"large N" $\mathcal{N} = 2$ super-Chern-Simons-matter theory

Anirbit

DTP, TIFR

July 10, 2011

Coming up next!

Studies done during the last 1 year of MSc. internship

Studies done during the last 1 year of MSc. internship

- Officially my MSc. internship started from August 2010. During the semester before that I had done a preliminary course in elementary QFT and in parallel had been studying CFT under the guidance of Prof.Shiraz.

Studies done during the last 1 year of MSc. internship

- Officially my MSc. internship started from August 2010. During the semester before that I had done a preliminary course in elementary QFT and in parallel had been studying CFT under the guidance of Prof.Shiraz.
- During the first half of my MSc. internship, I was learning about supersymmetry and superconformality.

Studies done during the last 1 year of MSc. internship

- Officially my MSc. internship started from August 2010. During the semester before that I had done a preliminary course in elementary QFT and in parallel had been studying CFT under the guidance of Prof.Shiraz.
- During the first half of my MSc. internship, I was learning about supersymmetry and superconformality.
- From the papers of Kao and Lee, I learnt about the technique of "dimensional reduction" to construct $\mathcal{N} = 2$ supersymmetric Chern-Simons theories in $2 + 1$ dimensions.

Studies done during the last 1 year of MSc. internship

- Officially my MSc. internship started from August 2010. During the semester before that I had done a preliminary course in elementary QFT and in parallel had been studying CFT under the guidance of Prof.Shiraz.
- During the first half of my MSc. internship, I was learning about supersymmetry and superconformality.
- From the papers of Kao and Lee, I learnt about the technique of "dimensional reduction" to construct $\mathcal{N} = 2$ supersymmetric Chern-Simons theories in $2+1$ dimensions.
- Along with the above I was trying to familiarize myself with the various previous works of Prof.Shiraz about calculating the superconformal index and the various foundational papers by Dunne, Chen, Semenoff, Wu, Kao and Lee about renormalization of supersymmetric Chern-Simons theory.

Studies done during the last 1 year of MSc. internship

- Officially my MSc. internship started from August 2010. During the semester before that I had done a preliminary course in elementary QFT and in parallel had been studying CFT under the guidance of Prof.Shiraz.
- During the first half of my MSc. internship, I was learning about supersymmetry and superconformality.
- From the papers of Kao and Lee, I learnt about the technique of "dimensional reduction" to construct $\mathcal{N} = 2$ supersymmetric Chern-Simons theories in $2 + 1$ dimensions.
- Along with the above I was trying to familiarize myself with the various previous works of Prof.Shiraz about calculating the superconformal index and the various foundational papers by Dunne, Chen, Semenoff, Wu, Kao and Lee about renormalization of supersymmetric Chern-Simons theory.

Studies done during the last 1 year of MSc. internship...contd.

Studies done during the last 1 year of MSc. internship...contd.

- During the second half of my project I started working on some calculations about counting the total number of states in certain superconformal theories and their BPS spectrum/"Q-cohomology".

Studies done during the last 1 year of MSc. internship...contd.

- During the second half of my project I started working on some calculations about counting the total number of states in certain superconformal theories and their BPS spectrum/"Q-cohomology". (This pursuit shall be the main part of this seminar)

Studies done during the last 1 year of MSc. internship...contd.

- During the second half of my project I started working on some calculations about counting the total number of states in certain superconformal theories and their BPS spectrum/"Q-cohomology". (This pursuit shall be the main part of this seminar)
- I also learnt methods of Geometric Invariant Theory (GIT) in trying to find a description of the conformal manifold of a fixed kind of matter coupling to $\mathcal{N} = 2$ super-Chern-Simons theory. This was motivated by a recent geometric prescription for this given by Daniel Green, Zohar Komargodski, Nathan Seiberg, Yuji Tachikawa and Brian Wecht.

Studies done during the last 1 year of MSc. internship...contd.

- During the second half of my project I started working on some calculations about counting the total number of states in certain superconformal theories and their BPS spectrum/"Q-cohomology". (This pursuit shall be the main part of this seminar)
- I also learnt methods of Geometric Invariant Theory (GIT) in trying to find a description of the conformal manifold of a fixed kind of matter coupling to $\mathcal{N} = 2$ super-Chern-Simons theory. This was motivated by a recent geometric prescription for this given by Daniel Green, Zohar Komargodski, Nathan Seiberg, Yuji Tachikawa and Brian Wecht.

Studies done during the last 1 year of MSc. internship...contd.

- During the second half of my project I started working on some calculations about counting the total number of states in certain superconformal theories and their BPS spectrum/"Q-cohomology". (This pursuit shall be the main part of this seminar)
- I also learnt methods of Geometric Invariant Theory (GIT) in trying to find a description of the conformal manifold of a fixed kind of matter coupling to $\mathcal{N} = 2$ super-Chern-Simons theory. This was motivated by a recent geometric prescription for this given by Daniel Green, Zohar Komargodski, Nathan Seiberg, Yuji Tachikawa and Brian Wecht.

My work during this internship was strongly influenced by the recent paper "Supersymmetric States in Large N Chern-Simons-Matter Theories" [arXiv:hep-th/1104.0680] by Shiraz Minwalla, Prithvi Narayan, Tarun Sharma, V. Umesh and Xi Yin.

Studies done during the last 1 year of MSc. internship...contd.

- During the second half of my project I started working on some calculations about counting the total number of states in certain superconformal theories and their BPS spectrum/"Q-cohomology". (This pursuit shall be the main part of this seminar)
- I also learnt methods of Geometric Invariant Theory (GIT) in trying to find a description of the conformal manifold of a fixed kind of matter coupling to $\mathcal{N} = 2$ super-Chern-Simons theory. This was motivated by a recent geometric prescription for this given by Daniel Green, Zohar Komargodski, Nathan Seiberg, Yuji Tachikawa and Brian Wecht.

My work during this internship was strongly influenced by the recent paper "Supersymmetric States in Large N Chern-Simons-Matter Theories" [arXiv:hep-th/1104.0680] by Shiraz Minwalla, Prithvi Narayan, Tarun Sharma, V. Umesh and Xi Yin.I shall be largely using the notations and formalisms of that paper.

Coming up next!

Motivations from recent research developments

Motivations from recent research developments

- In a beautiful paper in 2004 John Schwartz had pointed out the possibility of being able to supersymmetrize the Chern-Simons theory coupled to matter. With even low amounts of supersymmetry this theory already gives a natural example of the phenomenon of superconformality.

Motivations from recent research developments

- In a beautiful paper in 2004 John Schwartz had pointed out the possibility of being able to supersymmetrize the Chern-Simons theory coupled to matter. With even low amounts of supersymmetry this theory already gives a natural example of the phenomenon of superconformality. Otherwise in $3+1$ dimensions the ubiquitous scenario is of conformality being broken by quantum corrections though with high amounts of supersymmetry like in $\mathcal{N}=4$ SYM that can be saved.

Motivations from recent research developments

- In a beautiful paper in 2004 John Schwartz had pointed out the possibility of being able to supersymmetrize the Chern-Simons theory coupled to matter. With even low amounts of supersymmetry this theory already gives a natural example of the phenomenon of superconformality. Otherwise in $3+1$ dimensions the ubiquitous scenario is of conformality being broken by quantum corrections though with high amounts of supersymmetry like in $\mathcal{N}=4$ SYM that can be saved.
- Superconformal theories because of their high degree of symmetry give an increased analytic control for calculating quantities which are very likely to be invariant of the 'tHooft coupling and hence can provide a way of checking conjectured strong/weak coupling dualities like the "AdS/CFT" conjecture by Maldacena.

Motivations from recent research developments

- In a beautiful paper in 2004 John Schwartz had pointed out the possibility of being able to supersymmetrize the Chern-Simons theory coupled to matter. With even low amounts of supersymmetry this theory already gives a natural example of the phenomenon of superconformality. Otherwise in $3+1$ dimensions the ubiquitous scenario is of conformality being broken by quantum corrections though with high amounts of supersymmetry like in $\mathcal{N}=4$ SYM that can be saved.
- Superconformal theories because of their high degree of symmetry give an increased analytic control for calculating quantities which are very likely to be invariant of the 'tHooft coupling and hence can provide a way of checking conjectured strong/weak coupling dualities like the "AdS/CFT" conjecture by Maldacena.

Motivations from recent research developments...contd.

Motivations from recent research developments...contd.

- In a seminal paper in 2007 Davide Gaiotto and Xi Yin among other things had brought to light the exciting properties of a particular superconformal field theory namely the $\mathcal{N} = 2$ super-Chern-Simons-matter theory in $2 + 1$ dimensions and that will be of central interest in this article.

Motivations from recent research developments...contd.

- In a seminal paper in 2007 Davide Gaiotto and Xi Yin among other things had brought to light the exciting properties of a particular superconformal field theory namely the $\mathcal{N} = 2$ super-Chern-Simons-matter theory in $2 + 1$ dimensions and that will be of central interest in this article.One of the interesting things that they had argued in that paper is that the matter coupling in their Lagrangian flows under renormalization such that there is a conformal fixed point and there the supersymmetry enhances to $\mathcal{N} = 3$!

Motivations from recent research developments...contdd.

- In a seminal paper in 2007 Davide Gaiotto and Xi Yin among other things had brought to light the exciting properties of a particular superconformal field theory namely the $\mathcal{N} = 2$ super-Chern-Simons-matter theory in $2 + 1$ dimensions and that will be of central interest in this article.One of the interesting things that they had argued in that paper is that the matter coupling in their Lagrangian flows under renormalization such that there is a conformal fixed point and there the supersymmetry enhances to $\mathcal{N} = 3$!

Motivating the specific objective of my studies

Motivating the specific objective of my studies

- It is believed that the spectrum of BPS states is invariant under the change of the t'Hooft coupling in the t'Hooft limit and hence being able to calculate that in superconformal theories has been the focus of many enlightening recent papers like those by Christian Romelsberger, F.A.Dolan, Bo Feng, Amihay Hanany, Yang-Hui He etc.

Motivating the specific objective of my studies

- It is believed that the spectrum of BPS states is invariant under the change of the t'Hooft coupling in the t'Hooft limit and hence being able to calculate that in superconformal theories has been the focus of many enlightening recent papers like those by Christian Romelsberger, F.A.Dolan, Bo Feng, Amihay Hanany, Yang-Hui He etc.
- Many of the combinatorial techniques which permeate these papers and the related insights about the simplifications that happen in the "large N" limit were pointed out by Polyakov in an extremely insightful paper in 2002.

Motivating the specific objective of my studies

- It is believed that the spectrum of BPS states is invariant under the change of the t'Hooft coupling in the t'Hooft limit and hence being able to calculate that in superconformal theories has been the focus of many enlightening recent papers like those by Christian Romelsberger, F.A.Dolan, Bo Feng, Amihay Hanany, Yang-Hui He etc.
- Many of the combinatorial techniques which permeate these papers and the related insights about the simplifications that happen in the "large N" limit were pointed out by Polyakov in an extremely insightful paper in 2002.

Motivating the specific objective of my studies...contd.

Motivating the specific objective of my studies...contd.

- If one chooses a supercharge then in superconformal theories it acts as an endomorphism on the space of states at a fixed value of "energy" (scaling dimension $+$ $z-$component of spin).

Motivating the specific objective of my studies...contd.

- If one chooses a supercharge then in superconformal theories it acts as an endomorphism on the space of states at a fixed value of "energy" (scaling dimension + $z-$component of spin). There are reasons to believe that for a chosen supercharge (say Q) and an energy level, the equivalence classes, $Ker(Q)/Im(Q)$, of the above action, calculated in the classical theory are in bijection with the BPS states at that energy level.

Motivating the specific objective of my studies...contd.

- If one chooses a supercharge then in superconformal theories it acts as an endomorphism on the space of states at a fixed value of "energy" (scaling dimension + $z-$component of spin).There are reasons to believe that for a chosen supercharge (say Q) and an energy level, the equivalence classes, $Ker(Q)/Im(Q)$, of the above action, calculated in the classical theory are in bijection with the BPS states at that energy level.Similar motivational reasons, as behind the works cited above, lead one to believe that this "Q-cohomology" is also invariant under deformations as mentioned above and hence calculating that paves another way to test various dualities.

Motivating the specific objective of my studies...contd.

- If one chooses a supercharge then in superconformal theories it acts as an endomorphism on the space of states at a fixed value of "energy" (scaling dimension + $z-$component of spin).There are reasons to believe that for a chosen supercharge (say Q) and an energy level, the equivalence classes, $Ker(Q)/Im(Q)$, of the above action, calculated in the classical theory are in bijection with the BPS states at that energy level.Similar motivational reasons, as behind the works cited above, lead one to believe that this "Q-cohomology" is also invariant under deformations as mentioned above and hence calculating that paves another way to test various dualities.
- During this project I found an in-principle method of being able to write down the "exact" number of states at a fixed value of "energy".

Motivating the specific objective of my studies...contd.

- If one chooses a supercharge then in superconformal theories it acts as an endomorphism on the space of states at a fixed value of "energy" (scaling dimension + $z-$component of spin).There are reasons to believe that for a chosen supercharge (say Q) and an energy level, the equivalence classes, $Ker(Q)/Im(Q)$, of the above action, calculated in the classical theory are in bijection with the BPS states at that energy level.Similar motivational reasons, as behind the works cited above, lead one to believe that this "Q-cohomology" is also invariant under deformations as mentioned above and hence calculating that paves another way to test various dualities.
- During this project I found an in-principle method of being able to write down the "exact" number of states at a fixed value of "energy".An explicit expression will be given to achieve it and though I don't know of a closed form solution, an approximation technique to evaluate it will be explained.

Motivating the specific objective of my studies...contd.

- If one chooses a supercharge then in superconformal theories it acts as an endomorphism on the space of states at a fixed value of "energy" (scaling dimension + $z-$component of spin). There are reasons to believe that for a chosen supercharge (say Q) and an energy level, the equivalence classes, $Ker(Q)/Im(Q)$, of the above action, calculated in the classical theory are in bijection with the BPS states at that energy level. Similar motivational reasons, as behind the works cited above, lead one to believe that this "Q-cohomology" is also invariant under deformations as mentioned above and hence calculating that paves another way to test various dualities.
- During this project I found an in-principle method of being able to write down the "exact" number of states at a fixed value of "energy". An explicit expression will be given to achieve it and though I don't know of a closed form solution, an approximation technique to evaluate it will be explained.

Motivating the specific objective of my studies...contd.

Motivating the specific objective of my studies...contd.

- One way to test my quite simple approximation technique will be to see that the Hagedorn transition temperature that it predicts agrees very closely with the exact results for this temperature (obtained by much more sophisticated techniques) as obtained in a paper by by Ofer Aharony, Joseph Marsano, Shiraz Minwalla, Kyriakos Papadodimas and Mark Van Raamsdonk.

Motivating the specific objective of my studies...contd.

- One way to test my quite simple approximation technique will be to see that the Hagedorn transition temperature that it predicts agrees very closely with the exact results for this temperature (obtained by much more sophisticated techniques) as obtained in a paper by by Ofer Aharony, Joseph Marsano, Shiraz Minwalla, Kyriakos Papadodimas and Mark Van Raamsdonk.
- I shall also report some data that I calculated about the first few "Q-cohomologies". Here I do not yet have any exact result like for the above.

Coming up next!

The quantum numbers

The quantum numbers

In a superconformal theory one can choose as the simultaneously specifiable quantum numbers the following attributes of the fields/operators, namely scaling dimension (Δ), the "z" component of the spin (j) and the $R-$charge (h).

The quantum numbers

In a superconformal theory one can choose as the simultaneously specifiable quantum numbers the following attributes of the fields/operators, namely scaling dimension (Δ), the "z" component of the spin (j) and the $R-$charge (h).

For every operator one can write the $3-$tuple of the above in the order, (Δ, j, h). I shall be writing all my supersymmetry equations on-shell and hence I have only a scalar field ϕ and a chosen fermionic field component χ.

The quantum numbers

In a superconformal theory one can choose as the simultaneously specifiable quantum numbers the following attributes of the fields/operators, namely scaling dimension (Δ), the "z" component of the spin (j) and the $R-$charge (h).

For every operator one can write the $3-$tuple of the above in the order, (Δ, j, h). I shall be writing all my supersymmetry equations on-shell and hence I have only a scalar field ϕ and a chosen fermionic field component χ.Similarly the letter Q and D shall denote a particular supercharge and superderivative.

The quantum numbers

In a superconformal theory one can choose as the simultaneously specifiable quantum numbers the following attributes of the fields/operators, namely scaling dimension (Δ), the "z" component of the spin (j) and the $R-$charge (h).

For every operator one can write the $3-$tuple of the above in the order, (Δ, j, h). I shall be writing all my supersymmetry equations on-shell and hence I have only a scalar field ϕ and a chosen fermionic field component χ.Similarly the letter Q and D shall denote a particular supercharge and superderivative.

The charge assignments are as follows,

The quantum numbers

In a superconformal theory one can choose as the simultaneously specifiable quantum numbers the following attributes of the fields/operators, namely scaling dimension (Δ), the "z" component of the spin (j) and the $R-$charge (h).

For every operator one can write the $3-$tuple of the above in the order, (Δ, j, h). I shall be writing all my supersymmetry equations on-shell and hence I have only a scalar field ϕ and a chosen fermionic field component χ.Similarly the letter Q and D shall denote a particular supercharge and superderivative.

The charge assignments are as follows,

operator	(Δ, j, h)
ϕ	$(\frac{1}{2}, 0, \frac{1}{2})$
χ	$(1, \frac{1}{2}, \frac{1}{2})$
Q	$(\frac{1}{2}, -\frac{1}{2}, 1)$
D	$(1, 1, 0)$

The conserved quantity

One notes the following,

The conserved quantity

One notes the following,

- These "charges" are additive over taking product of operators.

The conserved quantity

One notes the following,

- These "charges" are additive over taking product of operators.
- Though the particular sequence in which the operators occur in a composite operator is important (up to cyclic permutation because of the overall trace over the gauge index in the actual operators) for the sake of counting its quantum number it is not.

The conserved quantity

One notes the following,

- These "charges" are additive over taking product of operators.
- Though the particular sequence in which the operators occur in a composite operator is important (up to cyclic permutation because of the overall trace over the gauge index in the actual operators) for the sake of counting its quantum number it is not. Hence for the sake of counting quantum numbers the most general operator can be thought of as "$\phi^n \chi^m D^k$" with the quantum numbers $(\frac{n}{2} + m + k, \frac{m}{2} + k, \frac{n}{2} + \frac{m}{2})$. Then after the action of the supersymmetry operator the quantum numbers of $Q(\phi^n \chi^m D^k)$ are $(\frac{n}{2} + m + k + \frac{1}{2}, \frac{m}{2} + k - \frac{1}{2}, \frac{n}{2} + \frac{m}{2} + 1)$.

The conserved quantity

One notes the following,

- These "charges" are additive over taking product of operators.
- Though the particular sequence in which the operators occur in a composite operator is important (up to cyclic permutation because of the overall trace over the gauge index in the actual operators) for the sake of counting its quantum number it is not. Hence for the sake of counting quantum numbers the most general operator can be thought of as "$\phi^n \chi^m D^k$" with the quantum numbers $(\frac{n}{2} + m + k, \frac{m}{2} + k, \frac{n}{2} + \frac{m}{2})$. Then after the action of the supersymmetry operator the quantum numbers of $Q(\phi^n \chi^m D^k)$ are $(\frac{n}{2} + m + k + \frac{1}{2}, \frac{m}{2} + k - \frac{1}{2}, \frac{n}{2} + \frac{m}{2} + 1)$.
- The sum $\Delta + j$ is conserved by the action of supersymmetry. Hence the action of Q preserves the value of $\Delta + j$ as $\frac{n}{2} + \frac{3m}{2} + 2k$. Coupled to the fact that Q is linear over "words" (states/operators) in the "letters" ϕ, χ and D it is consistent to talk of the matrix of Q at every value of $\Delta + j$.

Some definitions

Some definitions

- $\Delta + j$ will be the parameter by which the Witten Index is usually weighted and also the parameter using which I will organize the list of all possible operators.

Some definitions

- $\Delta + j$ will be the parameter by which the Witten Index is usually weighted and also the parameter using which I will organize the list of all possible operators.
- If given an operator O_1 there exists another operator O_2 such that $O_1 = Q(O_2)$ then O_1 will be called $Q - exact$. If for an operator O it is true that $Q(O) = 0$ then O will be called $Q - closed$.

A combinatorial formulation of the theory

A combinatorial formulation of the theory

In line with the paper by Shiraz et.al cited earlier, I shall give a purely combinatorial formulation of the theory which shall be enough to capture the essential phenomenon.

A combinatorial formulation of the theory

In line with the paper by Shiraz et.al cited earlier, I shall give a purely combinatorial formulation of the theory which shall be enough to capture the essential phenomenon.

- For the theory at hand the action of the supersymmetry operator on the fields is given as,
  - $Q\phi = [Q, \phi] = 0$
  - $Q\chi = \{Q, \chi\} = 0$(no superpotential)(or $= \lambda\phi^3$with a superpotential)
  - $Q(DO) = [[\phi, \chi], O\} + D(QO)$

  where,
  - O is any arbitrary operator made up of only ϕ and χ.
  - The notation $[, \}$ means that it will be a commutator or an anti-commutator depending on whether O is bosonic or fermionic respectively. O will be bosonic or fermionic depending on whether it has even or odd number of χs in it.

A combinatorial formulation of the theory..contd.

A combinatorial formulation of the theory..contd.

- If A and B are two operators made up of ϕ, χ and D only and $f(A)$ be the number of fermions (χs) in A then D acts as a normal derivative on the product AB and Q acts like a graded derivative as follows,

$$D(AB) = (DA)B + A(DB)$$
$$Q(AB) = (QA)B + (-1)^{f(A)}A(QB)$$

A combinatorial formulation of the theory..contd.

- If A and B are two operators made up of ϕ, χ and D only and $f(A)$ be the number of fermions (χs) in A then D acts as a normal derivative on the product AB and Q acts like a graded derivative as follows,

$$D(AB) = (DA)B + A(DB)$$
$$Q(AB) = (QA)B + (-1)^{f(A)} A(QB)$$

It was shown using inductive combinatorial techniques that the above axioms guarantee that Q^2 annihilates any state made up of arbitrary number of ϕs, χs and D's.

A combinatorial formulation of the theory..contd.

- If A and B are two operators made up of ϕ, χ and D only and $f(A)$ be the number of fermions (χs) in A then D acts as a normal derivative on the product AB and Q acts like a graded derivative as follows,

$$D(AB) = (DA)B + A(DB)$$
$$Q(AB) = (QA)B + (-1)^{f(A)}A(QB)$$

It was shown using inductive combinatorial techniques that the above axioms guarantee that Q^2 annihilates any state made up of arbitrary number of ϕs, χs and D's.

The matrix of Q

The matrix of Q

- The result of the action of Q on various members of an equivalence class of strings of letters up to cyclic permutation can only differ by an overall sign. This will be tantamount to flipping all the signs of one or more columns in the matrix for Q at any value of $\Delta + j$. But doing this does not change the rank or the dimension of the kernel of the matrix - the only two properties of the matrix that will be of relevance here. Hence it is consistent to live with the sign ambiguity resulting from arbitrarily picking the representative from each equivalence class under cyclic permutation of the "words" in the "letters".

The matrix of Q

- The result of the action of Q on various members of an equivalence class of strings of letters up to cyclic permutation can only differ by an overall sign. This will be tantamount to flipping all the signs of one or more columns in the matrix for Q at any value of $\Delta + j$. But doing this does not change the rank or the dimension of the kernel of the matrix - the only two properties of the matrix that will be of relevance here. Hence it is consistent to live with the sign ambiguity resulting from arbitrarily picking the representative from each equivalence class under cyclic permutation of the "words" in the "letters".

Coming up next!

Organizing the state counting

Organizing the state counting

- If O is any arbitrary state/operator then one can show that $Q(DO) = 0$ iff $Q(O) = 0$. This implies that at any value of $\Delta + j$ one can restrict to only those states which are not total derivatives since any $Q-$closed total derivative state is necessarily a "descendant" of some $Q-$closed state at $\Delta + j - 2$ and hence in some sense it is not a "new" state.

Organizing the state counting

- If O is any arbitrary state/operator then one can show that $Q(DO) = 0$ iff $Q(O) = 0$. This implies that at any value of $\Delta + j$ one can restrict to only those states which are not total derivatives since any $Q-$closed total derivative state is necessarily a "descendant" of some $Q-$closed state at $\Delta + j - 2$ and hence in some sense it is not a "new" state. But this fact need not be taken into account while doing a counting of the total number of states though this refinement might help in understanding the "Q-cohomology".

Organizing the state counting

- If O is any arbitrary state/operator then one can show that $Q(DO) = 0$ iff $Q(O) = 0$. This implies that at any value of $\Delta + j$ one can restrict to only those states which are not total derivatives since any $Q-$closed total derivative state is necessarily a "descendant" of some $Q-$closed state at $\Delta + j - 2$ and hence in some sense it is not a "new" state. But this fact need not be taken into account while doing a counting of the total number of states though this refinement might help in understanding the "Q-cohomology".
- In this listing if two operators are produced such that one when expanded by using the Leibnitz property of the D gives some scalar multiple of the other then only one representative form of the term will be listed. For the ease of counting purposes I will always list the term where D is acting on only one operator.

Organizing the state counting

- If O is any arbitrary state/operator then one can show that $Q(DO) = 0$ iff $Q(O) = 0$. This implies that at any value of $\Delta + j$ one can restrict to only those states which are not total derivatives since any $Q-$closed total derivative state is necessarily a "descendant" of some $Q-$closed state at $\Delta + j - 2$ and hence in some sense it is not a "new" state. But this fact need not be taken into account while doing a counting of the total number of states though this refinement might help in understanding the "Q-cohomology".
- In this listing if two operators are produced such that one when expanded by using the Leibnitz property of the D gives some scalar multiple of the other then only one representative form of the term will be listed. For the ease of counting purposes I will always list the term where D is acting on only one operator.

Organizing the state counting...contd.

Organizing the state counting...contd.

The generic operator will be gotten by various permutations (up to cyclic permutations) of all the strings of the form $\phi^m \chi^m D^k$ such that it has the $\Delta + j$ being listed then i.e they satisfy, $\frac{n}{2} + \frac{3m}{2} + 2k = \Delta + j$

Organizing the state counting...contd.

The generic operator will be gotten by various permutations (up to cyclic permutations) of all the strings of the form $\phi^m \chi^m D^k$ such that it has the $\Delta + j$ being listed then i.e they satisfy, $\frac{n}{2} + \frac{3m}{2} + 2k = \Delta + j$

$\Delta + j$	linearly independent $Q - closed$	linearly independent not $Q - closed$	linearly dependent not $Q - exact$
$\frac{1}{2}$	ϕ		
1	ϕ^2		
$\frac{3}{2}$	ϕ^3, χ		
2	$\phi\chi$, ϕ^4, (D ?)		
$\frac{5}{2}$	$\phi^2\chi$, ϕ^5, $D\phi$		
3	χ^2, $\phi^3\chi$, ϕ^6, $\phi D(\phi) = \frac{1}{2}D(\phi^2)$		
$\frac{7}{2}$	$\phi\chi^2$, $\phi^4\chi$, ϕ^7, $D(\chi)$, $\phi^2 D(\phi)$		
4	ϕ^8, $\phi^5\chi$, $\phi^2\chi^2$, $\chi\phi\chi\phi$, $\phi^3 D(\phi)$, $\phi D(\chi)$	$\chi D(\phi)$	$D(\phi\chi)$, D^2 (?)
$\frac{9}{2}$	ϕ^9, $\phi^6\chi$, $\phi^3\chi^2$, $\phi\chi\phi\chi\phi$, χ^3, $\phi^4 D(\phi)$, $\phi^2 D(\chi)$, $D^2\phi$	$\phi\chi D(\phi)$, $\chi\phi D(\phi)$	$\chi D(\phi^2)$, $\phi D(\chi\phi)$, $\phi D(\phi\chi)$, $D(\phi^2\chi)$, $D(\chi\phi^2)$
5	ϕ^{10}, $\phi^7\chi$, $\phi^4\chi^2$, $\phi^3\chi\phi\chi$, $\phi^2\chi\phi^2\chi$, $\phi\chi^3$, $\phi^5 D(\phi)$, $\chi D(\chi)$, $\phi^3 D(\chi)$	$\phi^2\chi D(\phi)$, $\chi\phi^2 D(\phi)$, $\phi\chi\phi D(\phi)$, $(D\phi)^2$, $\phi D^2(\phi)$(?)	$\phi^2 D(\phi\chi)$, $\phi^2 D(\chi\phi)$, $\phi D(\phi^2\chi)$, $\phi D(\phi\chi\phi)$, $\phi D(\chi\phi^2)$, $D(\chi\phi^3)$, $D(\phi^3\chi)$, $D(\phi\chi\phi^2)$, $D(\phi^2\chi\phi)$

An approximation in counting the number of states at any given $\Delta + j$

An approximation in counting the number of states at any given $\Delta + j$

For the sake of analytic tractability, in the counting of number of linearly independent states at every $\Delta + j$, I shall ignore the fermionic nature of χ. One can possibly account for this detail but the point is that eventually the Hagedorn transition temperature depends only on the size of the vector space dimension in the large $\Delta + j$ limit. This vector space dimension is expected to show an exponential growth and the claim is that this asymptotic behaviour will remain unaffected by the inclusion of these small number of extra states which are actually 0.

An approximation in counting the number of states at any given $\Delta + j$

For the sake of analytic tractability, in the counting of number of linearly independent states at every $\Delta + j$, I shall ignore the fermionic nature of χ. One can possibly account for this detail but the point is that eventually the Hagedorn transition temperature depends only on the size of the vector space dimension in the large $\Delta + j$ limit. This vector space dimension is expected to show an exponential growth and the claim is that this asymptotic behaviour will remain unaffected by the inclusion of these small number of extra states which are actually 0.

Steps towards counting the number of states at any given $\Delta + j$

Steps towards counting the number of states at any given $\Delta + j$

The key ideas that go into doing this counting are as follows,

Steps towards counting the number of states at any given $\Delta + j$

The key ideas that go into doing this counting are as follows,

- At every $\Delta + j$ the basis consists of only such terms where there are either no D's or the D acts on just one field ϕ or χ.

Steps towards counting the number of states at any given $\Delta + j$

The key ideas that go into doing this counting are as follows,

- At every $\Delta + j$ the basis consists of only such terms where there are either no D's or the D acts on just one field ϕ or χ.
- One notes that if a term consists of n ϕs, m χs and k Ds then its $\Delta + j$ is given by the equation, $n + 3m + 4k = 2(\Delta + j)$. Hence at each $\Delta + j$ the terms can be classified by the number of operators it has of each type i.e by the positive integral solutions of the above Diophantine equation.

Steps towards counting the number of states at any given $\Delta + j$

The key ideas that go into doing this counting are as follows,

- At every $\Delta + j$ the basis consists of only such terms where there are either no D's or the D acts on just one field ϕ or χ.
- One notes that if a term consists of n ϕs, m χs and k Ds then its $\Delta + j$ is given by the equation, $n + 3m + 4k = 2(\Delta + j)$. Hence at each $\Delta + j$ the terms can be classified by the number of operators it has of each type i.e by the positive integral solutions of the above Diophantine equation.

Steps towards counting the number of states at any given $\Delta + j$...contd.

Steps towards counting the number of states at any given $\Delta + j$...contd.

- Given any one such term corresponding to a fixed positive integral solution of the above Diophantine equation, all the cyclic permutations of a string of letters correspond to the same operator since everything is under a trace. More importantly this is true even with a D. Hence it suffices to count permutations of strings of letters with fixed number of ϕs, χs and Ds up to cyclic permutation. (This could be reminiscent of what is called the "inclusion-exclusion" technique in combinatorics.)

Steps towards counting the number of states at any given $\Delta + j$...contd.

- Given any one such term corresponding to a fixed positive integral solution of the above Diophantine equation, all the cyclic permutations of a string of letters correspond to the same operator since everything is under a trace. More importantly this is true even with a D. Hence it suffices to count permutations of strings of letters with fixed number of ϕs, χs and Ds up to cyclic permutation. (This could be reminiscent of what is called the "inclusion-exclusion" technique in combinatorics.) This shall be achieved by Polya's theory.

The expression for the counting of the number of states at any given $\Delta + j$

The expression for the counting of the number of states at any given $\Delta + j$

Doing the above calculation (ignoring the fermionic nature of χ) the dimension of the vector space of operators at a given $\Delta + j$ is,

The expression for the counting of the number of states at any given $\Delta + j$

Doing the above calculation (ignoring the fermionic nature of χ) the dimension of the vector space of operators at a given $\Delta + j$ is,

$$\sum_{(n,m,k)\ s.t\ n+3m+4k=2(\Delta+j)} \frac{1}{n+m+k} \sum_{(d|n,d|m,d|k)} \phi(d) \frac{\left(\frac{n+m+k}{d}\right)!}{\left(\frac{n}{d}\right)!\left(\frac{m}{d}\right)!\left(\frac{k}{d}\right)!}$$

The expression for the counting of the number of states at any given $\Delta + j$

Doing the above calculation (ignoring the fermionic nature of χ) the dimension of the vector space of operators at a given $\Delta + j$ is,

$$\sum_{(n,m,k)\ s.t\ n+3m+4k=2(\Delta+j)} \frac{1}{n+m+k} \sum_{(d|n,d|m,d|k)} \phi(d) \frac{\left(\frac{n+m+k}{d}\right)!}{\left(\frac{n}{d}\right)!\left(\frac{m}{d}\right)!\left(\frac{k}{d}\right)!}$$

Approximating the expression for the counting of the number of states at any given $\Delta + j$

Approximating the expression for the counting of the number of states at any given $\Delta + j$

Here on I shall explain a way of getting an asymptotic estimate of this expression.

Approximating the expression for the counting of the number of states at any given $\Delta + j$

Here on I shall explain a way of getting an asymptotic estimate of this expression. The central observation is that such multinomial coefficients maximize "near the middle" or around when they are balanced.

Approximating the expression for the counting of the number of states at any given $\Delta + j$

Here on I shall explain a way of getting an asymptotic estimate of this expression. The central observation is that such multinomial coefficients maximize “near the middle” or around when they are balanced. Also common divisors, $d > 1$ will have their contributions to the sum hugely suppressed compared to the contribution coming from the term $d = 1$.

Approximating the expression for the counting of the number of states at any given $\Delta + j$

Here on I shall explain a way of getting an asymptotic estimate of this expression.The central observation is that such multinomial coefficients maximize "near the middle" or around when they are balanced. Also common divisors, $d > 1$ will have their contributions to the sum hugely suppressed compared to the contribution coming from the term $d = 1$.

One can always write the right hand side of the Diophantine equation as, $2(\Delta + j) = 8p + r$ for some $p \in \mathbb{Z}^+$ and $r \in \{0, 1, 2, ..., 7\}$.

Approximating the expression for the counting of the number of states at any given $\Delta + j$

Here on I shall explain a way of getting an asymptotic estimate of this expression.The central observation is that such multinomial coefficients maximize "near the middle" or around when they are balanced. Also common divisors, $d > 1$ will have their contributions to the sum hugely suppressed compared to the contribution coming from the term $d = 1$.

One can always write the right hand side of the Diophantine equation as, $2(\Delta + j) = 8p + r$ for some $p \in \mathbb{Z}^+$ and $r \in \{0, 1, 2, ..., 7\}$.In that case there is always a countably infinite family of solutions of the form, $n = p + r$, $m = p$ and $k = p$. (Of course these are not all the solutions.)

Approximating the expression for the counting of the number of states at any given $\Delta + j$

Here on I shall explain a way of getting an asymptotic estimate of this expression.The central observation is that such multinomial coefficients maximize "near the middle" or around when they are balanced. Also common divisors, $d > 1$ will have their contributions to the sum hugely suppressed compared to the contribution coming from the term $d = 1$.

One can always write the right hand side of the Diophantine equation as, $2(\Delta + j) = 8p + r$ for some $p \in \mathbb{Z}^+$ and $r \in \{0, 1, 2, ..., 7\}$.In that case there is always a countably infinite family of solutions of the form, $n = p + r$, $m = p$ and $k = p$. (Of course these are not all the solutions.)

Approximating the expression for the counting of the number of states at any given $\Delta + j$...contd.

Approximating the expression for the counting of the number of states at any given $\Delta + j$...contd.

The convenient thing about the particular family of solutions stated above is that they scale linearly with the right hand side $2(\Delta + j)$.

Approximating the expression for the counting of the number of states at any given $\Delta + j$...contd.

The convenient thing about the particular family of solutions stated above is that they scale linearly with the right hand side $2(\Delta + j)$.Since the series one is looking at is a sum of positive terms the growth of these terms will always give a lower bound and these are expected to be the dominant contributors.

Approximating the expression for the counting of the number of states at any given $\Delta + j$...contd.

The convenient thing about the particular family of solutions stated above is that they scale linearly with the right hand side $2(\Delta + j)$.Since the series one is looking at is a sum of positive terms the growth of these terms will always give a lower bound and these are expected to be the dominant contributors.

The method is not rigorously founded but it also turns out that these are enough to get some of the physics correct!

Approximating the expression for the counting of the number of states at any given $\Delta + j$...contd.

The convenient thing about the particular family of solutions stated above is that they scale linearly with the right hand side $2(\Delta + j)$.Since the series one is looking at is a sum of positive terms the growth of these terms will always give a lower bound and these are expected to be the dominant contributors.

The method is not rigorously founded but it also turns out that these are enough to get some of the physics correct!

So the term corresponding to this solution family and $d = 1$ is,

$$\frac{1}{3p+r}\frac{(3p+r)!}{(p+r)!p!p!}$$

Approximating the expression for the counting of the number of states at any given $\Delta + j$...contd.

Approximating the expression for the counting of the number of states at any given $\Delta + j$...contd.

Since in the large $\Delta + j$ limit all of the above numbers will asymptotically grow similarly one can always use the Stirling approximation for them and the resulting expression will be,

Approximating the expression for the counting of the number of states at any given $\Delta + j$...contd.

Since in the large $\Delta + j$ limit all of the above numbers will asymptotically grow similarly one can always use the Stirling approximation for them and the resulting expression will be,

$$\frac{1}{2\sqrt{3}\pi\left(1+\frac{r}{3p}\right)}\frac{3^{3p\left(1+\frac{r}{3p}\right)}}{p^2}\sqrt{\frac{1+\frac{r}{3p}}{1+\frac{r}{p}}}\frac{\left(1+\frac{r}{3p}\right)^{3p+r}}{\left(1+\frac{r}{p}\right)^{p+r}}$$

Approximating the expression for the counting of the number of states at any given $\Delta + j$...contd.

Since in the large $\Delta + j$ limit all of the above numbers will asymptotically grow similarly one can always use the Stirling approximation for them and the resulting expression will be,

$$\frac{1}{2\sqrt{3}\pi\left(1+\frac{r}{3p}\right)}\frac{3^{3p\left(1+\frac{r}{3p}\right)}}{p^2}\sqrt{\frac{1+\frac{r}{3p}}{1+\frac{r}{p}}}\frac{\left(1+\frac{r}{3p}\right)^{3p+r}}{\left(1+\frac{r}{p}\right)^{p+r}}$$

Since r is bounded and p can grow unboundedly, one can drop powers of $\frac{r}{p}$ and the above expression is asymptotically expected to grow as,

Approximating the expression for the counting of the number of states at any given $\Delta + j$...contd.

Since in the large $\Delta + j$ limit all of the above numbers will asymptotically grow similarly one can always use the Stirling approximation for them and the resulting expression will be,

$$\frac{1}{2\sqrt{3}\pi\left(1+\frac{r}{3p}\right)} \frac{3^{3p\left(1+\frac{r}{3p}\right)}}{p^2} \sqrt{\frac{1+\frac{r}{3p}}{1+\frac{r}{p}}} \frac{\left(1+\frac{r}{3p}\right)^{3p+r}}{\left(1+\frac{r}{p}\right)^{p+r}}$$

Since r is bounded and p can grow unboundedly, one can drop powers of $\frac{r}{p}$ and the above expression is asymptotically expected to grow as,

$$\frac{3^{3p}}{2\sqrt{3}\pi p^2}$$

Approximating the expression for the counting of the number of states at any given $\Delta + j$...contd.

Approximating the expression for the counting of the number of states at any given $\Delta + j$...contd.

But by definition $p \sim \frac{2(\Delta+j)}{8}$ and hence up to overall constants a lower bound on the asymptotic growth rate of the number of linearly independent states is like,

Approximating the expression for the counting of the number of states at any given $\Delta + j$...contd.

But by definition $p \sim \frac{2(\Delta+j)}{8}$ and hence up to overall constants a lower bound on the asymptotic growth rate of the number of linearly independent states is like,

$$\frac{3^{\frac{3(\Delta+j)}{4}}}{(\Delta+j)^2}$$

Approximating the expression for the counting of the number of states at any given $\Delta + j$...contd.

But by definition $p \sim \frac{2(\Delta+j)}{8}$ and hence up to overall constants a lower bound on the asymptotic growth rate of the number of linearly independent states is like,

$$\frac{3^{\frac{3(\Delta+j)}{4}}}{(\Delta+j)^2}$$

In cases like theory with two non-commuting scalar fields and each of $\Delta + j = 1$ or in a theory with only ϕ and D, a similar analysis like above gives the Hagedorn transition temperature exactly the same and very close (respectively) to those obtained by more sophisticated methods in the paper "The Hagedorn/Deconfinement Phase Transition in Weakly Coupled Large N Gauge Theories", Adv.Theor.Math.Phys.8:603-696 (2004), [arXiv:hep-th/0310285v6] by Ofer Aharony, Joseph Marsano, Shiraz Minwalla, Kyriakos Papadodimas and Mark Van Raamsdonk.

Approximating the expression for the counting of the number of states at any given $\Delta + j$...contd.

But by definition $p \sim \frac{2(\Delta+j)}{8}$ and hence up to overall constants a lower bound on the asymptotic growth rate of the number of linearly independent states is like,

$$\frac{3^{\frac{3(\Delta+j)}{4}}}{(\Delta+j)^2}$$

In cases like theory with two non-commuting scalar fields and each of $\Delta + j = 1$ or in a theory with only ϕ and D, a similar analysis like above gives the Hagedorn transition temperature exactly the same and very close (respectively) to those obtained by more sophisticated methods in the paper "The Hagedorn/Deconfinement Phase Transition in Weakly Coupled Large N Gauge Theories", Adv.Theor.Math.Phys.8:603-696 (2004), [arXiv:hep-th/0310285v6] by Ofer Aharony, Joseph Marsano, Shiraz Minwalla, Kyriakos Papadodimas and Mark Van Raamsdonk.

Coming up next!

Setting up the "Q-cohomology" calculation

Setting up the "Q-cohomology" calculation

The following observations are made to set-up the calculation of "Q-cohomology",

Setting up the "Q-cohomology" calculation

The following observations are made to set-up the calculation of "Q-cohomology",

- As was earlier noted that the action of Q conserves $\Delta + j$ and hence it maps the vector space of states at every $\Delta + j$ back to itself.

Setting up the "Q-cohomology" calculation

The following observations are made to set-up the calculation of "Q-cohomology",

- As was earlier noted that the action of Q conserves $\Delta + j$ and hence it maps the vector space of states at every $\Delta + j$ back to itself. So one can write down a matrix for Q in a basis of states chosen at every $\Delta + j$. A natural candidate for such a basis was given in the table earlier which listed the linearly independent Q-closed or Q-not-closed states in the second and third columns.

Setting up the "Q-cohomology" calculation

The following observations are made to set-up the calculation of "Q-cohomology",

- As was earlier noted that the action of Q conserves $\Delta + j$ and hence it maps the vector space of states at every $\Delta + j$ back to itself. So one can write down a matrix for Q in a basis of states chosen at every $\Delta + j$. A natural candidate for such a basis was given in the table earlier which listed the linearly independent Q-closed or Q-not-closed states in the second and third columns.
- It has been combinatorial proven that as an operator $Q^2 = 0$.

Setting up the "Q-cohomology" calculation

The following observations are made to set-up the calculation of "Q-cohomology",

- As was earlier noted that the action of Q conserves $\Delta + j$ and hence it maps the vector space of states at every $\Delta + j$ back to itself. So one can write down a matrix for Q in a basis of states chosen at every $\Delta + j$. A natural candidate for such a basis was given in the table earlier which listed the linearly independent Q-closed or Q-not-closed states in the second and third columns.
- It has been combinatorial proven that as an operator $Q^2 = 0$. Hence for these matrices Ker/Im is well defined and for them one can calculate $dim(Ker/Im) = dim(Ker) - dim(Im)$.

Setting up the "Q-cohomology" calculation

The following observations are made to set-up the calculation of "Q-cohomology",

- As was earlier noted that the action of Q conserves $\Delta + j$ and hence it maps the vector space of states at every $\Delta + j$ back to itself. So one can write down a matrix for Q in a basis of states chosen at every $\Delta + j$. A natural candidate for such a basis was given in the table earlier which listed the linearly independent Q-closed or Q-not-closed states in the second and third columns.
- It has been combinatorial proven that as an operator $Q^2 = 0$. Hence for these matrices Ker/Im is well defined and for them one can calculate $dim(Ker/Im) = dim(Ker) - dim(Im)$. At any value of $\Delta + j$ this integer would be called the "Q-cohomology" at that $\Delta + j$.

Setting up the "Q-cohomology" calculation

The following observations are made to set-up the calculation of "Q-cohomology",

- As was earlier noted that the action of Q conserves $\Delta + j$ and hence it maps the vector space of states at every $\Delta + j$ back to itself. So one can write down a matrix for Q in a basis of states chosen at every $\Delta + j$. A natural candidate for such a basis was given in the table earlier which listed the linearly independent Q-closed or Q-not-closed states in the second and third columns.
- It has been combinatorial proven that as an operator $Q^2 = 0$. Hence for these matrices Ker/Im is well defined and for them one can calculate $dim(Ker/Im) = dim(Ker) - dim(Im)$. At any value of $\Delta + j$ this integer would be called the "Q-cohomology" at that $\Delta + j$. But a topological characterization of these numbers is still not known and that is an extremely important question that needs to be looked into in future.

Setting up the "Q-cohomology" calculation

The following observations are made to set-up the calculation of "Q-cohomology",

- As was earlier noted that the action of Q conserves $\Delta + j$ and hence it maps the vector space of states at every $\Delta + j$ back to itself. So one can write down a matrix for Q in a basis of states chosen at every $\Delta + j$. A natural candidate for such a basis was given in the table earlier which listed the linearly independent Q-closed or Q-not-closed states in the second and third columns.
- It has been combinatorial proven that as an operator $Q^2 = 0$. Hence for these matrices Ker/Im is well defined and for them one can calculate $dim(Ker/Im) = dim(Ker) - dim(Im)$. At any value of $\Delta + j$ this integer would be called the "Q-cohomology" at that $\Delta + j$. But a topological characterization of these numbers is still not known and that is an extremely important question that needs to be looked into in future.

Setting up the "Q-cohomology" calculation...contd.

Setting up the "Q-cohomology" calculation...contd.

In the previous sections when the objective was to just get a hold on the asymptotic growth of the number of states with $\Delta + j$, one could risk ignoring the fermionic nature of χ but here in the case of calculating the $Q - cohomology$ this can't be done.

Setting up the "Q-cohomology" calculation...contd.

In the previous sections when the objective was to just get a hold on the asymptotic growth of the number of states with $\Delta + j$, one could risk ignoring the fermionic nature of χ but here in the case of calculating the $Q - cohomology$ this can't be done. So some of the states in the table referred to earlier will not be counted since they are 0 if one accounts for the fermionic nature of χ.

Setting up the "Q-cohomology" calculation...contd.

Setting up the "Q-cohomology" calculation...contd.

To give an example of how the calculation of $Q - cohomology$ goes,

Setting up the "Q-cohomology" calculation...contd.

To give an example of how the calculation of $Q-cohomology$ goes,

The matrix of Q at $\Delta + j = \frac{9}{2}$ written in the ordered basis of $\{\phi^9, \phi^6\chi, \phi^3\chi^2, \phi\chi\phi\chi\phi, \chi^3, \phi^4 D\phi, \phi\chi D\phi, \chi\phi D\phi, \phi^2 D\chi, D^2\phi\}$ is a 10×10 matrix having 9 non-zero entries coming from the coefficients of the following 5 non-zero Q actions,

Setting up the "Q-cohomology" calculation...contd.

To give an example of how the calculation of $Q - cohomology$ goes,

The matrix of Q at $\Delta + j = \frac{9}{2}$ written in the ordered basis of $\{\phi^9, \phi^6\chi, \phi^3\chi^2, \phi\chi\phi\chi\phi, \chi^3, \phi^4 D\phi, \phi\chi D\phi, \chi\phi D\phi, \phi^2 D\chi, D^2\phi\}$ is a 10×10 matrix having 9 non-zero entries coming from the coefficients of the following 5 non-zero Q actions,

- $Q(\phi^6\chi) = \lambda\phi^9$

Setting up the "Q-cohomology" calculation...contd.

To give an example of how the calculation of $Q - \textit{cohomology}$ goes,

The matrix of Q at $\Delta + j = \frac{9}{2}$ written in the ordered basis of $\{\phi^9, \phi^6\chi, \phi^3\chi^2, \phi\chi\phi\chi\phi, \chi^3, \phi^4 D\phi, \phi\chi D\phi, \chi\phi D\phi, \phi^2 D\chi, D^2\phi\}$ is a 10×10 matrix having 9 non-zero entries coming from the coefficients of the following 5 non-zero Q actions,

- $Q(\phi^6\chi) = \lambda\phi^9$
- $Q(\chi^3) = 3\lambda\phi^3\chi^2$

Setting up the “Q-cohomology” calculation...contd.

To give an example of how the calculation of $Q - \textit{cohomology}$ goes,

The matrix of Q at $\Delta + j = \frac{9}{2}$ written in the ordered basis of $\{\phi^9, \phi^6\chi, \phi^3\chi^2, \phi\chi\phi\chi\phi, \chi^3, \phi^4 D\phi, \phi\chi D\phi, \chi\phi D\phi, \phi^2 D\chi, D^2\phi\}$ is a 10×10 matrix having 9 non-zero entries coming from the coefficients of the following 5 non-zero Q actions,

- $Q(\phi^6\chi) = \lambda\phi^9$
- $Q(\chi^3) = 3\lambda\phi^3\chi^2$
- $Q(\phi\chi D\phi) = \phi^3\chi^2 - 3\phi\chi\phi\chi\phi + \lambda\phi^4 D\phi$

Setting up the "Q-cohomology" calculation...contd.

To give an example of how the calculation of $Q - cohomology$ goes,

The matrix of Q at $\Delta + j = \frac{9}{2}$ written in the ordered basis of $\{\phi^9, \phi^6\chi, \phi^3\chi^2, \phi\chi\phi\chi\phi, \chi^3, \phi^4 D\phi, \phi\chi D\phi, \chi\phi D\phi, \phi^2 D\chi, D^2\phi\}$ is a 10×10 matrix having 9 non-zero entries coming from the coefficients of the following 5 non-zero Q actions,

- $Q(\phi^6\chi) = \lambda\phi^9$
- $Q(\chi^3) = 3\lambda\phi^3\chi^2$
- $Q(\phi\chi D\phi) = \phi^3\chi^2 - 3\phi\chi\phi\chi\phi + \lambda\phi^4 D\phi$
- $Q(\chi\phi D\phi) = -\phi^3\chi^2 + 3\phi\chi\phi\chi\phi - \lambda\phi^4 D\phi$

Setting up the "Q-cohomology" calculation...contd.

To give an example of how the calculation of $Q - cohomology$ goes,

The matrix of Q at $\Delta + j = \frac{9}{2}$ written in the ordered basis of $\{\phi^9, \phi^6\chi, \phi^3\chi^2, \phi\chi\phi\chi\phi, \chi^3, \phi^4 D\phi, \phi\chi D\phi, \chi\phi D\phi, \phi^2 D\chi, D^2\phi\}$ is a 10×10 matrix having 9 non-zero entries coming from the coefficients of the following 5 non-zero Q actions,

- $Q(\phi^6\chi) = \lambda\phi^9$
- $Q(\chi^3) = 3\lambda\phi^3\chi^2$
- $Q(\phi\chi D\phi) = \phi^3\chi^2 - 3\phi\chi\phi\chi\phi + \lambda\phi^4 D\phi$
- $Q(\chi\phi D\phi) = -\phi^3\chi^2 + 3\phi\chi\phi\chi\phi - \lambda\phi^4 D\phi$
- $Q(\phi^2 D\chi) = 3\lambda\phi^4 D\phi$

Setting up the "Q-cohomology" calculation...contd.

To give an example of how the calculation of $Q - cohomology$ goes,

The matrix of Q at $\Delta + j = \frac{9}{2}$ written in the ordered basis of $\{\phi^9, \phi^6\chi, \phi^3\chi^2, \phi\chi\phi\chi\phi, \chi^3, \phi^4 D\phi, \phi\chi D\phi, \chi\phi D\phi, \phi^2 D\chi, D^2\phi\}$ is a 10×10 matrix having 9 non-zero entries coming from the coefficients of the following 5 non-zero Q actions,

- $Q(\phi^6\chi) = \lambda\phi^9$
- $Q(\chi^3) = 3\lambda\phi^3\chi^2$
- $Q(\phi\chi D\phi) = \phi^3\chi^2 - 3\phi\chi\phi\chi\phi + \lambda\phi^4 D\phi$
- $Q(\chi\phi D\phi) = -\phi^3\chi^2 + 3\phi\chi\phi\chi\phi - \lambda\phi^4 D\phi$
- $Q(\phi^2 D\chi) = 3\lambda\phi^4 D\phi$

Data that was calculated about $Q - cohomology$ for the first few levels of $\Delta + j$

Data that was calculated about $Q - cohomology$ for the first few levels of $\Delta + j$

For the first 10 "energy" levels the calculated data is as follows,

Data that was calculated about $Q - cohomology$ for the first few levels of $\Delta + j$

For the first 10 “energy” levels the calculated data is as follows,

$\Delta + j$	$\lambda \neq 0$ dim(Ker)-dim(Im) = dim(Ker/Im)	$\lambda = 0$ dim(Ker)-dim(Im) = dim(Ker/Im)
5	$7 - 6 = 1$	$11 - 2 = 9$
$\frac{9}{2}$	$6 - 4 = 2$	$9 - 1 = 8$
4	$4 - 2 = 2$	$6 - 0 = 6$
$\frac{7}{2}$	$3 - 2 = 1$	$5 - 0 = 5$
3	$2 - 1 = 1$	$3 - 0 = 3$
$\frac{5}{2}$	$2 - 1 = 1$	$3 - 0 = 3$
2	$1 - 1 = 0$	$2 - 0 = 2$
$\frac{3}{2}$	$1 - 1 = 0$	$2 - 0 = 2$
1	$1 - 1 = 0$	$1 - 0 = 1$
$\frac{1}{2}$	$1 - 0 = 1$	$1 - 0 = 0$

Data that was calculated about $Q - cohomology$ for the first few levels of $\Delta + j$

For the first 10 "energy" levels the calculated data is as follows,

$\Delta + j$	$\lambda \neq 0$ dim(Ker)-dim(Im) = dim(Ker/Im)	$\lambda = 0$ dim(Ker)-dim(Im) = dim(Ker/Im)
5	$7-6=1$	$11-2=9$
$\frac{9}{2}$	$6-4=2$	$9-1=8$
4	$4-2=2$	$6-0=6$
$\frac{7}{2}$	$3-2=1$	$5-0=5$
3	$2-1=1$	$3-0=3$
$\frac{5}{2}$	$2-1=1$	$3-0=3$
2	$1-1=0$	$2-0=2$
$\frac{3}{2}$	$1-1=0$	$2-0=2$
1	$1-1=0$	$1-0=1$
$\frac{1}{2}$	$1-0=1$	$1-0=0$

The number of "Q-cohomology" classes seems to change for the slightest of non-zero values of λ from its values in the free case ($\lambda = 0$) and then it remains constant irrespective of the strength of the deformation. The above can be understood from the fact that in the defining equations for Q, the λ can always be scaled to 1 by a variable redefinition of the fields.

Coming up next!

Future directions

Future directions

- As has been said in the introduction, the main objective is to be able to get an analytic control over features of these theories like their BPS spectrum and "Q-cohomology" which are being believed to be independent of the 'tHooft parameter (λ). Added to the above is also the assumption that the "Q-cohomology" calculated in the classical theory is in bijection with the BPS spectrum. Given how important these intuitions are in paving a way towards strong/weak coupling dualities, it becomes extremely important to be able to find a proof or at least stronger evidence in their favour.

Future directions

- As has been said in the introduction, the main objective is to be able to get an analytic control over features of these theories like their BPS spectrum and "Q-cohomology" which are being believed to be independent of the 'tHooft parameter (λ). Added to the above is also the assumption that the "Q-cohomology" calculated in the classical theory is in bijection with the BPS spectrum. Given how important these intuitions are in paving a way towards strong/weak coupling dualities, it becomes extremely important to be able to find a proof or at least stronger evidence in their favour.
- In the same strain one should point out the need to find a topological understanding of the quantity that is being called "Q-cohomology"

Future directions

- As has been said in the introduction, the main objective is to be able to get an analytic control over features of these theories like their BPS spectrum and "Q-cohomology" which are being believed to be independent of the 'tHooft parameter (λ). Added to the above is also the assumption that the "Q-cohomology" calculated in the classical theory is in bijection with the BPS spectrum. Given how important these intuitions are in paving a way towards strong/weak coupling dualities, it becomes extremely important to be able to find a proof or at least stronger evidence in their favour.
- In the same strain one should point out the need to find a topological understanding of the quantity that is being called "Q-cohomology"

Future directions

Future directions

- There is increasing evidence for the decrease of the R-charge (from its original value of $\frac{1}{2}$) for the chiral fields as a function of the the ‘tHooft parameter. In perturbation theory this was pointed out by Davide Gaiotto and Xi Yin and in recent months it also has been converted into possibly an exact result by Daniel Jafferis. These give strong reasons to believe that the scaling dimensions of the operators will also decrease exactly in the same way so as to preserve the BPS nature of these operators. Hence the challenge is to show that the BPS spectrum remains invariant with the ‘tHooft coupling though the scaling dimension and R-charge (the very same quantities defining the BPS condition!) are decreasing.

Future directions

- There is increasing evidence for the decrease of the R-charge (from its original value of $\frac{1}{2}$) for the chiral fields as a function of the the 'tHooft parameter. In perturbation theory this was pointed out by Davide Gaiotto and Xi Yin and in recent months it also has been converted into possibly an exact result by Daniel Jafferis. These give strong reasons to believe that the scaling dimensions of the operators will also decrease exactly in the same way so as to preserve the BPS nature of these operators. Hence the challenge is to show that the BPS spectrum remains invariant with the 'tHooft coupling though the scaling dimension and R-charge (the very same quantities defining the BPS condition!) are decreasing.

Future directions

Future directions

- Here it is appropriate to mention that in the recent work by Shiraz Minwalla, Prithvi Narayan, Tarun Sharma, V. Umesh, Xi Yin, they have shown numerical evidence for the R-charge value to asymptote from above to $\frac{1}{2(\lambda+1)}$. They have also shown that in some theories the R-charge goes to zero as the 'tHooft coupling goes to infinity. That means that in that limit one can add arbitrary number of scalar fields to the operators with no increase in R-charge and this would mean that the theory is developing a continuum of states. This is indicative of there being a description in terms of a theory with a non-compact dimension though there was no such geometrical feature initially! These exciting results need to be understood analytically and hopefully a non-perturbative understanding would eventually emerge.

Coming up next!

Acknowledgements

Acknowledgements

- I would like to thank Prof.Shiraz for starting me in these directions and for the extremely insightful discussions.

Acknowledgements

- I would like to thank Prof.Shiraz for starting me in these directions and for the extremely insightful discussions.
- A lot of thanks are due to Prof.Jaikumar Radhakrishnan for his enlightening lectures on various aspects of combinatorics and especially for his discussions on Polya theory.

Acknowledgements

- I would like to thank Prof.Shiraz for starting me in these directions and for the extremely insightful discussions.
- A lot of thanks are due to Prof.Jaikumar Radhakrishnan for his enlightening lectures on various aspects of combinatorics and especially for his discussions on Polya theory.
- Thanks are due to Tarun for giving me a lot of his time in explaining his work which went into the paper, "Supersymmetric States in Large N Chern-Simons-Matter Theories", [arXiv:hep-th/1104.0680] by him, Shiraz Minwalla, Prithvi Narayan, Tarun Sharma, V. Umesh and Xi Yin

Acknowledgements

- I would like to thank Prof.Shiraz for starting me in these directions and for the extremely insightful discussions.
- A lot of thanks are due to Prof.Jaikumar Radhakrishnan for his enlightening lectures on various aspects of combinatorics and especially for his discussions on Polya theory.
- Thanks are due to Tarun for giving me a lot of his time in explaining his work which went into the paper, "Supersymmetric States in Large N Chern-Simons-Matter Theories", [arXiv:hep-th/1104.0680] by him, Shiraz Minwalla, Prithvi Narayan, Tarun Sharma, V. Umesh and Xi Yin

Acknowledgements

Acknowledgements

- I would like to thank Shiroman for having shared his insights over the many discussions we have had on these topics and he also kindly agreed to check through the writings.

Acknowledgements

- I would like to thank Shiroman for having shared his insights over the many discussions we have had on these topics and he also kindly agreed to check through the writings.
- Huge amount of thanks is due to Onkar (at UIUC) for having had enormous amount of discussion on various aspects of supersymmetry while we both were almost simultaneously learning the subject.

Acknowledgements

- I would like to thank Shiroman for having shared his insights over the many discussions we have had on these topics and he also kindly agreed to check through the writings.
- Huge amount of thanks is due to Onkar (at UIUC) for having had enormous amount of discussion on various aspects of supersymmetry while we both were almost simultaneously learning the subject.
- I would like to thank the contributors at physicsstackexchange.com for the many discussions on superconformality and super-Chern-Simons theories. Thanks are especially due to Lubos Motl and Olof for their extensive explanations.

Acknowledgements

- I would like to thank Shiroman for having shared his insights over the many discussions we have had on these topics and he also kindly agreed to check through the writings.
- Huge amount of thanks is due to Onkar (at UIUC) for having had enormous amount of discussion on various aspects of supersymmetry while we both were almost simultaneously learning the subject.
- I would like to thank the contributors at physicsstackexchange.com for the many discussions on superconformality and super-Chern-Simons theories. Thanks are especially due to Lubos Motl and Olof for their extensive explanations.

Acknowledgements

Acknowledgements

- I would like to thank Yuji Tachikawa for his detailed explanations on mathoverflow.net about the representation theory of superconformal algebra.

Acknowledgements

- I would like to thank Yuji Tachikawa for his detailed explanations on mathoverflow.net about the representation theory of superconformal algebra.
- On mathoverflow.net Abdelmalek Abdesselam helped understand aspects of GIT.

Acknowledgements

- I would like to thank Yuji Tachikawa for his detailed explanations on mathoverflow.net about the representation theory of superconformal algebra.
- On mathoverflow.net Abdelmalek Abdesselam helped understand aspects of GIT.David Bar Moshe on the same forum gave me interesting references about understanding the notion of partition function in the language of representation theory.

Acknowledgements

- I would like to thank Yuji Tachikawa for his detailed explanations on mathoverflow.net about the representation theory of superconformal algebra.
- On mathoverflow.net Abdelmalek Abdesselam helped understand aspects of GIT.David Bar Moshe on the same forum gave me interesting references about understanding the notion of partition function in the language of representation theory.
- I also owe a lot to the deep insights and beautiful references on these subjects that were shared with me by Davide Gaiotto and F.A.Dolan.