Brief notes on the paper "Fine structure of the $1s3p\ ^3P_J$ level in atomic ^4He:Theory and Experiment" by P.Mueller, L.B.Wang, G.W.F. Drake, K.Bailey, Z-T.Lu and T.P.O'Connor, PRL 94, 133001 (2005)

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I. INTRODUCTION

This paper is concerned about measuring the fine structure splits between the the $J = 0, 1, 2$ of the $1s2p\ ^3P_J$ configurations of the ^4He. This measurement has attracted a lot of attention because this allows for accurate measurements of the fine structure constant α and hence provides a great platform to match theory against experiment. The authors estimate that a measurement of the split between the $J = 0$ and the $J = 1$ states of the order of $29617MHz$ to an accuracy of $\pm 1KHz$ will be sufficient to measure α to an accuracy of $\pm 16ppb$. The paperis also motivated by the fact that at that time of publication there existed a $250Khz$ discrepancy between the theoretical and experimetal data which could not be accounted for even by a fifth order QED calculation.

In this paper the authors close this gap by improving the experimental accuracy by intrioducing some subtle changes in the set up and they claim that their refiend theoretical calculations also close in whose details are left from this paper.

In this paper the most important experimental technique that gets demonstrated which gives them the clincher over other such similar experimental tests is their taking into account of the fact that the value of the fine structure constant measured can depend on the power fluctuation of the LASER and also on the presence of the magentic field in the region of interaction of the cooled ^4He beam with the $389nm$ LASER. They take this into account by studying the variation of the fine structure constant as a function of the LASER power and the residual magnetic field and then quoting the final values as obtained at 0 LASER power from regression analysis.

II. SOME COMMENTS ABOUT THE ^4He ENERGY LEVELS

The advantage of using ^4He is that its nuclear spin is 0 and hence there are no hyperfine structure to be taken care of.

The two electrons of ^4He can distribute amongst the energy levels in many ways and their spins can couple to form singlet and triplet states of the molecule. The first few energy levels of the molecule relevant to the experiment are shown in the diagram attached at the end.

One notes the following important features of the energy spectrum:

- The configuration $1s2s(2\ ^3S_1)$ of He_4 is metastable and it can remain in this configuration for about $2.3hrs$ if collissional perturbations are absent.
- The $1s2s(2\ ^3S_1)$ state cannot decay by any first order transition to the more stable $1s^2(1\ ^1S_0)$ state since it is spin and orbitalangular momentum forbidden.
- The gap between $1s2s(2\ ^3S_1)$ and $1s2p(2\ ^3P_J)$ is of $1083nm$ and is well studied because this is the gap on which the historically the BEC experiments involving cooling had been done.
- In this experiment further doppler free cooling is done on the $383nm$ gap between the $1s2s(2\ ^3S_1)$ and $1s3p(3\ ^3P_J)$.
- One can excite an usable fraction of ground state He atoms to the $2\ ^3S_1$ state by bombarding with high-energy electrons or atoms and collide with the He atoms.

III. A BRIEF SKETCH OF THE EXPERIMENT

The main effort of the experiment goes into cooling the ^4He atoms. The technique used is what had been studied by Wang in his PhD thesis at UIUC and gets called *laser induced fluorescence detection.*

The basic steps are as follows:

- The metastable He atoms are produced in a RF driven discharge source that is cooled to Liquid Nitrogen temperature.
- A 2-dimensional doppler free transverse cooling is done on the $1083nm$ gap between $1s2s\ ^3S_1$ and $1s2p\ ^3P_2$.
- This cooling process is facilitated by reducing the divergence of the atomic beam and hence increasing the forward intensity by a factor of 10. The importance of reducing velocity coponents of the atoms in directions parallel to the LASER will be indicated later.
- This collimated and cooled He beam is then made to intersect with a $389nm$ LASER beam inside an interaction region which is magnetically shielded by materials of attenuation factor of about 800. This is done to minimize the possibility of Zeeman shifts.
- The $389nm$ LASER does the doppler free cooling on the transition from $1s2s\ ^3S_1$ to $1s3p\ ^3P_J$.
- The fluorescence due to the downward transitions (after the excitation) is focussed onto a PMT with 0.1% efficiency.

Since $389nm$ LASERs are not directly available this is generated by a frequency doubling of the amplified output of an external cavity diode laser ($DL1$) at $778nm$. The frequency doubled lightray is pased through the following stages before it enters the magnetically shielded interaction region,

- The just produced $389nm$ beam is made to pass through an Acousto Optic Modulator (AOM) which allows for scanning of the beam from DL1.
- Then the output is locked to a high frequency Fabery-Perot Interferometer (FPI) for further filtering and the FPI is locked to another DL2 at $778nm$.
- This DL2 is referenced to a saturation absorption signal from an iodine cell. The iodine celll gives a frequency stability of better than $30KHz$ in $1min$ for $389nm$.
- The frequency stability of the $389nm$ beam is checked by beating it with itself at the two different stages. The partial beam from DL1 and DL2 are overlapped on a fast photodiode and the beat frequency is amplified and monitored by a microwave frequency referenced to a Rb disciplined crystal oscillator with relative frequency uncertainity of less than $1ppb$.
- The 2 counter propagagating $389nm$ are maintained by a retro-reflector to ensure matching. One monitors the reverse tranmission of teh retro-relfected beam through a spatial filter infromt of the emergence point of the original beam.
- The AOM scans on a range of about $\pm 15MHz$ to detect the resonance absorption peak.

IV. ERROR CONTROL

Here is listed out all the possible sources of error in the experiment and how the authors have handled them,

- The error in the photon counting is statistical and there is a $3-5\%$ contribution to it from the power fluctuation of the blue LASER when the counting gate is open for $0.5sec$.
- There could have been a dependance of the power outut of the LASER on the frequency but the authors state that they could not observe any statistically significant drifts.
- The center frequency of resonance as detected from the $\pm 15MHz$ scan of the AOM is determined by doing a fit of the Voigt profile to the observed resonance curve. Later I shall briefly indicate the origin of the Voigt function which is used to fit the curve.

- The power of the LASER is chosen to be very low so that only a few photons are scatterred by each atom.
- The authors have also checked for assymmetry between the 2 counter-propagating LASER beams since any difference will cause systematic shifts in the value of the fine structure constant. This efefct was also reduced by chosing low LASER power.
- The most important error control that these authors have done in comparison to all previous efforts is that they have checked how the value of the fine-structure constant varies with the power fo the LASER which is known to fluctuate. So they found the value of α for a range of power from $25 - 350 \mu W$ and theyobtain as the result of the experiment the value they get at 0 LASER power by doing a regression analysis of the obtained data.
- This similar technology was used by thema also to check the effect of the residual magnetic field in the zone where the $389nm$ LASER beam interacts with the atomic beam. The residual field was found to be $3mG$ and they artificially jacked up the field to $8G$ and they obtained a lince center shift up by $0.28 MHz/G$.Extrapolating this shift to the low field case they obtain a maximum effect caused by Zeman shift to be of the order of $0.8Kz$ which is neligible at their level of uncertainity.
- Similarly syastematic effect from light shift and pressure shiftare also well below the $1KHz$ level.
- The other remaining source is the error due to the relative alignment between the two counter propagating LASER beams and the stability provided by the the retro-reflector is better than $2.5 \times 10^{-3} mrad$. The maximum doppler shift under these conditions is $5KHz$.
- The level crossings of the $J = 0$ and $J = 1$ occurs at a considerably higher magnetic field than the $J = 0$ and $J = 2$ crossings. Thefore the measurement of the first split is more sensitive to systematic uncertainities than the next but their method avoids all these complications by measuring values by extrapolating the power to 0.

V. A BRIEF LOOK AT LASER COOLING

We note that following difference between stimulated and saturated emission,

- In stimulated emission the emitted photon is in the same direction as the incident photon and of the same frequency.
- In spontaneous emission the emitted photon is isotropicin direction and is of the same frequency.

Hence by stimulated emission there is no momentum transfer to the atom whereas by spontaneous emission teher is a net momentum transfer.

Using $\vec{k}$ for the wave-vector for the LASER light and $\delta\vec{p}$ for the momentum transferred to the atom in N cycles then we have $\delta\vec{p} = N\hbar\vec{k}$. Using the standard symbols of λ and c for the LASER light and defining $s = \frac{I}{I_0}$ where I is the LASER intensity and I_0 is the saturation intensity given as $I_0 = \frac{\pi hc\Gamma}{3\lambda^3}$ and δ_L as the LASER detuning frequency which is the difference in the circular frequency of the LASER and the resonance frequency.

Then the force $\vec{F}$ on the atoms is given by

$$F = \frac{\hbar s \Gamma \vec{k}}{2[1 + s + (2\delta_L/\Gamma)^2]}$$

By such estimates one gets that for the $1083nm$ transition mentioned earlier $\Gamma = 1.02 \times 10^7/sec$ and very large s with the LASER in tune one gets deceleartion by as much as $4.7 \times 10^5 m/s^2$.

This deceleration causes the cooling.

A. Transverse Cooling

Here 2 counter propagating LASER beams with s being very large interact with an atomic beam perpendicularly. The atoms with tarnsverse veleocity magnitude v will feel a retarding force of magnitude,

$$F(v) = \frac{\hbar k \Gamma s}{2} \{ \frac{1}{1 + s + 4\left(\frac{\delta_L - kv}{\Gamma}\right)^2} - \frac{1}{1 + s + 4\left(\frac{\delta_L + kv}{\Gamma}\right)^2} \}$$

One notes that a high intensity beam switches the population and teh force is not just a linear superposition. Hence to get resonance the LASER ferquency used for transverse cooling is red-detuned.

VI. VOIGT PROFILE

The basic set is as shown in the diagram attached that of collimating a diverging beam of atoms coming from an oven by a slit. We denote by b the width of the slit and θ be the half-angle of the emerging beams from the oven. Let the slit be at a distance of d from teh oven and at the point when the LASER intersects the beam let the velocity along the direction parallel to the beam be called v_x and the orthogonal component be called v_z .

Then one has the following defintion,

$$Collimation\ Ratio = \frac{v_x}{v_z} = tan\epsilon = \frac{\frac{b}{2}}{d}$$

Let $n(v)dv$ number of the molecules with velocity in the range $(v, v+dv)$ at thermal equilibrium which effuses out at the most probable velocity of $v_p = \sqrt{\frac{2kT}{m}}$ and then we have from Maxwell-Boltzman statistics,

$$n(v)dv = A\frac{Cos(\epsilon)}{r^2}nv^2 \exp^{-\left(\frac{v}{v_p}\right)^2} dv$$

where $A = \frac{4}{v_p^3\sqrt{\pi}}$ determined to normalize $\int n(v)dv = n$.

Molecular transition at ω_0 in the rest frame is shifted to $\omega_0' = \omega_0 + kv_x$.

One defines a model of loss of power through the function $\alpha(x,\omega)$ so that the if the LASER beam is travelling along the x-direction from x_1 to x_2 then its power decreases as,

$$P(\omega) = P_0 \exp^{-\int_{x_1}^{x_2}\alpha(x,\omega)dx}$$

Typically the dissipation in such experiments reduces power by a factor of 10^{-4} to 10^{-15}.

One defnes the $a$bsorption cross-section $\sigma(v_x,\omega)$ such that,

$$\alpha(x,\omega) = \int n(v_x, x)\sigma(v_x,\omega)dx$$

Then the spectral profile of the absorbed power is given as,

$$\delta P(\omega) = P_0\int_{-\infty}^{\infty}\left[\int_{x_1}^{x_2} n(v_x,x)\sigma(\omega,v_x)dx\right]dv_x$$

Now one can substitute $v_x = \frac{x}{r}v$, $dv_x = \frac{x}{r}dv$ and $cos(\theta) = \frac{z}{r}$ and one puts in a model for the absorption of the monochromatic wave of frequecny ω by a molecule with velocity component v_x.

Such models will generically give expressions like the one below with arbitary parameters like σ_0 and γ (damping factors),

$$\sigma(\omega, v_x) = \sigma_0\frac{(\frac{\gamma}{2})^2}{(\omega-\omega_0-kv_x)^2+(\frac{\gamma}{2})^2}$$

Now one does the following standard set of variable re-definitions,

$$a = \left(\frac{P_0 n\sigma_0\gamma c^3 z}{\sqrt{\pi}v_p^3\omega_0^3}\right)\left(\frac{c\gamma}{2z\omega_0}\right)^2$$

$$x_1 = -rsin(\epsilon)$$

$$x_2 = rsin(\epsilon)$$

$$\omega_0' = \omega_0 + kv_x$$

Then one has,

$$\delta P(\omega) = a \int_{-\infty}^{\infty} \frac{\exp^{-\left(\frac{c(\omega-\omega_0')}{\omega_0' v_p sin\epsilon}\right)^2}}{(\omega-\omega_0')^2 + \left(\frac{\gamma}{2}\right)^2} d\omega_0'$$

The above convolution product between the Lorentzian of half-width γ and the Doppler function is calld the **Voigt Profile**

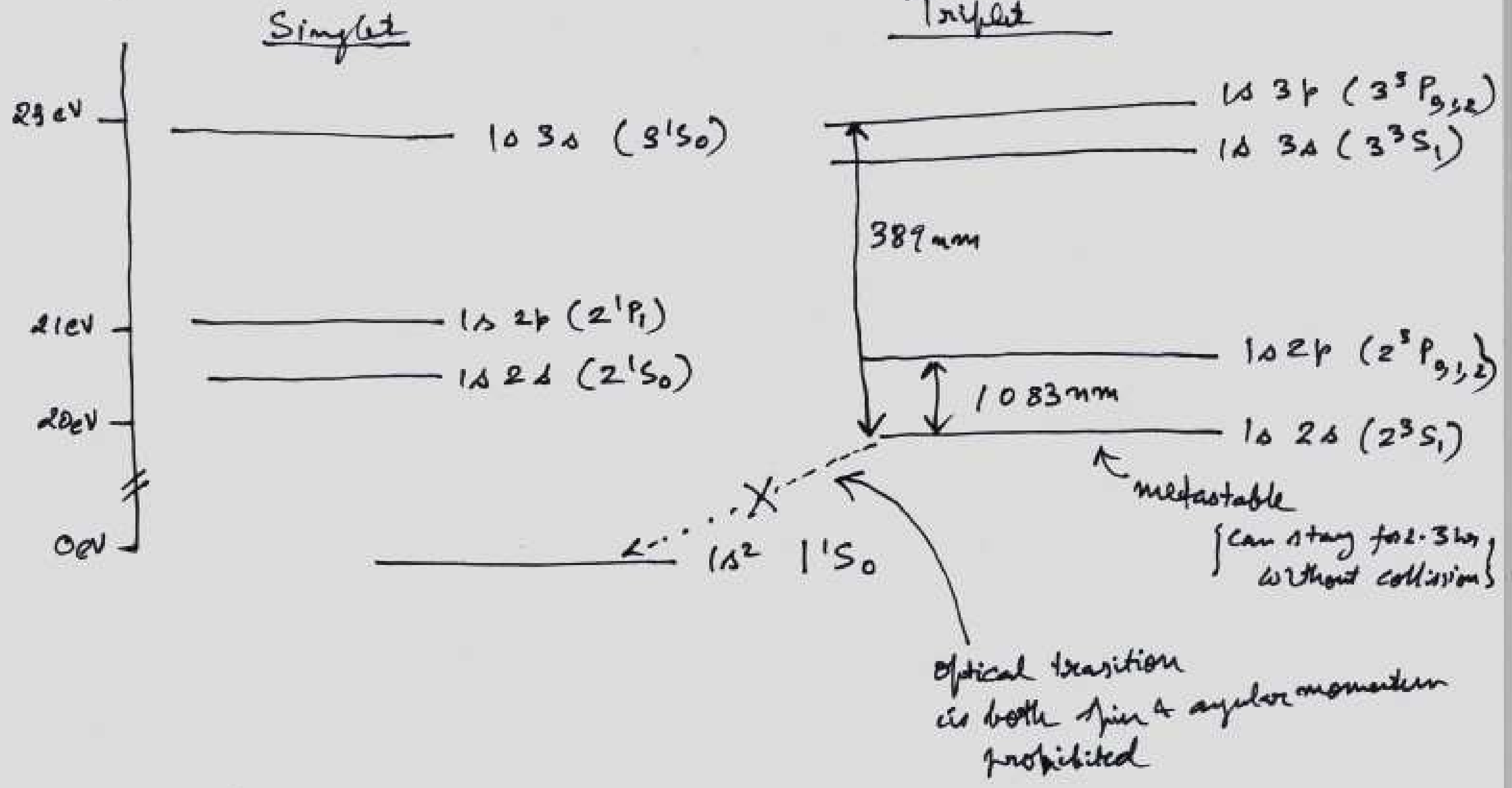
One can observe that the Doppler width is reduced by a factor of $sin\epsilon = \frac{v_x}{v} = \frac{b}{2d} = Collimation\ Ratio$

Hence one sees how collimation of the atomic beam reduces the Doppler width by a factor of $sin\epsilon$

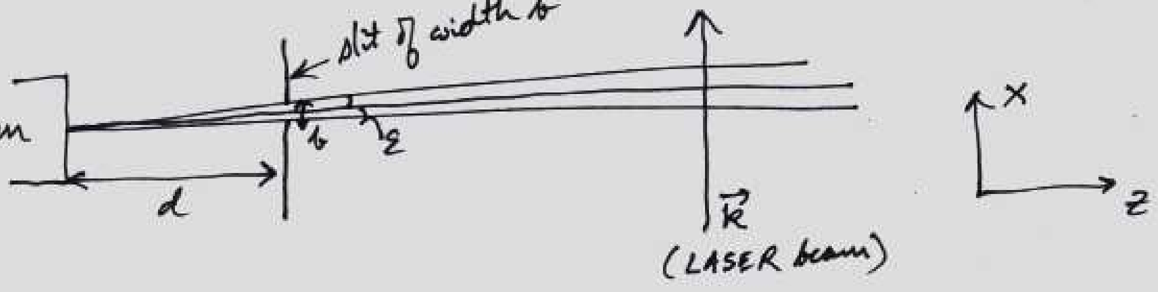
VII. REFERENCES

- PRL 94, 133001 (2005)
- Chapters 2, 3,4 and 9 of the Volume 1 of the book on Laser Spectroscopy by Demtrodder.
- The PhD thesis of L.B.Wang at UIUC in 2004.

(II) First few energy levels of ^4He



(VI) Set up to understand Voigt Profile.



(III) Experimental set-up.