### Questions raised by the discovery of CP violation

- Is it small or large? Is CP an approximate symmetry?
- Is the symmetry breaking spontaneous?
- Where does it come from ? Are there extra interactions ?

# CP violation and complex coupling

- CP violation:  $A(X \to Y) \neq A(\overline{X} \to \overline{Y})$
- If all amplitudes are real,  $|A(X \to Y)|^2 = |A(\overline{X} \to \overline{Y})|^2$
- CP violation possible if complex numbers involved in

$$|A(X \to Y)|^2 = |A(X \to w \to Y) + A(X \to z \to Y)|^2$$
  
$$|A(\overline{X} \to \overline{Y})|^2 = |A(\overline{X} \to \overline{w} \to \overline{Y}) + A(\overline{X} \to \overline{z} \to \overline{Y})|^2$$

CP violation ⇒ Amplitudes complex ⇒ Couplings complex

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#### The paper and the authors

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Progress of Theoretical Physics, Vol. 49, No. 2, February 1973

#### CP-Violation in the Renormalizable Theory of Weak Interaction

Makoto KOBAYASHI and Toshihide MASKAWA

Department of Physics, Kyoto University, Kyoto

(Received September 1, 1972)

In a framework of the renormalizable theory of weak interaction, problems of CP-violation are studied. It is concluded that no realistic models of CP-violation exist in the quartet scheme without introducing any other new fields. Some possible models of CP-violation are also discussed.





## Two generations of quarks are not enough

 The mixing matrix between up-type and down-type quarks has to be a 2 × 2 unitary matrix

$$\mathcal{L} \propto (\overline{u_L}, \overline{c_L}) \left( \begin{array}{cc} \cos \theta e^{i\phi_1} & \sin \theta e^{i\phi_2} \\ -\sin \theta e^{i\phi_3} & \cos \theta e^{i(\phi_2 + \phi_3 - \phi_1)} \end{array} \right) \left( \begin{array}{c} d_L \\ s_L \end{array} \right)$$

• Can change three relative phases of quarks to get rid of all three complex phases  $\phi_1, \phi_2, \phi_3$ 

$$\mathcal{L} \propto (\overline{u_L}, \overline{c_L} e^{i(\phi_3 - \phi_1)}) \left(egin{array}{cc} \cos heta & \sin heta \ -\sin heta & \cos heta \end{array}
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## Paremeter counting for two generations

- $2 \times 2$  complex matrix  $\Rightarrow$  4 real + 4 imaginary quantities
- Unitarity  $U^{\dagger}U = I$ : 3 real and 1 imaginary conditions
- 1 real and 3 imaginary parameters left
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- $3 \times 3$  matrix  $\Rightarrow 9$  real + 9 imaginary quantities
- Unitarity  $U^{\dagger}U = I$ : 6 real and 3 imaginary conditions
- 3 real and 6 imaginary parameters left
- Can choose the 5 relative phases between quarks to get rid of 5 imaginary parameters
- In addition to 3 real parameters (Euler angles of rotation), one imaginary quantity is unavoidable
- Mixing matrix complex ⇒ CP violation may be present

$$\begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix}$$

• Three families needed for the complex nature



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$$\left(egin{array}{cccc} c_1 & -s_1c_3 & -s_1s_3 \ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{array}
ight)$$

• Three families needed for the complex nature



- $3 \times 3$  matrix  $\Rightarrow$  9 real + 9 imaginary quantities
- Unitarity  $U^{\dagger}U = I$ : 6 real and 3 imaginary conditions
- 3 real and 6 imaginary parameters left
- Can choose the 5 relative phases between quarks to get rid of 5 imaginary parameters
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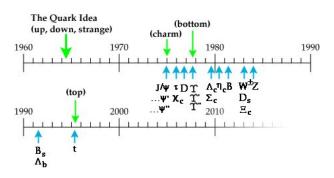
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Three families needed for the complex nature



### Experimental discovery of the third generation



- Discovery of τ: 1976
- $\Upsilon$ , B,  $B_s$ ,  $\lambda_b$  contain b quark
- Top quark: 1995
- The last element,  $\nu_{\tau}$ , discovered in 2000.

## CKM paradigm in modern language

Flavor basis vs. mass basis:

$$U' \equiv \left(\begin{array}{c} u \\ c \\ t \end{array}\right) \ , \quad D' \equiv \left(\begin{array}{c} d \\ s \\ b \end{array}\right)$$

Charged current in the basis of flavor eigenstates:

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \overline{U_L'} \gamma^{\mu} D_L' W_{\mu}^+ + h.c.$$

Charged current in the basis of mass eigenstates:

$$\mathcal{L}_{CC} = rac{g}{\sqrt{2}} \overline{U_L} \gamma^{\mu} (V_{UL}^{\dagger} V_{DL}) D_L W_{\mu}^{+} + H.c.$$

 $V_{UL}$ ,  $V_{DL}$ : unitary matrices that change the basis

#### Cabibbo-Kobayashi-Maskawa (CKM) matrix

Coupling between  $U_L$  and  $D_L$ :  $(g/\sqrt{2})V_{CKM}$ 

$$V_{CKM} \equiv V_{III}^{\dagger} V_{DI}$$



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$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- $\lambda \approx$  0.2: Cabibbo angle
- $\eta$ : the imaginary component of  $V_{CKM}$
- $\eta/\rho$  large  $\Rightarrow$  CP violation is large, not approximate
- All CP violation in terms of a single number: Jarlskog invariant  $J \equiv s_1 s_2 s_3 c_1^2 c_2 c_3 s_\delta = A^2 \lambda^6 \eta$



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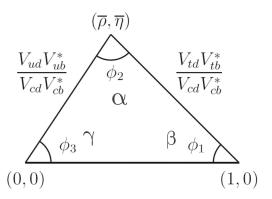
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## Unitarity relations and triangles

Unitarity relation  $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ 



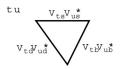
$$\alpha \equiv \mathrm{Arg}\left(-\frac{V_{td}\,V_{tb}^*}{V_{ud}\,V_{ub}^*}\right) \;, \quad \beta \equiv \mathrm{Arg}\left(-\frac{V_{cd}\,V_{cb}^*}{V_{td}\,V_{tb}^*}\right) \;, \quad \gamma \equiv \mathrm{Arg}\left(-\frac{V_{ud}\,V_{ub}^*}{V_{cd}\,V_{cb}^*}\right) \;,$$

## More unitarity triangles

$$\chi \underbrace{\begin{array}{c} v_{\rm ts} v_{\rm tb}^{\star} \\ v_{\rm cs} v_{\rm cb}^{\star} \end{array}}_{v_{\rm us} v_{\rm ub}^{\star}}$$

$$v_{cd}v_{td}^{\star}$$

$$v_{cb}v_{tb}^{\star}$$

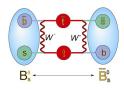


- All triangles have the same area, J/2
- $\Phi \ \chi \equiv \beta_{\rm S} \equiv \phi_{\rm S} \equiv {\rm Arg} \left( \frac{V_{\rm cs} \, V_{\rm cb}^*}{V_{\rm ts} \, V_{\rm tb}^*} \right) \sim \lambda^2 \; ,$
- $\chi' \equiv \beta_K \equiv \text{Arg}\left(-\frac{V_{cd}V_{cs}^*}{V_{cd}V_{cs}^*}\right) \sim \lambda^4$ ,

#### Outline

- Some historical perspective
- 2 CP violation: discovery and interpretation
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# $B_q - \overline{B_q}$ mixing: parametrization



ullet Oscillation and decay of  $a|B_q
angle+b|\overline{B}_q
angle$  :

$$\begin{split} i\frac{d}{dt} \left( \begin{array}{c} a \\ b \end{array} \right) &= \left( \mathbf{M} - \frac{i}{2} \Gamma \right) \left( \begin{array}{c} a \\ b \end{array} \right) \\ \mathbf{M} &\equiv \left( \begin{array}{c} M_{11} & M_{12} \\ M_{21} & M_{22} \end{array} \right) \quad , \quad \Gamma \equiv \left( \begin{array}{cc} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{array} \right) \\ \mathcal{CP} |B_q\rangle &= e^{i\varphi} |\bar{B}_q\rangle, \; \mathcal{CP} |\bar{B}_q\rangle = e^{-i\varphi} |B_q\rangle \end{split}$$

- CPT invariance :  $M_{11} = M_{22}$ ,  $\Gamma_{11} = \Gamma_{22}$
- Hermiticity:  $M_{21} = M_{12}^*$ ,  $\Gamma_{21} = \Gamma_{12}^*$



#### Mass difference and lifetime difference

• Mass eigenstates:

$$|B_{L,H}\rangle = p|B_q\rangle \pm q|\bar{B}_q\rangle \hspace{1cm} (|q|^2 + |p|^2 = 1)$$
• Mass difference Lifetime difference  $\Delta m = M_H - M_L \hspace{1cm} \Delta \Gamma = \Gamma_L - \Gamma_H (\rightarrow \Delta m > 0, \Delta \Gamma > 0 \text{ in SM})$ 

Eigenvalue equations:

$$(\Delta m)^2 - \frac{1}{4}(\Delta \Gamma)^2 = (4|M_{12}|^2 - |\Gamma_{12}|^2)$$
  
 $\Delta m \Delta \Gamma = -4 \text{Re}(M_{12}^* \Gamma_{12}).$ 

$$\Delta m = 2|M_{12}| + O(m_b^4/m_t^4)$$
  
$$\Delta \Gamma = -\frac{2\text{Re}(M_{12}^*\Gamma_{12})}{|M_{12}|} + O(m_b^4/m_t^4).$$

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200

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# Time evolution of a tagged $B_q$ or $\bar{B}_q$ decay

$$A_f \equiv \langle f|B_q\rangle, \ \overline{A}_f \equiv \langle f|\overline{B}_q\rangle, \ \lambda_f \equiv \frac{q}{p}\frac{\overline{A}_f}{A_f}$$

 $(\lambda_f \text{ independent of the unphysical phase } \varphi)$ 

$$\begin{split} \Gamma(B_q(t) \to f) &= \mathcal{N}_f |A_f|^2 \, \frac{1 + |\lambda_f|^2}{2} \, e^{-\Gamma t} \, \times \\ \left[ \cosh \frac{\Delta \Gamma_q \, t}{2} + \mathcal{A}_{\text{CP}}^{\text{dir}} \, \cos(\Delta m \, t) \right. &+ \left. \mathcal{A}_{\Delta \Gamma} \, \sinh \frac{\Delta \Gamma_q \, t}{2} + \mathcal{A}_{\text{CP}}^{\text{mix}} \, \sin(\Delta m \, t) \right], \end{split}$$

$$\begin{split} \Gamma(\overline{B}_q(t) \to f) &= \mathcal{N}_f \, |\overline{A}_f|^2 \, \frac{1 + |\lambda_f|^2}{2} \, e^{-\Gamma t} \, \times \\ \left[ \cosh \frac{\Delta \Gamma_q \, t}{2} - \mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}} \, \cos(\Delta m \, t) \right. &+ \left. \mathcal{A}_{\Delta \Gamma} \, \sinh \frac{\Delta \Gamma_q \, t}{2} - \mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}} \, \sin(\Delta m \, t) \right]. \end{split}$$

$$\mathcal{A}_{CP}^{dir} = \frac{1 - \left|\lambda_f\right|^2}{1 + \left|\lambda_f\right|^2}, \qquad \quad \mathcal{A}_{CP}^{mix} = -\frac{2\operatorname{Im}\lambda_f}{1 + \left|\lambda_f\right|^2} \quad \mathcal{A}_{\Delta\Gamma} = -\frac{2\operatorname{Re}\lambda_f}{1 + \left|\lambda_f\right|^2},$$

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## B factories: $B \equiv \bar{b}u, \bar{b}d, \bar{b}s, \bar{b}c, \quad \bar{B} \equiv \bar{u}b, \bar{d}b, \bar{s}b, \bar{c}b$





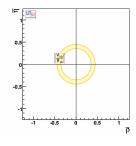
Babar (SLAC, USA)



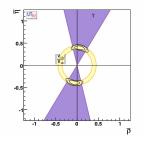
BELLE (KEK, Japan)



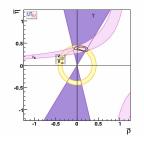
$$e^+e^- \to B\overline{B} \to \text{decayproducts}$$



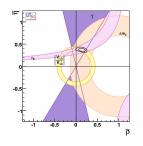
• Semileptonic decay  $B o D\ell 
u$ 



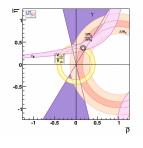
- Semileptonic decay  $B o D\ell \nu$
- ullet "Charmed" decays  $B \to DK$



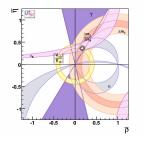
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- "Charmed" decays  $B \to DK$
- CP violation in K mesons



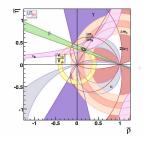
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- $\Delta M$  in  $B_d \overline{B_d}$  system



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- $\Delta M$  in  $B_s \overline{B}_s$  system



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- "Charmed" decays  $B \to DK$
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- $\Delta M$  in  $B_s \overline{B}_s$  system
- Decays to π and K



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- ullet "Charmed" decays B o DK
- CP violation in K mesons
- $\Delta M$  in  $B_d \overline{B_d}$  system
- $\Delta M$  in  $B_s \overline{B}_s$  system
- Decays to  $\pi$  and K
- CP asymmetry in  $B \to J/\psi K_S$

### Concluding remarks: end of Lecture 1

- No deviation from the CKM predictions has been observed
   ⇒ Constrains many new physics models
- CP violation required for baryon asymmetry, but the CKM is not enough, so there will be life beyond CKM
- Physics beyond the Standard Model must exist!
  - Where to look for it?
  - How to identify and quantify it?
  - ⇒ Lecture 2

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