On the Critical End Point of QCD

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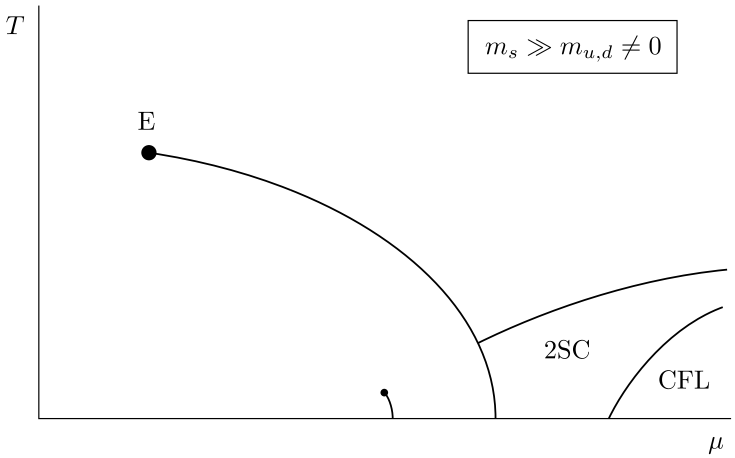
Introduction

Expected QCD Phase Diagram and Lattice Approaches to unravel it.



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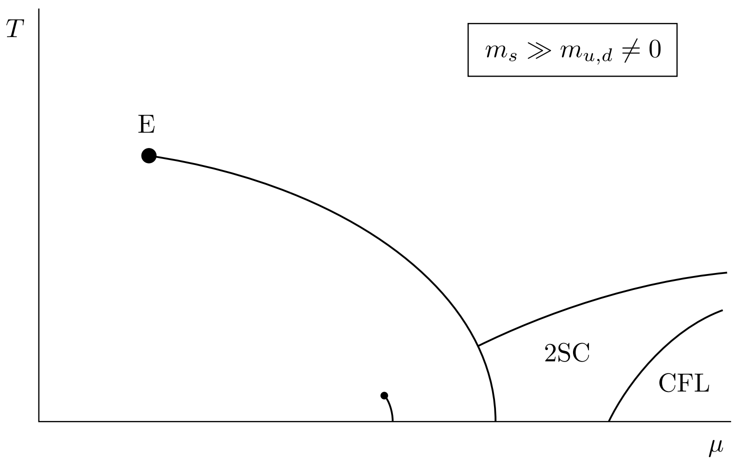

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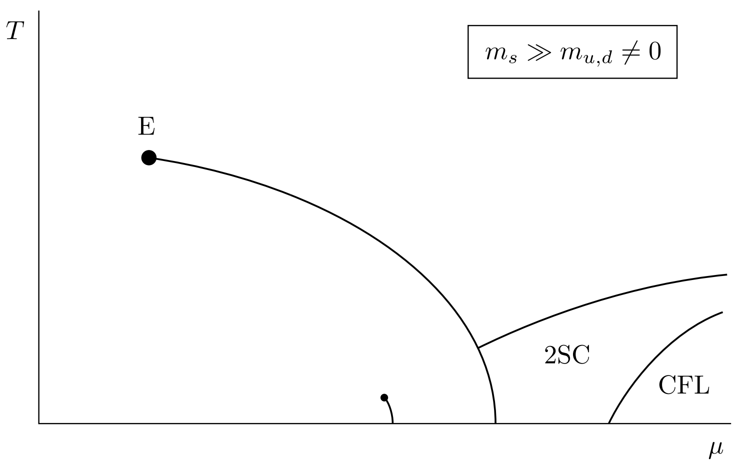

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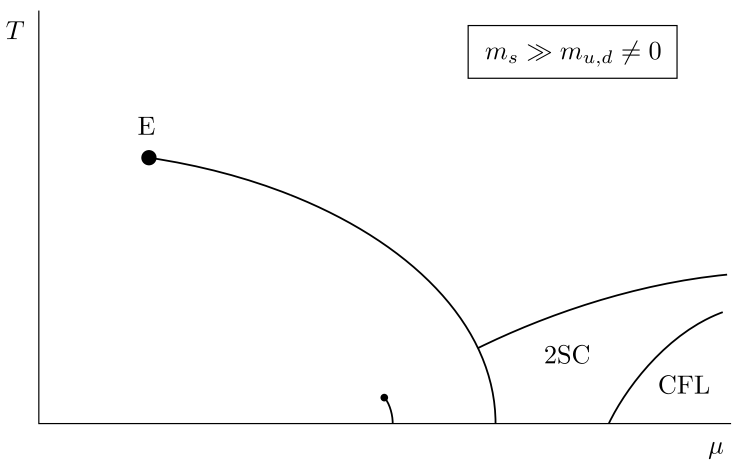

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- Imaginary Chemical Potential (Ph. de Frocrand & O. Philipsen, NP B642 (2002) 290; M.-P. Lombardo & M. D'Elia PR D67 (2003) 014505).
- Taylor Expansion (C. Allton et al., PR D66 (2002) 074507 & D68 (2003) 014507; R.V. Gavai and S. Gupta, PR D68 (2003) 034506).

Why Taylor series expansion?

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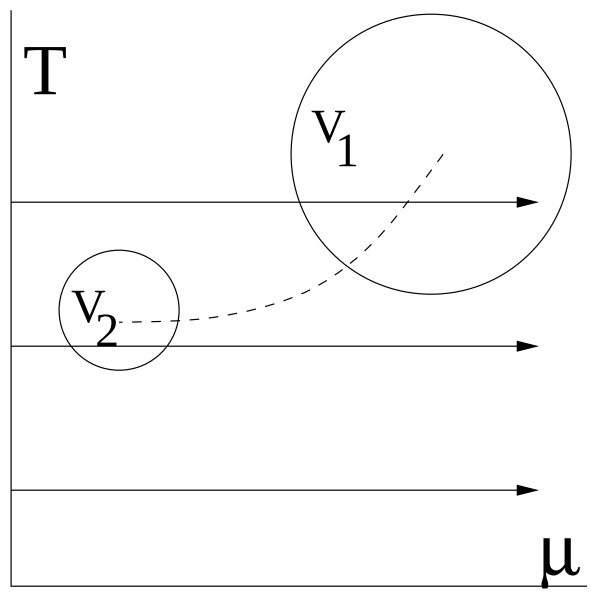

We study volume dependence at several T to i) bracket the critical region and then to ii) track its change as a function of volume.

Formalism

Assuming N_f flavours of quarks, and denoting by μ_f the corresponding chemical potentials, the QCD partition function is

$$\mathcal{Z} = \int DU \exp(-S_G) \ \prod_f \mathrm{Det} \ M(m_f, \mu_f) \quad .$$

Canonical definitions then yield various number densities and susceptibilities :

$$n_i = \frac{T}{V}\frac{\partial \ln \mathcal{Z}}{\partial \mu_i} \text{ and } \qquad \chi_{ij} = \frac{T}{V}\frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_i \partial \mu_j} \quad .$$

Higher order susceptibilities are defined by

$$\chi_{fg\cdots} = \frac{T}{V}\frac{\partial^n \log Z}{\partial \mu_f \partial \mu_g \cdots} = \frac{\partial^n P}{\partial \mu_f \partial \mu_g \cdots} \ . \qquad (1)$$

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These are Taylor coefficients of the pressure P in its expansion in μ.

$$\frac{\Delta P}{T^4} \equiv \frac{P(\mu,T)}{T^4} - \frac{P(0,T)}{T^4} = \sum_{n_u,n_d} \chi_{n_u,n_d} \frac{1}{n_u!} \left(\frac{\mu_u}{T}\right)^{n_u} \frac{1}{n_d!} \left(\frac{\mu_d}{T}\right)^{n_d} \tag{2}$$

From this a series for baryonic susceptibility can be constructed. Its radius of convergence gives the nearest critical point.

For 2 light flavours, its coefficients up to 6th order in $\mu_B/3 = \mu_u = \mu_d$ are

$$\chi_B^0 = \chi_{20}, \qquad \chi_B^4 = \frac{1}{4!}\left[\chi_{60} + 4\chi_{51} + 7\chi_{42} + 4\chi_{33}\right],$$
$$\chi_B^2 = \frac{1}{2!}\left[\chi_{40} + 2\chi_{31} + \chi_{22}\right], \qquad \chi_B^6 = \frac{1}{6!}\left[\chi_{80} + 6\chi_{71} + 16\chi_{62} + 26\chi_{53} + 15\chi_{44}\right]. \tag{3}$$

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Successive estimates for the radius of convergence can be obtained from these using $\sqrt{\frac{\chi_B^n}{\chi_B^{n+2}}}$.

Similar coefficients for the off-diagonal susceptibility are

$$\underline{\chi}_B^0 = \chi_{11}, \qquad \underline{\chi}_B^2 = \frac{1}{2!}\left[2\chi_{31} + 2\chi_{22}\right],$$
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♡ The ratio χ_{11}/χ_{20} can be shown to yield the ratio of widths of the measure in the imaginary and real directions at $\mu = 0$.

♡ Can be generalized to nonzero μ with some care and the coefficients above.

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The Susceptibilities

All susceptibilities can be written as traces of products of M^{-1} and various derivatives of M.
Two steps for getting NLS : 1) Writing down in terms of derivatives of Z and 2) obtaining these derivatives in terms of traces.

Setting $\mu_i = 0$,χ's are nontrivial for only even $N = n_u + n_d$. Thus at leading order

$$\chi_{20} = \left(\frac{T}{V}\right)\frac{Z_{20}}{Z} \qquad \chi_{11} = \left(\frac{T}{V}\right)\frac{Z_{11}}{Z} \tag{5}$$

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$\mathrm{Tr}\ A = \sum_{i=1}^{N_v} R_i^\dagger A R_i / 2N_v$, and $(\mathrm{Tr}\ A)^2 = 2\sum_{i>j=1}^{L} (\mathrm{Tr}\ A)_i (\mathrm{Tr}\ A)_j / L(L-1)$,
where R_i is a complex vector from a set of N_v subdivided in L independent sets.

Higher order NLS are more involved; systematic evaluation procedure helpful.
E.g., at the 8th order,

$$\chi_{80} = \frac{T}{V}\left[\frac{Z_{80}}{Z} - 28\frac{Z_{20}}{Z}\frac{Z_{60}}{Z} - 35\left(\frac{Z_{40}}{Z}\right)^2 + 420\left(\frac{Z_{20}}{Z}\right)^2\frac{Z_{40}}{Z} - 630\left(\frac{Z_{20}}{Z}\right)^4\right], \tag{7}$$

with the higher derivatives involving operators up to $\mathcal{O}_8$ which in turn have terms up to 8 quark propagators. In fact, the entire evaluation of the χ_{80} needs 20 inversions of Dirac matrix.

Problem of finding the minimum number inversions for a given order

— Akin to Steiner Problem in Computer Science $\rightsquigarrow$ our algorithm

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Our Simulations & Results

- Lattice used : 4 $\times N_s^3$, $N_s =$ 8, 10, 12, 16, 24
- Staggered fermions with $N_f = 2$ of $m/T_c = 0.1$; R-algorithm with traj. length of 1 MD time on $N_s = 8$, scaled $\propto N_s$ on larger ones.
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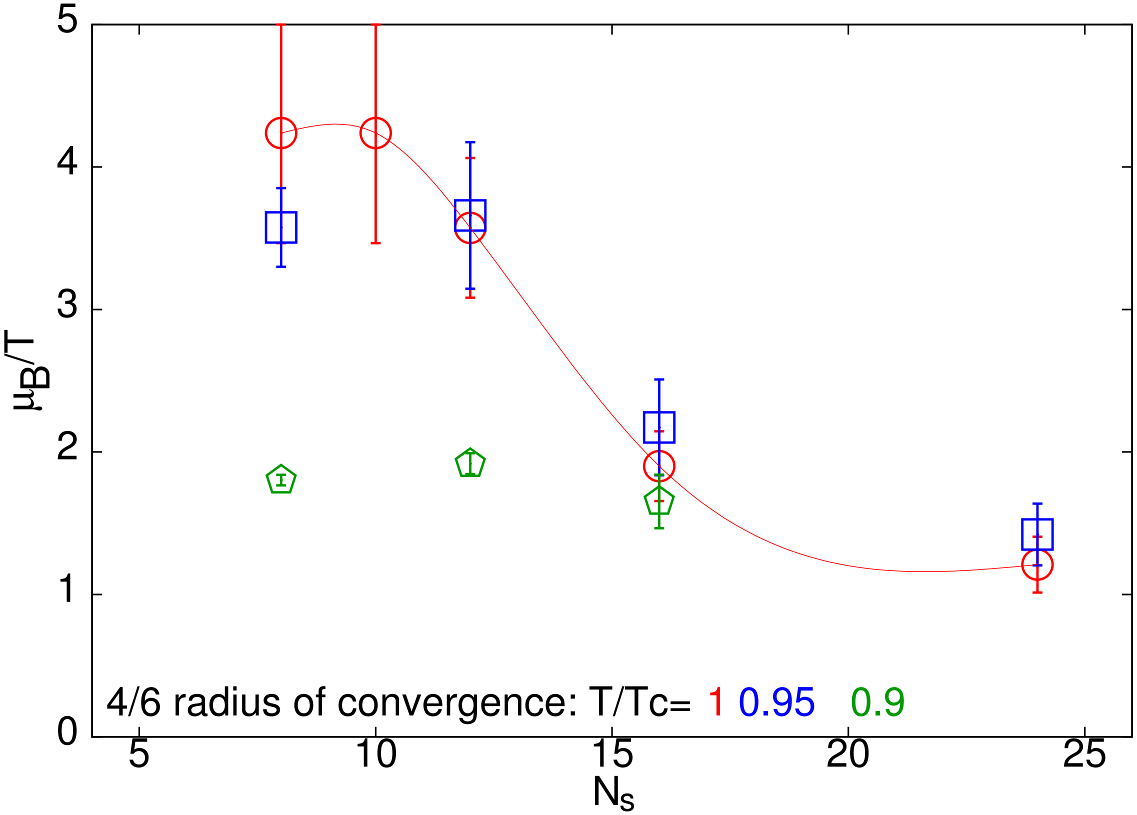
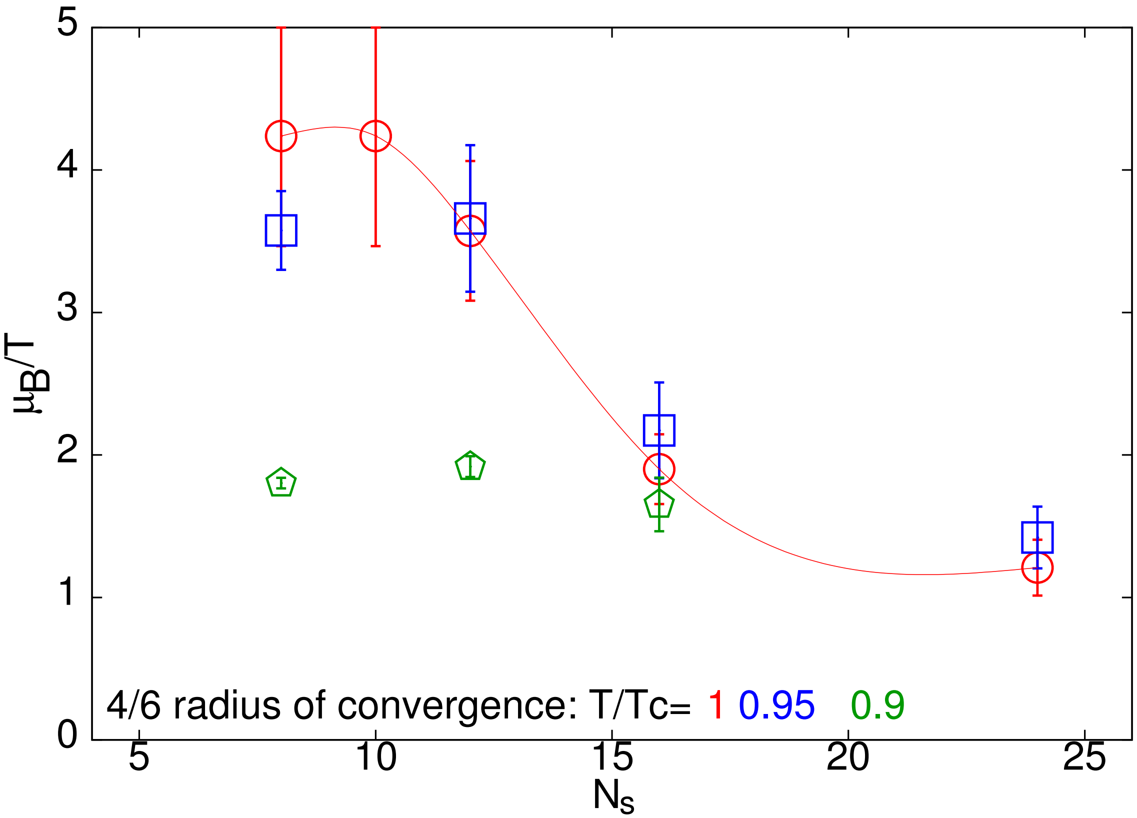
- Simulations made at T/T_c = 0.75(2), 0.80(2), 0.85(1), 0.90(1), 0.95(1), 0.975(10), 1.00(1), 1.05(1), 1.25(1), 1.65(6) and 2.15(10)
- Typical stat. 50-100 in max autocorrelation units.



μ_B/T
0
1
2
3
4
5
5
10
15
20
25
N_s
4/6 radius of convergence: T/Tc= 1 0.95 0.9



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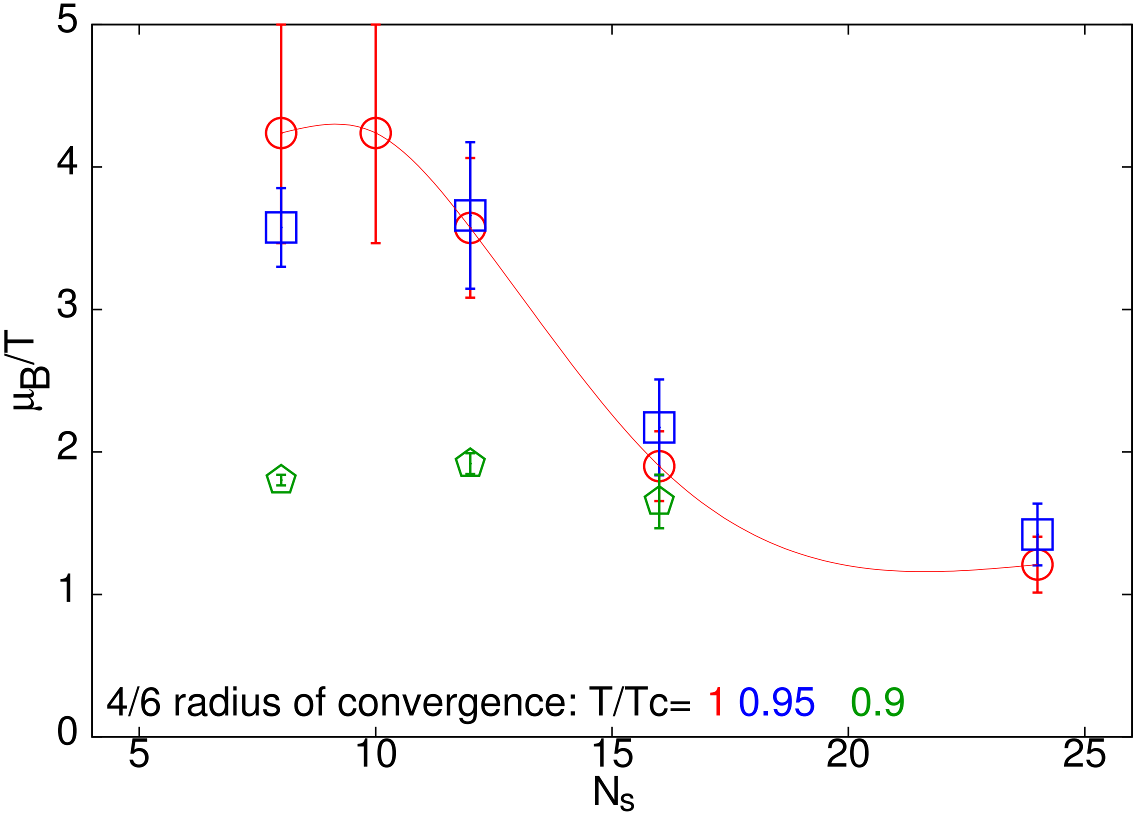

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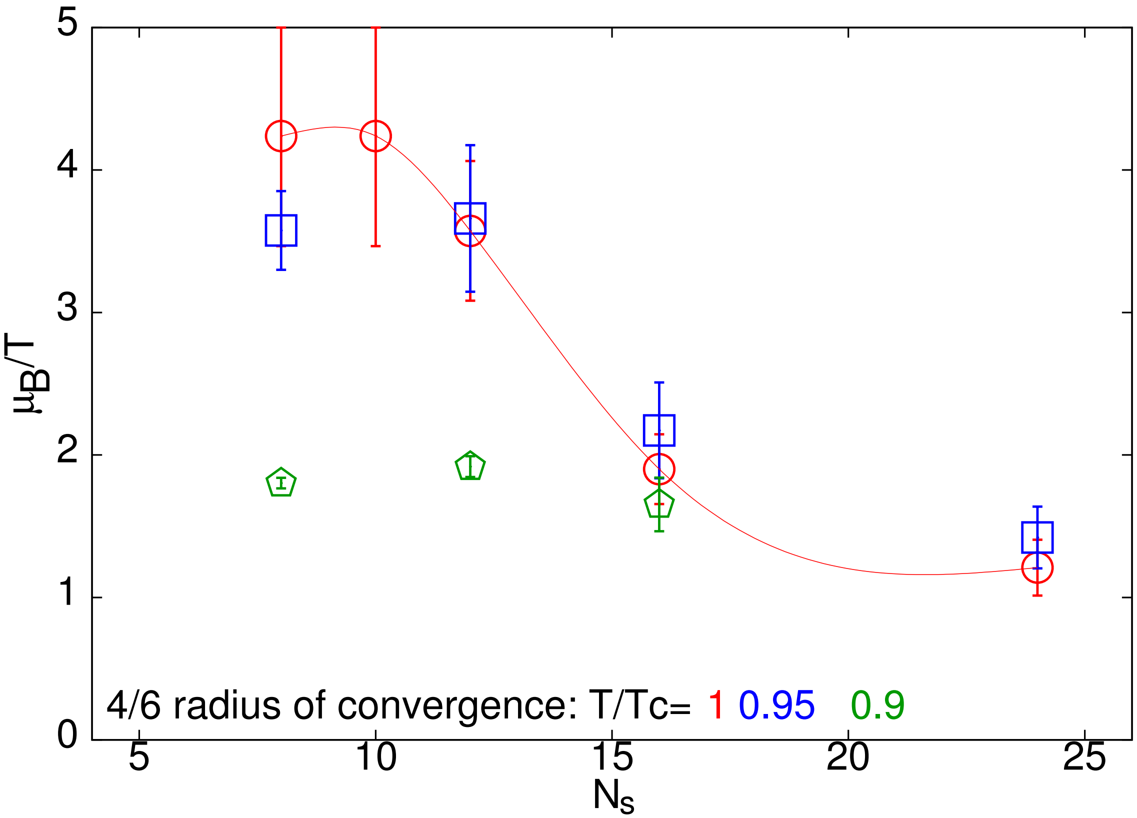

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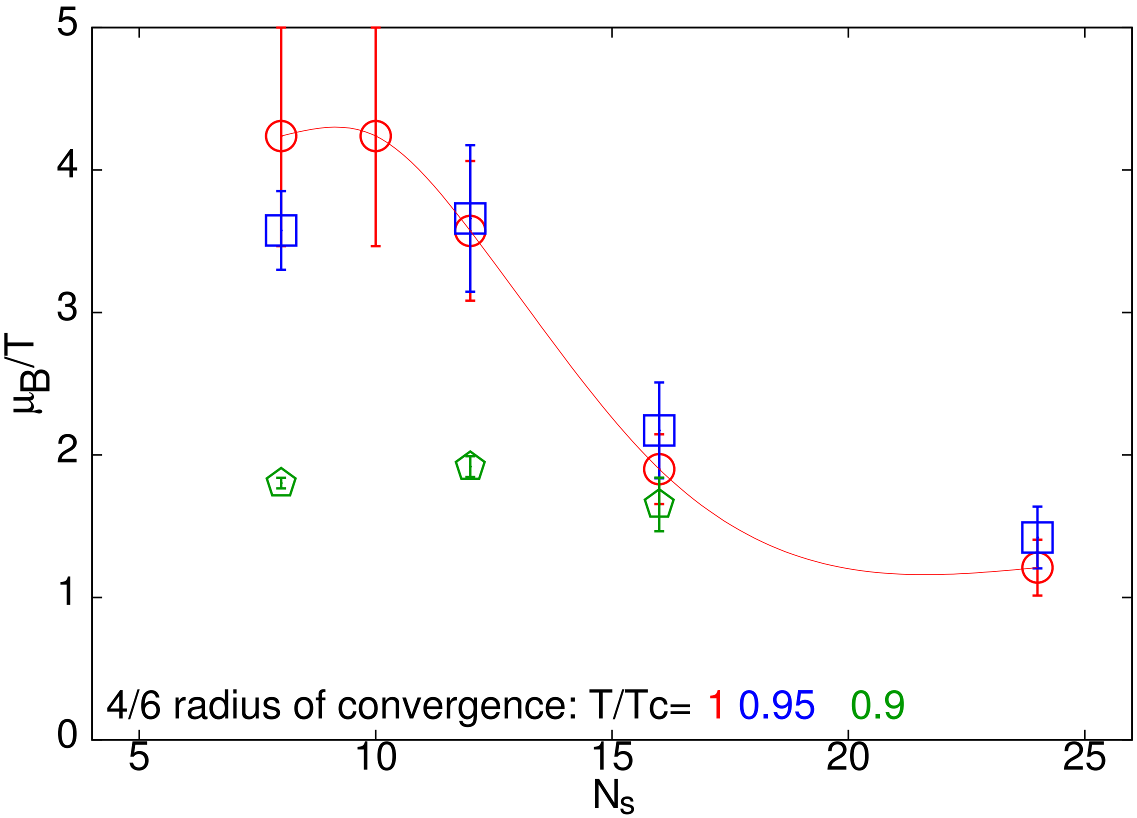

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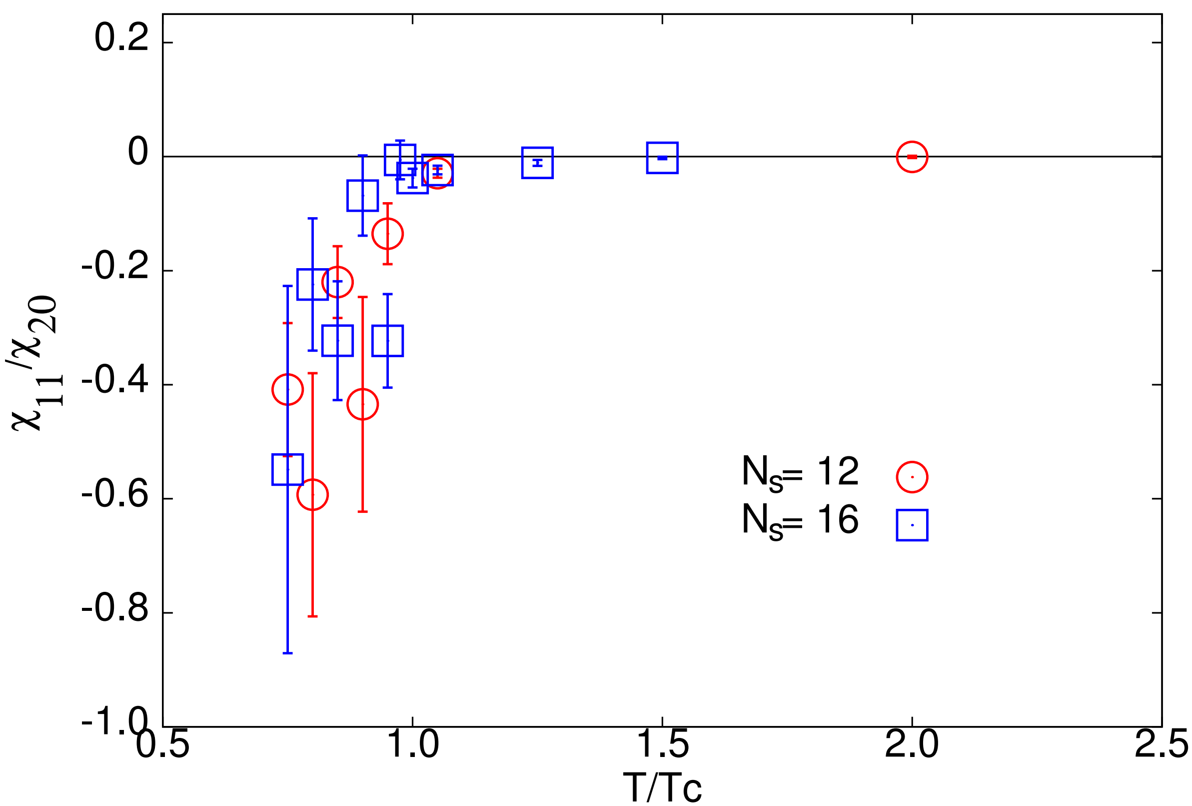
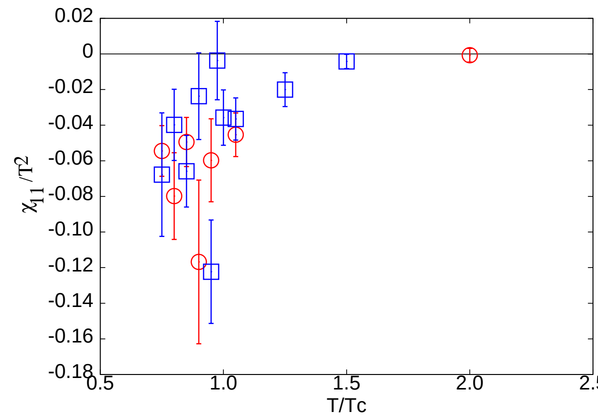
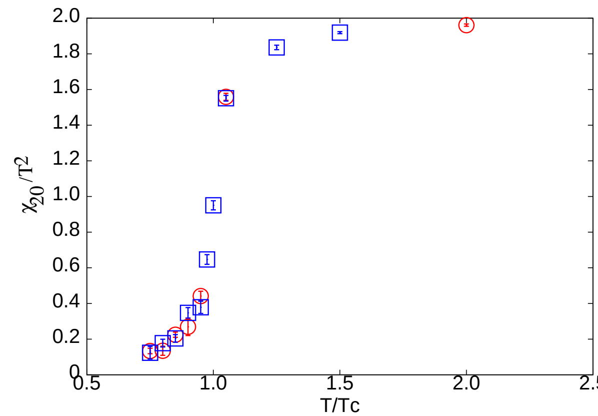
- Critical point shifted to smaller $\mu_B/T \sim 1-2$.

More Details

Measure of the seriousness of sign problem : Ratio χ_{11}/χ_{20}







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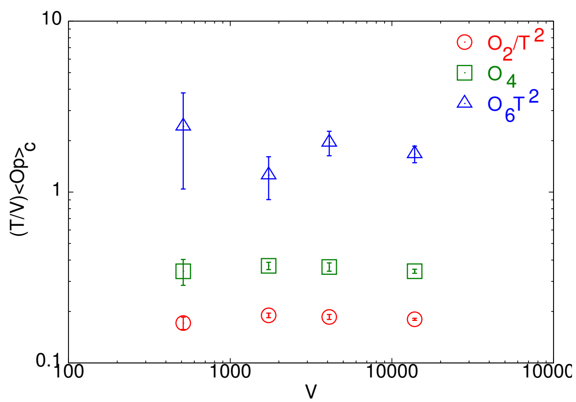
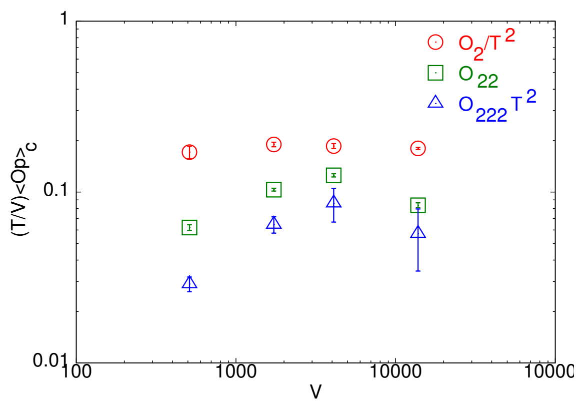
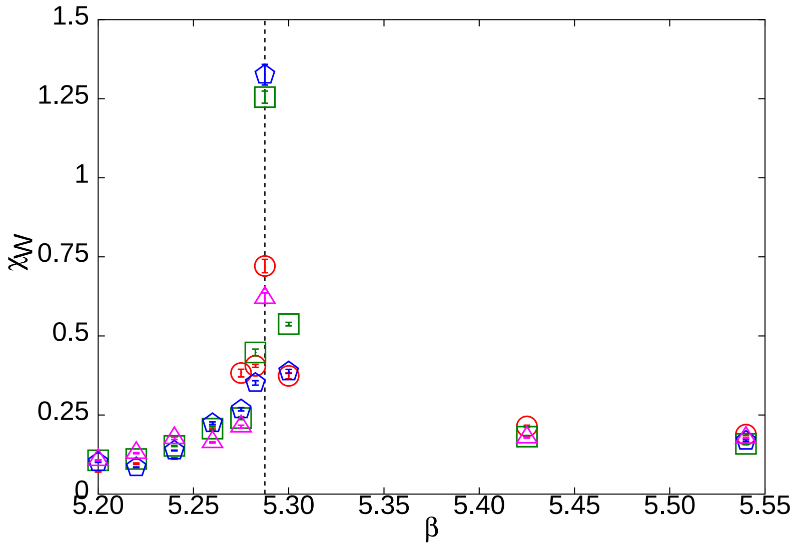
♠ We had earlier suggested to obtain more pairs of diverging terms by taking larger N_f.

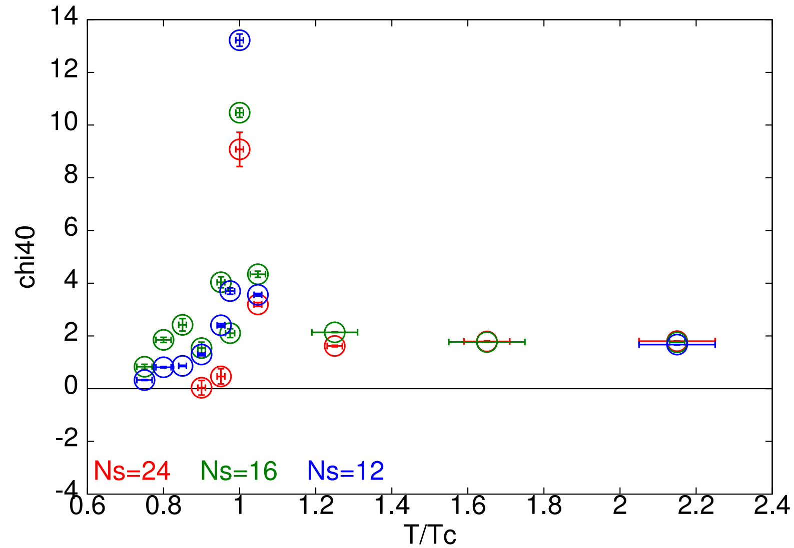
♠ E.g. $T/V \langle \mathcal{O}_{22} \rangle_c$ should be finite as it is a combination of Taylor Coeffs.





♠ Interesting to note that χ_{40} shows the same volume dependence at T_c as χ_L which in turn comes from the $\langle \mathcal{O}_{22} \rangle_c$.





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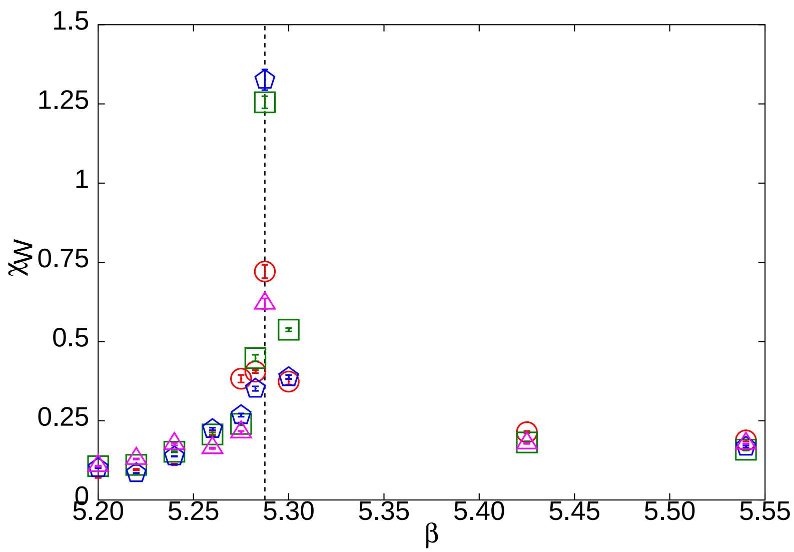

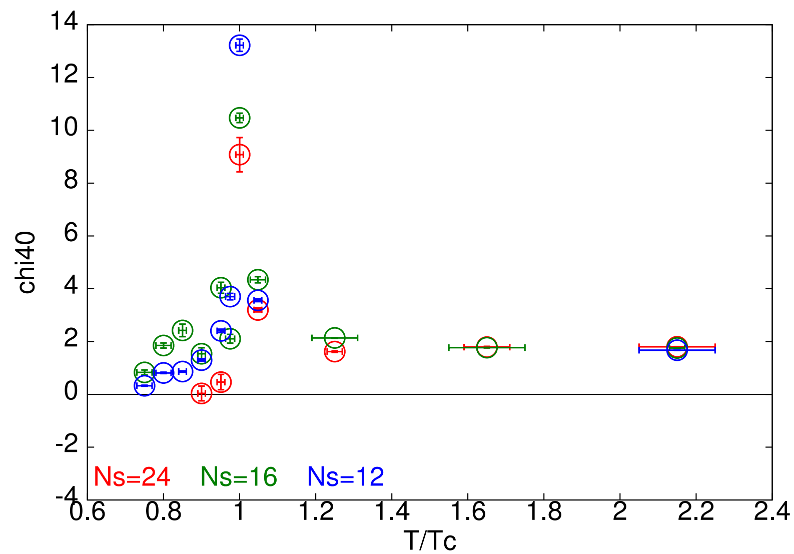

♠ Similar behaviour in higher order terms as well.

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Discussion Figure